Competition in the Presence of Individual Demand Uncertainty

Marc Möller*   Makoto Watanabe†

Abstract

This article offers a tractable model of (oligopolistic) competition in differentiated product markets characterized by individual demand uncertainty. The main result shows that, in equilibrium, firms offer advance purchase discounts and that these discounts are larger than in the monopolistic benchmark. Competition reduces welfare by increasing the fraction of consumers who purchase in advance, i.e. without (full) knowledge of their preferences.

Keywords: Competition, Price Discrimination, Individual Demand Uncertainty, Advance Purchase Discounts.

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*Department of Economics, University of Bern. Email: marc.moeller@vwi.unibe.ch

†Department of Economics, FEWEB, VU University Amsterdam, Tinbergen Institute. Email: makoto.wtnb@gmail.com

1 Introduction

In many markets, firms offer advance purchase discounts (APDs) to early customers. For example, automobile companies often announce special introductory prices that apply to buyers who sign up prior to the launch of a new model. Similarly, conferences and sports events frequently offer reduced participation fees to those participants who register before a certain deadline. Finally, airlines increase their ticket prices as the date of travel approaches or require an early booking to qualify for a low fare category.

A common feature of these markets is the presence of individual demand uncertainty. At the time of purchase, a test drive, the conference program, or the traveler's schedule might be unavailable, leaving consumers with imperfect knowledge about the match between their preferences and the product's characteristics. Consumers choose between an early, uninformed purchase at a low price and a late, informed purchase at a high price.

An emerging literature has shown that in the presence of individual demand uncertainty, an APD may constitute a firm's optimal selling strategy. An APD induces consumers with weak preferences or low degrees of uncertainty to purchase in advance, while deferring the purchase of consumers with strong preferences or high degrees of uncertainty. An APD thus enables a firm to price-discriminate between consumers of different types. While the existing literature focuses on the case of a monopolistic seller, a tractable model of competition is still missing. This article fills this gap by considering a duopoly.

In our model, two differentiated products are sold during two periods, an advance purchase period (1) and a consumption period (2). A continuum of consumers with unitary demands know their preferences in period 2 but face uncertainty in period 1. We model this uncertainty, by assuming that in period 1 each consumer receives only an imperfect (private) signal about the identity of his preferred product. The signals' precision is identical across consumers, i.e. all consumers face the same degree of uncertainty. Consumers are also identical with respect to their average valuation of the two products.

However, consumers differ in their "choosiness". More choosy consumers derive a higher consumption value from their preferred product and a lower consumption value from their non-preferred product. We compare the case in which products are sold by two competing firms with the monopolistic benchmark, in which both products are offered by a single seller. Our main analysis assumes that firms are able to commit to a price schedule in advance and focuses on the case in which market structure has no influence on the total quantity supplied.

We first show that, in equilibrium, firms offer APDs, thereby extending the insights of the existing literature to the case of competition. Our main result shows that, in any (symmetric pure-strategy) equilibrium competing firms must offer larger APDs than a monopolist, inducing a greater fraction of consumers to purchase in advance. This result is driven by the firms' incentive to capture those consumers in advance who might become their rival's customers in the future.[1] It holds under the fairly weak restriction that the distribution of consumer types has an increasing hazard rate.[2]

Price-commitment turns out to be essential for the occurrence of intertemporal price discrimination. We show that without commitment, intertemporal price discrimination ceases to occur. However, while competing firms serve all of their customers in advance, a monopolistic supplier maximizes profits by selling exclusively *after* demand uncertainty has been resolved. Hence our main result about the increase in advance sales becomes amplified in the absence of commitment.

The influence of competition on the intertemporal allocation of sales has an interesting welfare implication. Because advance purchases are subject to the risk of a consumer-product mismatch, an increase in the number of advance sales has a negative effect on

[1]This is similar to the occurrence of customer poaching in markets with switching costs (Chen (1997), Villas-Boas (1999), Fudenberg and Tirole (2000)) with the difference that consumers are captured ex ante rather than ex post.

[2]We prove the existence of a pure strategy equilibrium for two cases: An equilibrium exists, (1) if individual demand uncertainty is sufficiently strong, or, (2) if the distribution of types is uniform. In the general case, existence may require further restrictions on the distribution of types.

total surplus. Generally, this negative effect of competition might be compensated by an increase in the total quantity sold. Extending our analysis to the case where individual demand is elastic we show the perhaps surprising result that competition can lead to a reduction in welfare even when it increases the total quantity sold. To the best of our knowledge, we are the first to point out these negative welfare consequences of competition for markets characterized by individual demand uncertainty.

One may argue that, although detrimental for overall welfare, competition should be beneficial for consumers. We show that, for a uniform distribution of types, competition leads to a price decrease in the advance selling period but may result in a price *increase* in the consumption period. Hence, competition benefits the "unchoosy" consumers who purchase early but may harm the "choosy" consumers who purchase late. We show that the aggregate effect of competition on consumer surplus can be negative.

The plan of the paper is as follows. Section 2 introduces the model. In Section 3 we consider the case of a monopoly which serves as a benchmark for our subsequent analysis. Section 4 contains our main results about competition. Our final Section 5 considers the issue of price-commitment. The more technical proofs are relegated to Appendix A. Appendix B, available online, contains our extension to the case of elastic demand and the rather lengthy proof of equilibrium existence for a uniform distribution of consumer types.

Related literature

The existing literature on intertemporal price discrimination with individual demand uncertainty lacks the analysis of competition: DeGraba (1995), Courty and Li (2000), Courty (2003), Möller and Watanabe (2010), and Nocke, Peitz, and Rosar (2011) all consider the monopolist's problem.[3]

APDs have been derived as optimal selling mechanisms in other settings. Dana (1998)

[3]An exception is Gale (1993) who features a duopoly but assumes that products are homogeneous ex ante. In our model products are differentiated not only ex post but also ex ante.

derives an APD for a perfectly competitive industry characterized by *aggregate* demand uncertainty. His analysis suggests that market power may not be necessary to explain the observation of an APD. Firms use APDs in order to reduce the risk of holding unutilized capacity. Similarly, Gale and Holmes (1993) show that an airline may use APDs to divert consumers from a peak period where demand exceeds capacity to an off-peak period. In our setting, aggregate demand is certain and capacity is neither restricted nor costly. For a monopolist APDs act as a screening device, whereas competing firms offer APDs to capture customers.

The role of an APD as a screening device makes our model part of a broader literature on price discrimination in markets for differentiated products (see Stole (2007) for an overview). The influence of competition on a firm's ability to screen its customers has been an important issue in this literature.[4] Borenstein (1985) and Holmes (1989) were the first to challenge the common view that, with marginal cost pricing being a feature of a competitive market, competition should have a negative influence on price discrimination. They argued that if firms discriminate consumers with respect to their willingness to switch supplier, then competition reduces the low prices charged to high elasticity consumers even further, while relatively high prices can be maintained for those who are reluctant to switch. Our finding that competition may lead to a decrease in advance prices accompanied by an increase in spot prices resonates well with this "brand-loyalty effect". However, instead of being motivated by their loyalty to a particular brand, consumers are willing to pay a high price in order to be able to make an informed purchase.

The consumers' willingness to pay a premium for the ability to choose their preferred product relates our model to a literature determining the optimal selling strategy for a multi-product monopolist (Thanassoulis 2004, Pavlov 2011). This literature emphasizes

[4]While some empirical studies document a positive relationship between competition and price discrimination (Borenstein and Rose 1994, Stavins 1996, Busse and Rysman 2005, Asplund et al. 2008), others find this relation to be negative (Gerardi and Shapiro 2009, Gaggero and Piga 2011, Moon and Watanabe 2013).

the role of "product-lotteries" as a screening device. Consumers with weak preferences choose a lottery promising the delivery of a random product at a low price whereas consumers with strong preferences pay a high price for the right to choose their most preferred product. Consumer screening also explains the emergence of buy-now discounts in markets with search frictions. Armstrong and Zhou (forthcoming) offer the intuition that demand from consumers visiting a seller for the first time is more elastic than demand from returning consumers. This is similar in our model where a small price decrease is sufficient to make consumers switch products *before* but not *after* they have learned their preferences. While Armstrong and Zhou (forthcoming) include the analysis of duopoly, the optimality of product lotteries in the presence of competition is still an open issue.

Our model also allows the interpretation of the consumers' timing of purchase as a choice between a refundable (high quality) option and a non-refundable (low quality) option. This relates our article to the literature on non-linear pricing in which firms compete by offering quality-price menus (Stole (1995), Armstrong and Vickers (2001), and Rochet and Stole (2002)). Because in our setting demand uncertainty is the same for all consumers, unobserved preference heterogeneity is restricted to the horizontal dimension, making our setting most comparable to Stole (1995). Stole shows that competing firms will implement the same quality distortions as a (multi-product) monopolist. Competition has the mere effect of decreasing prices and as incentive compatibility requires all prices to decrease by the same amount, the premium payed for high quality remains unchanged. In our setting, with its two exogenously given "quality" levels, this result is no longer valid. Competition extends the set of consumer types who are offered the low quality (non-refundable) option and incentive compatibility thus requires the price of low quality to decrease by a larger amount than the price of high quality.

Finally, because APDs influence the timing of sales and hence the amount of information that is available at the time of purchase, our model is connected to the literature on information disclosure in market settings. Lewis and Sappington (1994) and Bar Isaac et al. (2010) consider the issue of whether a monopolist should provide buyers with information about their valuation of his product. Our model suggests that market structure may have a crucial influence on the amount of information consumers are supplied with.

2 Model

We consider a market with two differentiated products $i \in \{A, B\}$ which can be purchased in two periods; an advance purchase period (1) and a consumption period (2). As an example, one may think of a Thursday and a Friday flight between identical destinations. We assume that firms can commit to a price schedule $(p_{1,i}, p_{2,i}) \in \Re_+^2$ where $p_{1,i}$ and $p_{2,i}$ denote the prices of product i in period 1 and 2 respectively.[5] The unit cost of production is assumed to be constant and identical across products. For simplicity, we normalize unit costs to zero and abstract from discounting.

There is a continuum of consumers with mass 1. Consumers have unit demands. A consumer of type $\sigma \in [0, 1]$ obtains the value $s + \frac{t}{2}\sigma$ from consuming his preferred product and $s - \frac{t}{2}\sigma$ from consuming his non-preferred product. The parameter $s > 0$ denotes a consumer's average consumption value and is assumed to be identical across consumers.[6] The parameter $t > 0$ measures the general degree of product differentiation. Consumers differ only in their *choosiness*, σ, which constitutes their private information. In the eyes of more choosy consumers, differences in the products' characteristics weigh more heavily. For example, flying on a Thursday rather than on a Friday may imply a considerable degree of inconvenience for business travelers whereas leisure travelers may care less.

The consumers' choosiness σ is distributed in $[0, 1]$ with strictly positive and continuous density f and cumulative distribution function F. We require f to have an increasing

[5]Our focus on price-posting is motivated by its prevalence in many markets. The assumption of commitment is relaxed in Section 5. In the absence of commitment, firms face a time consistency problem, similar to the one in the durable goods literature (Coase (1972)).

[6]This assumption isolates individual demand uncertainty from other features of demand that may lead to intertemporal price discrimination. If consumers differed in their average valuations, a seller would have an incentive to discriminate between high value consumers and low value consumers.

hazard rate, i.e. we assume that $\frac{f}{1-F}$ is non-decreasing.[7] To keep the model symmetric we further assume that, for any degree of choosiness σ, the mass of consumers whose preferred product is A is the same as the mass of consumers whose preferred product is B.

The main feature of our model is the presence of individual demand uncertainty. In particular, we assume that, while in the consumption period preferences are known, in the advance purchase period, each consumer faces uncertainty about the identity of his preferred product. For example, a traveler may very well be able to judge the importance of flying on the correct date, but may not know the correct date in advance. We capture this by assuming that in period 1, each consumer receives a (private) signal $S \in \{A, B\}$ about the identity of his preferred product. We denote the product indicated by signal S as the consumer's *favorite product* in order to distinguish it from his (potentially different) *preferred product*. The signal's precision, i.e. the probability with which the consumer's favorite product turns out to be his preferred product, is given by $\gamma \in (\frac{1}{2}, 1)$. The parameter γ measures the level of individual demand uncertainty and is the same for all consumers. For $\gamma \to \frac{1}{2}$, consumers face complete uncertainty whereas for $\gamma \to 1$ preferences are certain even in advance.[8]

Our analysis abstracts from the possibility of an equilibrium in which (some) consumers fail to be served. Such an equilibrium can be ruled out by requiring the consumers' average consumption value to be sufficiently high. More specifically, we require that $s \geq \frac{\gamma(\gamma-\frac{1}{2})}{\gamma^2+(1-\gamma)^2}\frac{t}{f(0)}$. In addition, our analysis implicitly assumes that those consumers who purchase in advance find it optimal to consume even when they turn out to have purchased their non-preferred product. This can be guaranteed by requiring that $s \geq \frac{t}{2}$.

[7]This holds, for example, when f is non-decreasing or log-concave. Log-concavity is satisfied by most commonly used density functions (Bagnoli and Bergstrom, 2005).

[8]We excluded the case $\gamma = \frac{1}{2}$ from the model's general formulation. For $\gamma = \frac{1}{2}$, product's are homogeneous from the consumers' viewpoint in period 1, making a firm's demand in period 1 a discontinuous function of its price. The analysis of this special case forms part of the proof of Proposition 4.

In summary we therefore make the following parametric restriction:

$$\frac{s}{t} \geq \max\left(\frac{\gamma(\gamma-\frac{1}{2})}{\gamma^2+(1-\gamma)^2}\frac{1}{f(0)}, \frac{1}{2}\right). \tag{A1}$$

We further assume that, when indifferent, consumers purchase in period 2 rather than in period 1. Finally, we assume that each consumer can purchase at most one product. This rules out the possibility that consumers purchase both products in advance or switch product after purchasing the wrong product.[9]

In the following we first consider the monopoly case in which both products are offered by a single supplier. This case will serve as a benchmark for a comparison with the case of competition in which products are offered by two separate firms.

3 Monopolistic benchmark

In this section, we consider the case where both products are offered by the same (monopolistic) supplier. This market structure may be the outcome of a merger by two duopolists, making this case a natural benchmark to consider.

Due to symmetry, a monopolist will choose the same price schedule (p_1, p_2) for both products. If the monopolist commits to a decreasing price schedule then all consumers would prefer to purchase in period 2 rather than in period 1. Hence we can assume without loss of generality, that the monopolist sets $p_1 \leq p_2$. In the proof of Proposition 1, we show that under Assumption (A1), the monopolist maximizes profits by selling to *all* consumers. Here we offer a derivation of the intertemporal allocation of sales which makes the interpretation of the subsequent results more intuitive.

For this purpose, consider a consumer with choosiness $\sigma \in [0, 1]$. If the consumer buys his favorite product $S \in \{A, B\}$ in period 1 then with probability γ this product will turn

[9] This assumption is made to simplify the exposition. It becomes redundant when equilibrium prices are sufficiently high to make multiple purchases sub-optimal. Introducing a parameter $c > 0$ for the unit cost of production, we have confirmed that multiple purchases are sub-optimal in equilibrium when c is above a certain threshold. Details are available on request.

out to be his preferred product in period 2 whereas with probability $1-\gamma$ he will prefer the other product. The consumer's expected utility from purchasing his favorite product in period 1 is thus given by

$$U(\sigma|1,S) = s + \gamma\frac{t}{2}\sigma - (1-\gamma)\frac{t}{2}\sigma - p_1. \tag{1}$$

Instead, the consumer may wait until period 2 in order to guarantee the purchase of his preferred product, giving the utility

$$U(\sigma|2) = s + \frac{t}{2}\sigma - p_2. \tag{2}$$

Waiting pays off if the consumer's choosiness is relatively large in comparison to the discount $\Delta p = p_2 - p_1$:

$$U(\sigma|2) \geq U(\sigma|1,S) \Leftrightarrow \sigma \geq \frac{\Delta p}{t(1-\gamma)} \equiv \sigma_W. \tag{3}$$

Given a discount of size $\Delta p \in (0, t(1-\gamma))$, consumers with low choosiness $\sigma \in [0, \sigma_W)$ purchase in advance at price p_1 whereas consumers with high choosiness $\sigma \in [\sigma_W, 1]$ buy in period 2 at price $p_2 = p_1 + \Delta p$ (see Figure 1).

By choosing the discount, Δp, the monopolist determines the intertemporal allocation of sales, σ_W. He will choose σ_W to maximize total surplus minus the sum of consumer rents. For an early buyer, surplus is given by $s + \gamma\frac{t}{2}\sigma - (1-\gamma)\frac{t}{2}\sigma = s + t(\gamma - \frac{1}{2})\sigma$. He obtains information rents $t(\gamma - \frac{1}{2})\sigma$ from pooling with consumers of the lowest type. The monopolist can extract the rent s from each type of consumer in $[0, \sigma_W)$ by setting $p_1 = s$. For a late buyer surplus is $s + \frac{t}{2}\sigma$. In addition to the rent $t(\gamma - \frac{1}{2})\sigma_W$ obtained by type σ_W, late buyers receive the informational rent $\frac{t}{2}(\sigma - \sigma_W)$ from pooling with the cutoff. Hence, the monopolist can extract the rent $s + \frac{t}{2}\sigma - t(\gamma - \frac{1}{2})\sigma_W - \frac{t}{2}(\sigma - \sigma_W) = s + t(1-\gamma)\sigma_W$ from each type of consumer in $[\sigma_W, 1]$ by setting $p_2 = s + t(1-\gamma)\sigma_W$.

The optimal cutoff σ_W trades off the surplus gain from the elimination of potential mismatches with the loss in consumer rents. A low cutoff is good for total surplus due to the elimination of the potential product mismatch for early buyers. However, a low cutoff also leads to high consumer rents because it enables late buyers to pool with consumers characterized by relatively low degrees of choosiness. Formally, σ_W maximizes the monopolist's profit

$$\Pi^M = F(\sigma_W)s + [1 - F(\sigma_W)][s + t(1-\gamma)\sigma_W] = s + t(1-\gamma)\sigma_W[1 - F(\sigma_W)]. \tag{4}$$

From (4) it is immediate that selling to all consumers in the same period ($\sigma_W = 0$ or $\sigma_W = 1$) cannot be optimal. The increasing hazard rate of the distribution f guarantees the existence of a unique optimum $\sigma_W^M \in (0,1)$ defined by the first order condition

$$\frac{1 - F(\sigma_W^M)}{f(\sigma_W^M)} - \sigma_W^M = 0. \tag{5}$$

Proposition 1. *The profit maximizing monopolistic price schedule is given by $p_1^M = s$ and $p_2^M = s + t(1-\gamma)\sigma_W^M$ where $\sigma_W^M \in (0,1)$ is the unique solution to (5). At these prices, all consumers participate in the market. The discount $\Delta p^M = t(1-\gamma)\sigma_W^M > 0$ induces a fraction $F(\sigma_W^M) \in (0,1)$ of consumers to buy in advance.*

Proof: See Appendix A.

Proposition 1 will serve as our benchmark when we consider the case of competition in the following section.

4 Competition

To analyze the effect of competition on the intertemporal allocation of sales we assume for the remainder that products A and B are offered by two competing firms. Each firm $i \in \{A, B\}$ chooses a price schedule $(p_{1,i}, p_{2,i})$. Without loss of generality, we can restrict the firms' strategy space by requiring prices to be non-decreasing. This is because if $p_{1,i} > p_{2,i}$, then firm i's first period demand is zero and the firm can obtain the same profit by lowering $p_{1,i}$ until it becomes equal to $p_{2,i}$.

Given the symmetry of the setup, we focus on symmetric pure-strategy equilibria in which firms offer the same deterministic price schedule (p_1^*, p_2^*). In the following we will denote such a (p_1^*, p_2^*) simply as an equilibrium. Taking the existence of a symmetric pure strategy equilibrium as given, we first derive properties that have to be satisfied by any such equilibrium. Subsequently, we establish equilibrium existence for two cases: (1) a sufficiently high degree of individual demand uncertainty; and (2) a uniform distribution of types.

Time-invariant pricing

Consider the possibility of an equilibrium in which firms choose a price that is constant across periods. We have the following:

Proposition 2. *Time invariant pricing $p_1 = p_2$ cannot be an equilibrium. Hence, in any equilibrium (p_1^*, p_2^*), competing firms must offer an advance purchase discount, $\Delta p^* = p_2^* - p_1^* > 0$.*

Proof: Suppose that firms set prices $p_1 = p_2 = p$ and consider a deviation by firm A to a lower first period price $p_{1,A} < p$. In response to this discount, consumers with sufficiently low degrees of choosiness will purchase product A in period 1 at price $p_{1,A}$. A consumer whose favorite is $S = A$ would have become firm A's customer in period 2 at price p with probability γ. Similarly, a consumer whose favorite is $S = B$ would have become firm A's customer in period 2 at price p with probability $1 - \gamma$. This implies that as long as the discount is not too large, firm A obtains an additional profit of size $p_{1,A} - \gamma p > 0$ from any advance customer whose favorite product is A and $p_{1,A} - (1 - \gamma)p > 0$ from any advance customer whose favorite product is B. Hence there exists a profitable deviation, i.e. time-invariant pricing cannot be an equilibrium. QED.

The intuition for Proposition 2 is straightforward. Firms offer APDs in order to secure a purchase by consumers, who could become the rival firm's customers in the future. Although this shows that prices must be increasing, Proposition 2 does not necessarily imply that firms practice price discrimination. Instead, firms may offer APDs that are sufficiently large to induce consumers to buy *exclusively* in advance (at the same price). In the following we therefore consider the possibility of an equilibrium in which firms offer an APD and sales are positive in both periods.

Intertemporal price discrimination

In this section, we consider the possibility that firms practice price-discrimination by inducing different consumers to pay different prices. To be precise we make the following:

Definition 1. *An equilibrium* (p_1^*, p_2^*) *is denoted as a price-discrimination equilibrium if* $p_1^* < p_2^*$ *and firms sell a positive quantity in both periods.*

Because a consumer knows his preferences only imperfectly, his expected utility (1) from purchasing his favorite product early is increasing less strongly in his choosiness σ than his utility (2) from purchasing his preferred product late. As a consequence, the consumers' behavior in a price discrimination equilibrium can be characterized with the help of two thresholds σ_0^* and σ_W^* satisfying $0 \leq \sigma_0^* < \sigma_W^* < 1$: Consumers with $\sigma \in [\sigma_W^*, 1]$ buy in period 2; consumers with $\sigma \in [\sigma_0^*, \sigma_W^*)$ buy in period 1; and consumers with $\sigma \in [0, \sigma_0^*)$ do not buy in any period. In the proof of Proposition 3 we first show that in any equilibrium, the market must be covered. Hence in a price-discrimination equilibrium consumers behave as depicted in Figure 2. The difference to the monopoly case is that (off equilibrium) the cutoff σ_W may depend on the identity of the consumer's favorite product. This is why in Figure 2 we distinguish between consumers whose favorite is $S = A$ and consumers whose favorite is $S = B$. Another difference is that, as first period prices may differ across firms, the least choosy consumers will prefer the cheaper product over their favorite product. Hence there exists an additional cutoff $\bar{\sigma} \geq 0$ such that all advance customers with $\sigma > \bar{\sigma}$ will purchase their favorite product whereas all advance customers with $\sigma \leq \bar{\sigma}$ will purchase the cheaper product (Figure 2 depicts the case in which $p_{1,A} > p_{1,B}$). In equilibrium, $p_{1,A}^* = p_{1,B}^*$ implies that $\bar{\sigma}^* = 0$.

In order to determine the thresholds $\sigma_W(A)$, $\sigma_W(B)$, and $\bar{\sigma}$, suppose that firm B chooses the equilibrium price schedule (p_1^*, p_2^*) and consider a small deviation by firm A to a price schedule $(p_{1,A}, p_{2,A}) \neq (p_1^*, p_2^*)$. For a consumer whose favorite is $S = A$, purchasing A in advance gives (expected) utility

$$U(\sigma, A|1, A) = s + \gamma \frac{t}{2}\sigma - (1-\gamma)\frac{t}{2}\sigma - p_{1,A}. \tag{6}$$

Any consumer who postpones his purchase must condition his product choice in period 2 on the identity of his preferred product. Otherwise he could have purchased the product he buys in period 2 already in period 1, at a lower price. Waiting until period 2 therefore gives the (expected) utility

$$U(\sigma, A|2) = s + \frac{t}{2}\sigma - \gamma p_{2,A} - (1-\gamma)p_2^*. \tag{7}$$

Waiting is preferable if and only if the (expected) gain in consumption value, $t(1-\gamma)\sigma$, exceeds the (expected) price premium $(1-\gamma)p_2^* + \gamma p_{2,A} - p_{1,A}$, or equivalently

$$\sigma \geq \sigma_W(A) \equiv \frac{(1-\gamma)p_2^* + \gamma p_{2,A} - p_{1,A}}{t(1-\gamma)}. \tag{8}$$

For a consumer whose favorite product is $S = B$ the gain in consumption value is identical, but the price premium is given by $\gamma p_2^* + (1-\gamma)p_{2,A} - p_1^*$. Waiting is preferable if

$$\sigma \geq \sigma_W(B) \equiv \frac{\gamma p_2^* + (1-\gamma)p_{2,A} - p_1^*}{t(1-\gamma)}. \tag{9}$$

Finally, consider an advance customer whose favorite product happens to be more expensive than his non-favorite product. Purchasing his favorite product is preferable if the (expected) gain in consumption value $s+\gamma\frac{t}{2}\sigma-(1-\gamma)\frac{t}{2}\sigma-[s+(1-\gamma)\frac{t}{2}\sigma-\gamma\frac{t}{2}\sigma] = t(2\gamma-1)\sigma$ exceeds the price difference $|p_{1,A} - p_{1,B}|$ or equivalently $\sigma > \bar{\sigma}$ with

$$\bar{\sigma} = \frac{|p_{1,A} - p_1^*|}{t(2\gamma - 1)}. \tag{10}$$

Firm A's profits $\Pi_A = \Pi_{1,A} + \Pi_{2,A}$ consist of first period profits

$$\Pi_{1,A} = \begin{cases} \frac{p_{1,A}}{2}[F(\sigma_W(A)) - F(\bar{\sigma})] & \text{if} \quad p_{1,A} > p_1^* \\ \frac{p_{1,A}}{2}[F(\sigma_W(A)) + F(\bar{\sigma})] & \text{if} \quad p_{1,A} \leq p_1^* \end{cases} \tag{11}$$

and second period profits

$$\Pi_{2,A} = \frac{p_{2,A}}{2}\left\{\gamma[1 - F(\sigma_W(A)] + (1-\gamma)[1 - F(\sigma_W(B))]\right\}. \tag{12}$$

First period profits depend on whether $p_{1,A}$ is smaller or larger than p_1^*. For $p_{1,A} > p_1^*$ firm A's first period demand consists of all consumers with favorite $S = A$ who are not choosy enough to wait but choosy enough to pay a higher price for product A. This case is depicted in Figure 2. For $p_{1,A} < p_1^*$ firm A's first period demand consists of all consumers with favorite $S = A$ who are not choosy enough to wait and consumers with favorite B who are sufficiently unchoosy to be attracted by firm A's lower first period price.

Firm A's second period profits also originate from two distinct groups of consumers. The first group are consumers who were too choosy to buy their favorite A in period 1 and prefer A in period 2. The second group are consumers who were too choosy to buy their favorite B and turned out to actually prefer A.

Marginal deviations from a price discrimination equilibrium (p_1^*, p_2^*) must not be profitable. Differentiating Π_A with respect to $p_{1,A}$ and $p_{2,A}$ and substituting $(p_{1,A}, p_{2,A}) = (p_1^*, p_2^*)$ therefore gives the following two necessary conditions for a price discrimination equilibrium:

$$0 = F(\sigma_W^*) + (\gamma p_2^* - p_1^*)\frac{f(\sigma_W^*)}{t(1-\gamma)} - p_1^*\frac{f(0)}{t(2\gamma-1)} \tag{13}$$

$$0 = 1 - F(\sigma_W^*) + \{\gamma p_1^* - [\gamma^2 + (1-\gamma)^2]p_2^*\}\frac{f(\sigma_W^*)}{t(1-\gamma)}, \tag{14}$$

with

$$\sigma_W^* = \frac{p_2^* - p_1^*}{t(1-\gamma)}. \tag{15}$$

Proposition 3. *In any equilibrium the market must be covered. If (p_1^*, p_2^*) is a price-discrimination equilibrium, then prices must satisfy conditions (13) and (14), and $\Delta p^* > \Delta p^M$, i.e. competing firms offer a larger APD than a monopolist.*

Proof: See Appendix A.

Propositions 1 - 3 have as an immediate consequence the following:

Corollary 1. *In any (symmetric pure-strategy) equilibrium, competing firms induce a larger fraction of consumers to buy in advance than a monopolist. Hence, competition has a negative effect on welfare.*

To understand the intuition for this result, recall that a monopolist benefits from lowering his APD due to the elimination of a potential product-mismatch for those consumers who switch from buying in advance to waiting. In the presence of competition, firms fail to internalize fully the corresponding increase in consumer surplus. This is because only a fraction γ of the consumers who are induced to postpone their purchase under the ADP of firm A, will eventually become customers of this firm. The remaining fraction $1-\gamma$ will purchase from firm B and the increment in these consumers' surplus will be extracted by firm A's rival. Under competition firms induce less consumers to postpone their purchase than under monopoly because they fail to internalize the positive externality of an improved consumer–product matching on the rival firm.

The welfare effects of an increase in advance sales are straightforward. Since consumers have unitary demands and the market must be covered, competition has no effect on the total quantity supplied. As individual preferences are uncertain, advance purchases are subject to the risk of consumer-product mismatches. A consumer who purchases in advance and turns out to prefer the other product experiences a surplus loss. Hence, an increase in the fraction of advance sales has a negative effect on welfare. This welfare loss is similar to the one resulting from customer poaching in markets with switching costs (Chen (1997), Villas-Boas (1999), Fudenberg and Tirole (2000)). In both cases competition increases the mismatch between consumer preferences and product characteristics.

In general, we do not expect this welfare reduction to persist in the presence of quantity effects. However, in Appendix B, available online, we provide an example where competition reduces welfare even when it increases the total quantity supplied. There we abandon our assumption of unitary demands and show that when consumers' demand schedules are linear, the allocative inefficiency resulting from an increase in advance sales can outweigh the welfare gain from a more efficient production.

Equilibrium existence

So far, our analysis has ignored the question of whether a symmetric pure-strategy equilibrium actually exists. In general, existence may require further restrictions on the distribution, f, of consumer types. Below we determine the (unique) price discrimination equilibrium for the case where f is uniform. However, before moving to the uniform case, we let f remain general and consider the limit as $\gamma \to \frac{1}{2}$. Our next result shows that if individual demands are sufficiently uncertain then a symmetric pure-strategy equilibrium exists under no additional restrictions on the distribution of consumer types:

Proposition 4. *Suppose that individual demand uncertainty is sufficiently strong, i.e. γ is close to $\frac{1}{2}$. Then there exists a (unique) price-discrimination equilibrium (p_1^*, p_2^*). In the limit as $\gamma \to \frac{1}{2}$ it holds that $p_1^* \to 0$ and $p_2^* \to \frac{t}{2}\sigma_W^*$ where $\sigma_W^* \in (0,1)$ is the unique solution to*

$$\frac{1 - F(\sigma_W^*)}{f(\sigma_W^*)} - \frac{1}{2}\sigma_W^* = 0. \tag{16}$$

Proof: See Appendix A.

Intuitively, for $\gamma \to \frac{1}{2}$, the firms' products become homogeneous from the buyers' viewpoint in period 1. As a consequence, equilibrium first period prices p_1^* converge towards marginal costs which we normalized to zero. A deviation to a $p_1 > p_1^*$ has the sole effect of reducing the deviating firm's first period demand to zero. It fails to increase second period demand, because by homogeneity only the *lowest* first period price is relevant for the consumer's choice between buying early and buying late. Hence we only have to check for profitable deviations to price schedules of the form $(p_1^*, p_2) \neq (p_1^*, p_2^*)$. This makes the proof of existence tractable.

Uniform distribution of consumer types

We close this section by considering the special case in which the distribution of consumer types, f, is uniform. In this case, the equilibrium conditions (13) and (14) become linear equations which allows us to derive an explicit solution:

$$p_1^* = \frac{(1+\gamma)(2\gamma-1)}{-4\gamma^2+7\gamma-1} t \in (0,t) \tag{17}$$

$$p_2^* = \frac{3\gamma-1}{-4\gamma^2+7\gamma-1} t \in (p_1^*, t). \tag{18}$$

The price schedule (p_1^*, p_2^*) constitutes the unique candidate for a price discrimination equilibrium. Its explicit form allows us to confirm the non-profitability of all potential deviations to price schedules $(p_1, p_2) \neq (p_1^*, p_2^*)$. This analysis is rather lengthy and has therefore been moved to Appendix B which is available online.

In order to guarantee that at (p_1^*, p_2^*), consumers obtain positive utility we need to tighten the first part of Assumption (A1) by requiring that $p_1^* \leq s$.[10] However, the second part of Assumption (A1) can be relaxed because in the uniform case, we can use (17) and (18) to determine an explicit solution $\sigma_W^* = \frac{2\gamma}{-4\gamma^2+7\gamma-1} \in (0,1)$ and negative consumption values are ruled out already when $\frac{s}{t} \geq \frac{1}{2}\sigma_W^*$. For the uniform case we therefore substitute Assumption (A1) by

$$\frac{s}{t} \geq \max\left(\frac{(1+\gamma)(2\gamma-1)}{-4\gamma^2+7\gamma-1}, \frac{\gamma}{-4\gamma^2+7\gamma-1}\right). \tag{A1'}$$

The explicit form of (17) and (18) allows us to derive some additional results which we could not obtain for a general distribution:

[10] Assumption (A1) ruled out the possibility of an uncovered market equilibrium but did not guarantee that a covered market equilibrium exists. For Proposition 4, $p_1^* \leq s$ was satisfied automatically as advance prices converge to zero when $\gamma \to \frac{1}{2}$.

Proposition 5. *Suppose that f is uniform and Assumption (A1') holds. The price schedule (p_1^*, p_2^*) given by (17) and (18) constitutes the unique price discrimination equilibrium. An increase in the level of individual demand uncertainty leads to a decrease in the fraction of consumers served in advance. Competition decreases advance prices for all parameter values: $p_1^* < p_1^M$. However, there exist parameter values for which competition increases spot prices and decreases aggregate consumer surplus: $p_2^* > p_2^M \Leftrightarrow \frac{s}{t} < T_P(\gamma)$ and $CS^* < CS^M \Leftrightarrow \frac{s}{t} < T_{CS}(\gamma)$.*

Proof: See Appendix A.

The explicit expressions for the thresholds $T_P(\gamma)$ and $T_{CS}(\gamma)$ are derived in the proof of Proposition 5. The thresholds are depicted in Figure 3. As can be seen from the figure, if individual demand uncertainty is not too strong, then there exist values of $\frac{s}{t}$ satisfying Assumption (A1') for which competition leads to an increase in spot prices and to a decrease in aggregate consumer surplus.

To understand why competition may lead to an increase in spot prices, note that, relative to the monopolistic benchmark, spot prices apply to a smaller and hence more select group of consumers with high valuations for their preferred product. These consumers are willing to pay a larger premium $\Delta p^* > \Delta p^M$ for the ability to purchase their preferred product. When the level of preference uncertainty is sufficiently high, the increment in the premium can be large enough to overcome the reduction in the price level $p_1^* < p_1^M = s$, leading to $p_2^* > p_2^M$.[11] This happens when the difference in first period prices is small, i.e. when products are sufficiently differentiated ex ante. Note, however, that, although spot prices can be higher, the *average* price paid must be lower under competition because a monopolist could always implement the prices that competing firms choose in equilibrium.

The consequences for consumer surplus are straightforward. When spot prices are

[11]It seems surprising that competition may lead to an *increase* in (spot) prices. However, there exists empirical evidence which is in line with this finding. Borenstein (1989) shows that more competitive airline routes are characterized by lower 20th percentile fares but *higher* 80th percentile fares. Proposition 5 provides a potential explanation for this finding.

decreased, competition has a positive effect on the surplus of *all* consumers. Otherwise, only the "unchoosy" consumers benefit from lower advance prices whereas the "choosy" consumers suffer from higher spot prices. When products are sufficiently differentiated, advance prices become comparable under both market structures. The surplus loss of the choosy consumers then exceeds the surplus gain of the unchoosy consumers.

Finally, in order to understand the comparative statics contained in Proposition 5 first note that a higher level of uncertainty (smaller γ) makes consumers less willing to buy in advance. As a response, firms will offer a larger APD. However, in the uniform case, the discount chosen in equilibrium is not sufficient to offset the consumers' reduced willingness to buy in advance. As a consequence, the number of units sold in advance goes down. This stands in sharp contrast to the monopoly case in which the number of units sold in advance is independent of γ.

5 Price commitment

Our model follows the literature on (monopolistic) markets with individual demand uncertainty in assuming that firms are able to commit to future prices in advance. In many settings this assumption is indeed justified. For example, during the launch of a new product, firms often announce introductory and standard prices together with a "commitment" to increase their price from one level to the other at a pre-specified point in time. Similarly, the organizers of conferences or sport events often commit to prices by publishing a schedule of registration fees. However, in the absence of commitment, a firm has an incentive to adjust its prices in response to past period sales. For the case of a monopolist, this incentive has been shown to have an adverse effect on the use of APDs as a means of intertemporal price discrimination (Möller and Watanabe, 2010).

In this section, we relax our assumption about price-commitment by assuming that in period 1 firms cannot commit to period 2 prices. Second period prices are chosen *after* first period sales have taken place. We will show that, without price-commitment, the effect of competition on the intertemporal allocation of sales becomes amplified. While under commitment firms sell in both periods independently of market structure, in the absence of commitment price discrimination ceases to exist. Without commitment a monopolist sells exclusively *after* demand uncertainty has been resolved, whereas competing firms sell to all consumers in advance.

Because firms do not observe the consumers' types, the determination of second period prices requires the specification of firms' beliefs about the remaining consumers' types. We therefore resort to Perfect Bayesian equilibrium as the solution concept. We start our analysis with the following:

Lemma 1. *If firms cannot commit to future prices in advance, then, independently of market structure, price discrimination will not occur.*

Proof: Assume, to the contrary, that given prices $p_1 < p_2$, consumers with low choosiness purchase in period 1, resulting in a period 2 market populated by consumers with high choosiness $\sigma \in [\sigma_W, 1]$. Bayesian updating implies that a firm's belief about the remaining consumers' types must be given by the distribution $\frac{f(\sigma)}{1-F(\sigma_W)}$ with support $[\sigma_W, 1]$. We now argue, that it must hold that $p_2 \geq s+\frac{t}{2}\sigma_W$. Under both market structures, if $p_2 < s+\frac{t}{2}\sigma_W$, a firm could increase its second period price to $p_2+\epsilon$ without loosing any of its customers. Due to the absence of consumers with low degrees of choosiness, even competing firms possess some monopoly power in period 2. It follows that consumers with type σ_W must receive a zero payoff and, because lower types of consumers participate in the market, would have been better off by purchasing already in period 1, a contradiction. QED.

If price discrimination is not an option, it remains to consider whether consumers are induced to purchase *before* or *after* demand uncertainty has been resolved. In the former case, a complication arises from the fact that, in equilibrium, the second period is never reached. Because a Perfect Bayesian equilibrium puts no restrictions on firms' beliefs *off* the equilibrium path, this adds a degree of freedom to the determination of the second period price a deviating consumer should expect. In order to obtain a tighter description of equilibrium behavior we therefore resort to an equilibrium refinement in the spirit of trembling hand perfection. More specifically, we assume that with a small probability each consumer trembles by deviating from his equilibrium strategy. As a result, some consumers will always remain in the market and second period beliefs and hence prices are uniquely determined. Letting the probability of trembles go to zero allows us to determine an equilibrium which is robust to the possibility of such consumer mistakes. In the Appendix we prove the following:

Proposition 6. *Suppose that* $1 + F$ *is log-concave,* $f(0) < 2$, $\frac{s}{t} \geq \max(\frac{1}{f(0)}, (2\gamma - 1)(\frac{1}{2} + \frac{1}{f(0)}))$ *and firms cannot commit to prices in advance. Under competition there exists an equilibrium in which all consumers buy in advance. In this equilibrium, firms set* $p_1^* = \frac{t(2\gamma-1)}{f(0)}$ *in period 1 and would charge* $p_2^* = \frac{t}{f(0)}$ *to any consumer who postponed his purchase. There cannot exist an equilibrium in which competing firms sell exclusively in period* 2*. In contrast, a monopolist will set* $p_1^M = p_2^M = s$ *thereby inducing all consumers to purchase on the spot.*

Proof: See Appendix A.

The intuition for this result is as follows. Under monopoly, consumers must expect prices to remain constant over time when the firm is unable to update its beliefs about the consumers' type distribution from its observation of first period sales. This is because for a monopolist, price is determined by the reservation utility of the lowest participating type, which for a covered market is identical across periods. In contrast, under competition, the equilibrium price depends on the degree of product differentiation and, as products appear more differentiated after consumers have learned their preferences, consumers can expect period 2 prices to be higher than period 1 prices. This permits the existence of an equilibrium in which all consumers purchase in advance expecting a price increase in the future. The condition $f(0) < 2$ guarantees that the future price increase is large enough to make an advance purchase optimal for all consumers. When all consumers purchase in advance, our model collapses to a one-shot Hotelling-style competition. The log-concavity of $1 + F$ is necessary for the existence of a (covered market) equilibrium in this one-shot competition (Neven 1986). That the market is covered, independently of market structure, is guaranteed by $\frac{s}{t} \geq \max(\frac{1}{f(0)}, (2\gamma - 1)(\frac{1}{2} + \frac{1}{f(0)}))$. The intuition why selling exclusively on the spot cannot be an equilibrium under competition is the same as in the case with price commitment.

Proposition 6 extends our main result to the case in which firms are unable to commit to prices in advance. In particular, it generalizes Corollary 1 by showing that competition increases the fraction of advance purchases even in the absence of price-commitment.

6 Conclusion

In this article, we have provided a tractable model of competition in differentiated product markets characterized by individual demand uncertainty. Our main result shows that, in equilibrium, firms offer advance purchase discounts and that these discounts are larger than the ones chosen by a monopolistic supplier. Discounts induce the least choosy consumers to make a purchase without (full) knowledge of their preferences. Hence they result in a potential mismatch between consumer preferences and product characteristics. As competition leads to larger discounts and thus to a greater number of advance sales, our model reveals a potential drawback of competition. Competition may result in a welfare reduction when the increased mismatch due to advance selling fails to be overcome by the positive effect of price reductions on the total quantity supplied.

One limitation of our model is that we restrict the firms' selling strategies to consist of simple price-posting. A more general selling mechanism would specify a payment together with the probabilities with which the consumer obtains product A, product

B or no product, respectively. Payment and delivery probabilities would be contingent on the consumer's (announced) type (σ, S) and the identity of his preferred product. The determination of the optimal mechanism for a setting with multiple products and individual preference uncertainty is still an open question. It is beyond the scope of this article and is left for future research.

Appendix A - Proofs

Proof of Proposition 1: If $p_1 < s$ or if $p_2 < p_1$ the monopolist can increase prices without affecting demand. We can therefore restrict attention to price schedules (p_1, p_2) for which $p_1 \geq s$ and $p_2 \geq p_1$. For a consumer to derive positive utility from buying his favorite product in period 1 it has to hold that

$$U(\sigma|1, S) = s + t(\gamma - \frac{1}{2})\sigma - p_1 \geq 0 \Leftrightarrow \sigma \geq \frac{p_1 - s}{t(\gamma - \frac{1}{2})} \equiv \sigma_0. \tag{19}$$

For $\sigma \geq \sigma_0$, waiting is preferable if and only if

$$U(\sigma|2) = s + \frac{t}{2}\sigma - p_2 \geq U(\sigma|1, S) \Leftrightarrow \sigma \geq \frac{\Delta p}{t(1-\gamma)} \equiv \sigma_W. \tag{20}$$

The monopolist's problem can be stated as choosing $\sigma_0 \in [0, 1]$ (by setting p_1) and $\sigma_W \in [\sigma_0, 1]$ (by setting $\Delta p = p_2 - p_1$) in order to maximize his profit

$$\begin{aligned} \Pi^M &= p_1[1 - F(\sigma_0)] + \Delta p[1 - F(\sigma_W)] && (21) \\ &= [s + t(\gamma - \frac{1}{2})\sigma_0][1 - F(\sigma_0)] + t(1-\gamma)\sigma_W[1 - F(\sigma_W)]. && (22) \end{aligned}$$

Consider

$$\frac{\partial \Pi^M}{\partial \sigma_W} = t(1-\gamma) f(\sigma_W) \left\{ \frac{1 - F(\sigma_W)}{f(\sigma_W)} - \sigma_W \right\}. \tag{23}$$

As the hazard rate $\frac{f}{1-F}$ is increasing, the term in parenthesis is decreasing. It is positive for $\sigma_W = 0$ and negative for $\sigma_W = 1$. Hence, for a given σ_0, profit is maximized by setting $\sigma_W = \max(\sigma_0, \sigma_W^M)$ where $\sigma_W^M \in (0, 1)$ denotes the unique solution of equation (5).

For $\sigma_0 \in [0, \sigma_W^M)$ the monopolist therefore maximizes profit by selling in both periods by setting $\sigma_W = \sigma_W^M > \sigma_0$ and we have

$$\frac{d\Pi^M}{d\sigma_0} = t(\gamma - \frac{1}{2})f(\sigma_0)\left\{\frac{1-F(\sigma_0)}{f(\sigma_0)} - \sigma_0 - \frac{s}{t(\gamma - 1/2)}\right\}. \tag{24}$$

For $\sigma_0 \in [\sigma_W^M, 1]$, the monopolist maximizes profit by selling exclusively in period 2 and substitution of $\sigma_W = \sigma_0$ gives the profit $\Pi^M = (s + \frac{t}{2}\sigma_0)[1 - F(\sigma_0)]$ with the derivative

$$\frac{d\Pi^M}{d\sigma_0} = \frac{t}{2}f(\sigma_0)\left\{\frac{1-F(\sigma_0)}{f(\sigma_0)} - \sigma_0 - \frac{2s}{t}\right\}. \tag{25}$$

In both cases, the increasing hazard rate implies that the term in parenthesis is decreasing in σ_0. Moreover, $\frac{d\Pi^M}{d\sigma_0}$ is non-positive for $\sigma_0 = 0$ if and only if

$$s \geq \frac{t(\gamma - \frac{1}{2})}{f(0)}. \tag{26}$$

This holds by Assumption (A1), because $\gamma > \gamma^2 + (1-\gamma)^2$ for all $\gamma \in (\frac{1}{2}, 1)$. At $\sigma_0 = \sigma_W^M$, $\frac{d\Pi^M}{d\sigma_0}$ is negative by the definition of σ_W^M. We have therefore shown that profit is maximized by setting $\sigma_0^M = 0$ and $\sigma_W = \sigma_W^M$, or equivalently $p_1^M = s$ and $p_2^M = s + t(1-\gamma)\sigma_W^M$. QED.

Proof of Proposition 3: We first show that, in any symmetric pure strategy equilibrium, the market must be covered. Suppose, to the contrary, that there exists an equilibrium (p_1^*, p_2^*) in which consumers with $\sigma < \sigma_0^* \in (0,1)$ fail to participate in the market. We need to consider two possibilities: (1) a price-discrimination equilibrium and (2) an advance-selling equilibrium.

Consider a price-discrimination equilibrium first. If firm A chooses $(p_{1,A}, p_{2,A})$ then the consumer who is indifferent between buying A in advance and not buying at all is given by $\sigma_0 = \frac{p_{1,A} - s}{t(\gamma - \frac{1}{2})}$ and firm A obtains the profit

$$\Pi_A = \frac{p_{1,A}}{2}[F(\sigma_W(A)) - F(\sigma_0)] + \frac{p_{2,A}}{2}\{\gamma[1 - F(\sigma_W(A))] + (1-\gamma)[1 - F(\sigma_W(B))]\} \tag{27}$$

with the thresholds $\sigma_W(A)$ and $\sigma_W(B)$ as defined in (8) and (9). At $(p_{1,A}, p_{2,A}) = (p_1^*, p_2^*)$

the derivatives are

$$\frac{\partial \Pi_A}{\partial p_{1,A}} = \frac{F(\sigma_W^*) - F(\sigma_0^*)}{2} - \frac{p_1^*}{2}\left[\frac{f(\sigma_W^*)}{t(1-\gamma)} + \frac{2f(\sigma_0^*)}{t(2\gamma-1)}\right] + \frac{p_2^*}{2}\frac{\gamma f(\sigma_W^*)}{t(1-\gamma)} \tag{28}$$

$$\frac{\partial \Pi_A}{\partial p_{2,A}} = \frac{1 - F(\sigma_W^*)}{2} + \frac{f(\sigma_W^*)}{2t(1-\gamma)}\left\{\gamma p_1^* - p_2^*\left[\gamma^2 + (1-\gamma)^2\right]\right\} \tag{29}$$

with $\sigma_W^* = \frac{p_2^* - p_1^*}{t(1-\gamma)}$ and $\sigma_0^* = \frac{p_1^* - s}{t(\gamma - \frac{1}{2})}$. Solving $\frac{\partial \Pi_A}{\partial p_{2,A}} = 0$ from (29) for p_2^* and substituting into (28) gives

$$\begin{aligned} 2\frac{\partial \Pi_A}{\partial p_{1,A}} &= \frac{\gamma - (2\gamma-1)(1-\gamma)F(\sigma_W^*)}{\gamma^2 + (1-\gamma)^2} - F(\sigma_0^*) - \frac{p_1^*}{t}\left[\frac{f(\sigma_W^*)(1-\gamma)}{\gamma^2+(1-\gamma)^2} + \frac{f(\sigma_0^*)}{\gamma - \frac{1}{2}}\right] \\ &\leq f(\sigma_0^*)\left\{\frac{\gamma}{\gamma^2+(1-\gamma)^2}\frac{1-F(\sigma_0^*)}{f(\sigma_0^*)} - \sigma_0^* - \frac{s}{t}\frac{1}{\gamma - \frac{1}{2}}\right\}. \end{aligned} \tag{30}$$

For the inequality we substituted p_1^* by $s + t(\gamma - \frac{1}{2})\sigma_0^*$ and used the fact that $F(\sigma_W^*) \geq F(\sigma_0^*)$ in a price discrimination equilibrium. Because the hazard rate $\frac{f}{1-F}$ is increasing, the term in parenthesis is decreasing in σ_0^* and hence bounded from above by $\frac{\gamma}{\gamma^2+(1-\gamma)^2}\frac{1}{f(0)} - \frac{s}{t}\frac{1}{\gamma-\frac{1}{2}}$. This term is negative by Assumption (A1). Hence $\frac{\partial \Pi_A}{\partial p_{1,A}} < 0$ and firm A can increase its profit by lowering its price, i.e. (p_1^*, p_2^*) cannot be an equilibrium. We have therefore shown that, in a price discrimination equilibrium, the market must be covered.

For an advance-selling equilibrium the only difference is that $\Pi_A = \frac{p_{1,A}}{2}[1 - F(\sigma_0)]$ implying

$$2\frac{\partial \Pi_A}{\partial p_{1,A}}|_{p_{1,A}=p_1^*} = f(\sigma_0^*)\left\{\frac{1-F(\sigma_0^*)}{f(\sigma_0^*)} - \sigma_0^* - \frac{s}{t(\gamma - \frac{1}{2})}\right\} \tag{31}$$

which is negative for all $\sigma_0^* \geq 0$ if and only if $\frac{s}{t} > \frac{\gamma - \frac{1}{2}}{f(0)}$. This condition is weaker than the one derived above for the case of a price-discrimination equilibrium.

It remains to show that any solution to the system of equations (13) and (14) must satisfy $\Delta p^* > \Delta p^M$. Substituting $p_2^* = p_1^* + t(1-\gamma)\sigma_W^*$ into (13) and (14) gives:

$$0 = F(\sigma_W^*) + \gamma\sigma_W^* f(\sigma_W^*) - \frac{p_1^*}{t}[f(\sigma_W^*) + \frac{1}{2\gamma-1}f(0)] \tag{32}$$

$$0 = 1 - F(\sigma_W^*) - [\gamma^2 + (1-\gamma)^2]\sigma_W^* f(\sigma_W^*) + \frac{p_1^*}{t}(2\gamma-1)f(\sigma_W^*). \tag{33}$$

Solving (32) for p_1^* and substituting into (33) gives a condition on σ_W^* that has to be satisfied in any price discrimination equilibrium:

$$\frac{1-F(\sigma_W^*)}{f(\sigma_W^*)} - \sigma_W^* + \left\{ (2\gamma-1)\frac{F(\sigma_W^*)+\gamma\sigma_W^* f(\sigma_W^*)}{f(\sigma_W^*)+\frac{1}{2\gamma-1}f(0)} + 2\gamma(1-\gamma)\sigma_W^* \right\} = 0. \tag{34}$$

As the term in parenthesis is strictly positive, it follows from the comparison with the corresponding condition (5) under monopoly that $\sigma_W^* > \sigma_W^M$. This is equivalent to $\Delta p^* > \Delta p^M$. QED.

Proof of Proposition 4: We show that in the limit where $\gamma \to \frac{1}{2}$, $(p_1^*, p_2^*) = (0, \frac{t}{2}\sigma_W^*)$ constitutes a (unique) price-discrimination equilibrium. It then follows by continuity of the firms' profit functions that a unique price discrimination equilibrium also exists for all γ sufficiently close to $\frac{1}{2}$.

To see that for $\gamma \to \frac{1}{2}$, the price schedule $(p_1^*, p_2^*) = (0, \frac{t}{2}\sigma_W^*)$ solves the equilibrium conditions (13) and (14) note that (13) can be written as

$$p_1^*\frac{f(0)}{t} = (2\gamma-1)[F(\sigma_W^*) + (\gamma p_2^* - p_1^*)\frac{f(\sigma_W^*)}{t(1-\gamma)}]. \tag{35}$$

Since for $\gamma \to \frac{1}{2}$ the right hand side converges to zero, it must hold that in the limit $p_1^* = 0$. Substituting $p_1^* = 0$ and $p_2^* = t(1-\gamma)\sigma_W^*$ into (14) and letting $\gamma \to \frac{1}{2}$ gives (16). It follows from the increasing hazard rate of f that the expression in (16) is decreasing. It is positive for $\sigma_W^* = 0$ and negative for $\sigma_W^* = 1$. Hence the solution $\sigma_W^* \in (0,1)$ to (16) is well defined and unique. We have therefore shown that for $\gamma \to \frac{1}{2}$, the price schedule $(p_1^*, p_2^*) = (0, \frac{t}{2}\sigma_W^*)$ solves the equilibrium conditions (13) and (14) and that this solution is unique.

It remains to show that there exists no profitable deviation from $(p_1^*, p_2^*) = (0, \frac{t}{2}\sigma_W^*)$. Let $\gamma \to \frac{1}{2}$ and suppose that firm B chooses $(p_1^*, p_2^*) = (0, \frac{t}{2}\sigma_W^*)$. Consider a deviation of firm A to $(p_1, p_2) \neq (p_1^*, p_2^*)$. We first argue that we can restrict attention to deviations of the form $(p_1, p_2) = (p_1^*, p_2)$. This is obvious for $p_1 < p_1^*$ because decreasing price below marginal cost gives firm A a loss in period 1 and a demand reduction in period 2. For $p_1 > p_1^*$, the intertemporal allocation of sales is independent of p_1 because the consumers' choice between buying early and buying late depends only on the lowest price offered in period 1. The only effect of choosing $p_1 > p_1^*$ rather than $p_1 = p_1^*$ is to reduce firm A's first period demand to zero.

It therefore remains to consider deviations to (p_1^*, p_2). Using the indifferent consumer type $\sigma_W = \frac{p_2+p_2^*}{t}$ we can write $p_2 = t\sigma_W - p_2^* = t\sigma_W - \frac{t}{2}\sigma_W^*$ and substitute into firm A's profit $\Pi_A = \frac{1}{2}p_2[1 - F(\sigma_W)]$ to get $\Pi_A = \frac{1}{2}(t\sigma_W - \frac{t}{2}\sigma_W^*)[1 - F(\sigma_W)]$ for $p_2 \leq t - \frac{t}{2}\sigma_W^*$. For $p_2 > t - \frac{t}{2}\sigma_W^*$ we have $\sigma_W = 1$ and hence $\Pi_A = 0$. Choosing a $p_2 \in [0, t - \frac{t}{2}\sigma_W^*]$ is equivalent to choosing a $\sigma_W \in [\frac{\sigma_W^*}{2}, 1]$. We have

$$\frac{\partial \Pi_A}{\partial \sigma_W} = f(\sigma_W)\left\{\frac{t}{2}\frac{1 - F(\sigma_W)}{f(\sigma_W)} - \frac{t}{2}(\sigma_W - \frac{\sigma_W^*}{2})\right\}. \tag{36}$$

The term in parenthesis is positive at $\sigma_W = \frac{\sigma_W^*}{2}$. It is decreasing in σ_W due to the increasing hazard rate of f. Hence if σ_W^* is given by the unique solution to (16) then choosing $\sigma_W = \sigma_W^*$ or equivalently $p_2 = p_2^*$ maximizes the profit of the deviating firm. Hence we have shown that $(p_1^*, p_2^*) = (0, \frac{t}{2}\sigma_W^*)$ constitutes an equilibrium and, as the solution σ_W^* to (16) is unique, this equilibrium is the unique price discrimination equilibrium. QED.

Proof of Proposition 5: In order to show that the prices (p_1^*, p_2^*) given by (17) and (18) constitute an equilibrium we need to check the non-profitability of all potential deviations to price schedules $(p_1, p_2) \neq (p_1^*, p_2^*)$. This involves a lengthy distinction of various types of deviations which has been moved to Appendix B available online. In this proof we derive the comparative statics and perform the comparison with the monopolistic benchmark.

The following comparative statics are straightforward:

$$\frac{dp_1^*}{d\gamma} = [2\gamma^2 + (1-\gamma)^2]\frac{3(\sigma_W^*)^2}{2\gamma^2}t > 0, \quad \frac{dp_2^*}{d\gamma} = [2\gamma^2 + (1-\gamma)^2]\frac{(\sigma_W^*)^2}{\gamma^2}t > 0 \tag{37}$$

$$\frac{d(p_2^* - p_1^*)}{d\gamma} = -[2\gamma^2 + (1-\gamma)^2]\frac{(\sigma_W^*)^2}{2\gamma^2}t < 0, \quad \frac{d\sigma_W^*}{d\gamma} = (4\gamma^2 - 1)\frac{(\sigma_W^*)^2}{2\gamma^2} > 0. \tag{38}$$

Under monopoly, it follows from the first order condition $\frac{1-F(\sigma_W^M)}{f(\sigma_W^M)} - \sigma_W^M = 0$ for f uniform that $\sigma_W^M = \frac{1}{2}$ and thus $p_2^M = s + \frac{t}{2}(1-\gamma)$. Consider

$$p_2^* - p_2^M = \frac{3\gamma - 1}{-4\gamma^2 + 7\gamma - 1}t - [s + \frac{t}{2}(1-\gamma)] = \frac{t}{2}\frac{-4\gamma^3 + 11\gamma^2 - 2\gamma - 1}{-4\gamma^2 + 7\gamma - 1} - s. \quad (39)$$

Hence $p_2^* > p_2^M$ if and only if

$$\frac{s}{t} < \frac{1}{2}\frac{-4\gamma^3 + 11\gamma^2 - 2\gamma - 1}{-4\gamma^2 + 7\gamma - 1} \equiv T_P(\gamma). \quad (40)$$

The consumer surplus comparison gives

$$CS^* - CS^M = \sigma_W^M(p_1^M - p_1^*) + (1 - \sigma_W^*)(p_2^M - p_2^*) + \int_{\sigma_W^M}^{\sigma_W^*} p_2^M - p_1^* - (1-\gamma)t\sigma d\sigma. \quad (41)$$

Substituting prices and cutoffs we get

$$CS^* - CS^M = s - \frac{t}{8}\frac{48\gamma^5 - 216\gamma^4 + 259\gamma^3 - 29\gamma^2 - 35\gamma + 5}{(-4\gamma^2 + 7\gamma - 1)^2}. \quad (42)$$

Hence $CS^* < CS^M$ if and only if

$$\frac{s}{t} < \frac{1}{8}\frac{48\gamma^5 - 216\gamma^4 + 259\gamma^3 - 29\gamma^2 - 35\gamma + 5}{(-4\gamma^2 + 7\gamma - 1)^2} \equiv T_{CS}(\gamma). \quad (43)$$

Now compare the thresholds T_P and T_{CS} with the boundaries of the parameter set given by Assumption (A1'). It is easy to see that the relevant bound in (A1') is given by $\frac{(1+\gamma)(2\gamma-1)}{-4\gamma^2+7\gamma-1}$ if and only if $\gamma > \sqrt{\frac{1}{2}}$. We have

$$T_P(\gamma) > \frac{(1+\gamma)(2\gamma-1)}{-4\gamma^2 + 7\gamma - 1} \Leftrightarrow \frac{1}{2}(1-\gamma)\{4\gamma^2 - 3\gamma + 1\} > 0. \quad (44)$$

The term in parenthesis is increasing in $\gamma \in (\frac{1}{2}, 1)$ and becomes $\frac{1}{2}$ for $\gamma \to \frac{1}{2}$. Hence, for $\gamma > \sqrt{\frac{1}{2}}$, the threshold T_P lies strictly above the lower bound given by Assumption (A1'). This shows that there exist parameter values for which (p_1^*, p_2^*) is an equilibrium and $p_2^* > p_2^M$. A similar reasoning shows that there also exist parameter values for which (p_1^*, p_2^*) is an equilibrium and $CS^* < CS^M$. QED.

Proof of Proposition 6: In this proof we first consider the possibility of selling exclusively on the spot before dealing with the option of selling exclusively in advance.

Spot selling: Consider the case of a duopoly and suppose that (p_1^*, p_2^*) is an equilibrium in which no consumer purchases in advance, and a consumer buys his preferred product in period 2 iff $\sigma \geq \sigma_0^* = \frac{2(p_2^*-s)}{t}$. We first argue that in equilibrium the market must be covered. Suppose, instead, that $\sigma_0^* > 0$ and consider a deviation by firm A to a price $p_{2,A} \in (p_2^* - \epsilon, p_2^*)$. For small ϵ it follows from $\sigma_0^* > 0$ that firm A's deviation has no influence on the behavior of consumers preferring product B and firm A's profit from deviating is given by $\Pi_A = \frac{p_{2,A}}{2}[1 - F(\frac{2(p_{2,A}-s)}{t})]$. We have

$$\frac{\partial \Pi_A}{\partial p_{2,A}}|_{p_{2,A}=p_2^*} = \frac{1}{2}[1 - F(\frac{2(p_2^* - s)}{t})] - \frac{p_2^*}{t} f(\sigma_0^*) = f(\sigma_0^*) \left\{ \frac{1 - F(\sigma_0^*)}{2f(\sigma_0^*)} - \frac{s}{t} - \frac{\sigma_0^*}{2} \right\} \quad (45)$$

where we have substituted $p_2^* = s + \frac{t}{2}\sigma_0^*$. The term in parenthesis is decreasing in σ_0^* due to the increasing hazard rate of the distribution f. Moreover, as $\frac{s}{t} > \frac{1}{2f(0)}$ the term is negative for $\sigma_0^* \to 0$. Hence we have shown that for any price p_2^* such that $\sigma_0^* > 0$ it holds that $\frac{\partial \Pi_A}{\partial p_{2,A}}|_{p_{2,A}=p_2^*} < 0$, i.e. deviating to a price $p_{2,A} < p_2^*$ is profitable for firm A. Hence p_2^* must be such that all consumers buy in period 2.

Given that the market is covered in period 2, it must hold that $p_1^* \geq p_2^*$ because otherwise, the least choosy consumers would prefer to buy in period 1 rather than in period 2. Consider a deviation by firm A to $p_{1,A} = p_2^* - \epsilon < p_1^*$. As in equilibrium all consumers buy in period 2, there exists a non-empty interval of relatively unchoosy consumers who will become A's customers in period 1. Only a fraction of these consumers (γ of those favoring A and $(1-\gamma)$ of those favoring B) would have become A's customers in period 2. Hence, for ϵ sufficiently small, such a deviation is profitable for firm A. We have thus shown that there cannot exist an equilibrium in which consumers buy exclusively in period 2.

We now show that under monopoly $p_1^M = p_2^M = s$ and all consumers purchase in period 2. A consumer prefers to wait until period 2, given that all other consumers do so, because he expects price to remain constant and is able to make an informed rather than an uninformed purchase. If all consumers wait until period 2 then it follows from an argument similar to the one used in the duopoly case that the monopolist maximizes profit by selling to all consumers, i.e. $p_2^M = s$ is optimal. Choosing $p_1^M < s$ would lead to a decrease in monopoly profit, whereas for $p_1^M > s$ consumers would still prefer to wait. Hence $p_1^M = s$ is optimal.

Advance selling: For the analysis of an equilibrium in which all consumers prefer to purchase in advance, suppose that each consumer trembles with a small probability $\epsilon \in (0,1)$. A consumer who trembles fails to purchase in period 1 even when he prefers buying over waiting. In the spirit of trembling hand perfection we assume that the likelihood of a tremble is independent of a consumer's type and hence the same for each consumer. In an advance selling equilibrium, each consumer's strategy recommends a period 1 purchase and firms therefore face a mass ϵ of consumers in period 2 with types distributed according to the original distribution f.

Consider first the case of a monopolist. Suppose that the monopolist maximizes profit by selling to all consumers in advance at $p_1 = s$. As $\frac{s}{t} \geq \frac{1}{f(0)} > \frac{1}{2f(0)}$ it follows from the above that a monopolist will sell to all remaining consumers, i.e. those who trembled, by setting $p_2 = s$. However, if consumers expect the price to remain constant over time, then it cannot be optimal for consumers to purchase in advance without knowledge of their preferences, a contradiction. Hence we have shown that there cannot exist a Trembling Hand Perfect Bayesian equilibrium in which the monopolist sells to all consumers in advance.

Finally, consider the possibility of advance selling for a duopoly. The condition $f(0) < 2$ is equivalent to $p_2^* - p_1^* > t(1-\gamma)$, which implies that at the suggested equilibrium prices, all consumers prefer an early over a late purchase. Next, we show that $p_{2,A} = p_{2,B} = p_2^* = \frac{t}{f(0)}$ constitutes an equilibrium of the pricing subgame starting in period 2. For this purpose suppose that in period 2 firm B chooses p_2^* and consider a deviation by firm A

to a price $p_{2,A} \in (p_2^* - t, p_2^*)$. Firm A's second period profit is $\Pi_{2,A} = \frac{\epsilon}{2} p_{2,A}[1 + F(\frac{p_2^* - p_{2,A}}{t})]$ with

$$\frac{\partial \Pi_{2,A}}{\partial p_{2,A}} = \frac{\epsilon}{2} f(\frac{p_2^* - p_{2,A}}{t}) \left\{ \frac{1 + F(\frac{p_2^* - p_{2,A}}{t})}{f(\frac{p_2^* - p_{2,A}}{t})} - \frac{p_{2,A}}{t} \right\}. \tag{46}$$

If $1 + F$ is log-concave then the term in parenthesis is decreasing in $p_{2,A}$. It is zero at $p_{2,A} = p_2^*$. Hence $\frac{\partial \Pi_{2,A}}{\partial p_{2,A}} > 0$ for all $p_{2,A} \in (p_2^* - t, p_2^*)$. Because decreasing $p_{2,A}$ below $p_2^* - t$ does not increase A's demand any further we have therefore shown that a deviation to a lower second period price cannot be optimal. Consider a deviation to a higher second period price $p_{2,A} \in (p_2^*, p_2^* + t)$. Firm A's second period profit is $\Pi_{2,A} = \frac{\epsilon}{2} p_{2,A}[1 - F(\frac{p_{2,A} - p_2^*}{t})]$ whereas for $p_{2,A} \geq p_1^* + t$ profit is zero. We have

$$\frac{\partial \Pi_{2,A}}{\partial p_{2,A}} = \frac{\epsilon}{2} f(\frac{p_{2,A} - p_2^*}{t}) \left\{ \frac{1 - F(p_{2,A} - \frac{p_2^*}{t})}{f(\frac{p_{2,A} - p_2^*}{t})} - \frac{p_{2,A}}{t} \right\}. \tag{47}$$

As f has an increasing hazard rate, the term in parenthesis is decreasing in $p_{2,A}$. It is zero at $p_{2,A} = p_2^*$. Hence $\frac{\partial \Pi_{2,A}}{\partial p_{2,A}} < 0$ for all $p_{2,A} \in (p_2^*, p_2^* + t)$. We have therefore shown that a deviation to a higher second period price cannot be optimal. The condition $\frac{s}{t} \geq \frac{1}{f(0)}$ guarantees that given p_2^*, all consumers derive positive utility from purchasing in period 2.

It remains to consider the firms' first period behavior. It is clear that all consumers obtain positive utility from purchasing in period 1 as this is also true for all consumers purchasing at the higher second period price. Consider a deviation to a lower first period price $p_{1,A} < p_1^*$. No consumer will react by postponing his purchase. However some of the consumers whose favorite is B will start purchasing A in advance. Firm A's profit from deviating to $p_{1,A} \in (p_1^* - (2\gamma - 1)t, p_1^*)$ is $\Pi_A = (1 - \epsilon) \frac{p_{1,A}}{2}[1 + F(\frac{p_1^* - p_{1,A}}{(2\gamma - 1)t})] + \epsilon \frac{p_2^*}{2}$. The derivative is

$$\frac{\partial \Pi_A}{\partial p_{1,A}} = \frac{(1 - \epsilon)}{2} f(\frac{p_1^* - p_{1,A}}{(2\gamma - 1)t}) \left\{ \frac{1 + F(\frac{p_1^* - p_{1,A}}{(2\gamma - 1)t})}{f(\frac{p_1^* - p_{1,A}}{(2\gamma - 1)t})} - \frac{p_{1,A}}{(2\gamma - 1)t} \right\}. \tag{48}$$

If $1 + F$ is log-concave than the term in parenthesis is decreasing in $p_{1,A}$. It is zero at $p_{1,A} = p_1^*$. Hence $\frac{\partial \Pi_A}{\partial p_{1,A}} > 0$ for all $p_{1,A} \in (p_1^* - (2\gamma - 1)t, p_1^*)$. Because decreasing $p_{1,A}$

below $p_1^* - (2\gamma - 1)t$ does not increase A's demand any further we have therefore shown that a deviation to a lower first period price cannot be optimal.

Finally, consider a deviation of firm A to a higher first period price $p_{1,A} > p_1^*$. The behavior of consumers whose favorite is B is not affected by this deviation as these consumers prefer B over A in period 1 already when $p_{1,A} = p_1^*$. Consider a consumer whose favorite is A. The condition $\frac{s}{t} \geq (2\gamma - 1)(\frac{1}{2} + \frac{1}{f(0)})$ guarantees that a consumer's utility from purchasing his non-favorite product in period 1 at price p_1^* is positive even for the most choosy consumer. In particular, a consumer whose favorite is A obtains a positive utility from purchasing product B in period 1. Hence, an argument similar to the one in the proof of Lemma 1 shows that there cannot exist a consumer type who is indifferent between purchasing early and waiting. If such a type existed, firms would choose a second period price that leaves zero utility to this type. A consumer whose favorite is A will therefore either buy A or B in period 1 and firm A's profit from deviating to a price $p_{1,A} \in (p_1^*, p_1^* + (2\gamma - 1)t)$ is given by $\Pi_A = (1-\epsilon)\frac{p_{1,A}}{2}[1 - F(\frac{p_{1,A} - p_1^*}{(2\gamma-1)t})] + \epsilon \frac{p_2^*}{2}$. For $p_{1,A} \geq p_1^* + (2\gamma - 1)t$ firm A's profit is zero. We have

$$\frac{\partial \Pi_A}{\partial p_{1,A}} = \frac{(1-\epsilon)}{2} f(\frac{p_{1,A} - p_1^*}{(2\gamma - 1)t}) \left\{ \frac{1 - F(\frac{p_{1,A} - p_1^*}{(2\gamma-1)t})}{f(\frac{p_{1,A} - p_1^*}{(2\gamma-1)t})} - \frac{p_{1,A}}{(2\gamma - 1)t} \right\}. \tag{49}$$

As f has an increasing hazard rate the term in parenthesis is decreasing in $p_{1,A}$. It is zero at $p_{1,A} = p_1^*$. Hence $\frac{\partial \Pi_A}{\partial p_{1,A}} < 0$ for all $p_{1,A} \in (p_1^*, p_1^* + (2\gamma - 1)t)$. Thus, we have shown that a deviation to a higher first period price cannot be profitable. QED.

Appendix B - For publication online

In this online-appendix we prove the existence of a price-discrimination equilibrium for the case of a uniform type distribution and extend our analysis to a setting where consumers have elastic demands in order to consider the effect of competition on welfare in the presence of quantity effects.

Equilibrium existence - The uniform case

In the following we show that the price schedule (p_1^*, p_2^*) given by (17) and (18) constitutes an equilibrium by checking the non-profitability of any deviation $(p_{1,A}, p_{2,A}) = (p_1, p_2) \neq (p_1^*, p_2^*)$. The proof proceeds as follows. In Step 1, we identify the subset of deviations (denoted as $\mathbf{P}_a$ in Figure 4) for which the profit of the deviating firm A takes the form (11) derived in the main text. We show that, within this subset, profits are strictly concave and hence maximized at $(p_1, p_2) = (p_1^*, p_2^*)$. In Steps 2-6, we examine the remaining subsets of potential deviations for which the firm's profit takes a different form.

<u>Step 1:</u> In (11) and (12) we have derived a deviating firm's profits for a *small* deviation from the candidate equilibrium (p_1^*, p_2^*). In particular, we have implicitly assumed that

$$\sigma_W(A) \leq 1 \Leftrightarrow p_1 \geq \underline{p}_1' \equiv \gamma p_2 + (1-\gamma)p_2^* - t(1-\gamma) \quad (50)$$

$$\sigma_W(B) \leq 1 \Leftrightarrow p_2 \leq \bar{p}_2 \equiv \frac{(1-\gamma)t - \gamma p_2^* + p_1^*}{1-\gamma} \quad (51)$$

and that for $p_1 > p_1^*$

$$\bar{\sigma} \leq \sigma_W(A) \Leftrightarrow p_1 \leq \bar{p}_1 \equiv \frac{(1-\gamma)p_1^* + (2\gamma-1)(\gamma p_2 + (1-\gamma)p_2^*)}{\gamma} \quad (52)$$

whereas for $p_1 < p_1^*$

$$\bar{\sigma} \leq \sigma_W(B) \Leftrightarrow p_1 \geq \underline{p}_1 \equiv \frac{\gamma p_1^* - (2\gamma-1)(\gamma p_2^* + (1-\gamma)p_2)}{1-\gamma}. \quad (53)$$

Hence for the set of deviations

$$\mathbf{P}_a \equiv \left\{ (p_1, p_2) \in \mathbf{R}_+^2 \mid p_1 \in [\max\{\underline{p}_1, \underline{p}_1'\}, \bar{p}_1], p_2 \in [p_1, \bar{p}_2] \right\} \quad (54)$$

firm A's profits take the form in (11) and (12) which for the uniform distribution simplifies to:

$$\Pi_A = \frac{p_1}{2}[\sigma_W(A) - \frac{p_1 - p_1^*}{t(2\gamma-1)}] + \frac{p_2}{2}[\gamma(1-\sigma_W(A)) + (1-\gamma)(1-\sigma_W(B))]. \quad (55)$$

It is easy to see that $(p_1^*, p_2^*) \in \text{int } \mathbf{P}_a$ is the unique solution to the first order conditions (12) and (13). To see that (p_1^*, p_2^*) achieves not only a local but a *global* maximum in $\mathbf{P}_a$, consider the Hessian matrix given by

$$H \equiv \begin{pmatrix} \frac{\partial^2 \Pi_A}{\partial p_1^2} & \frac{\partial^2 \Pi_A}{\partial p_1 \partial p_2} \\ \frac{\partial^2 \Pi_A}{\partial p_2 \partial p_1} & \frac{\partial^2 \Pi_A}{\partial p_2^2} \end{pmatrix} = \begin{pmatrix} -\frac{\gamma}{t(2\gamma-1)(1-\gamma)} & \frac{\gamma}{t(1-\gamma)} \\ \frac{\gamma}{t(1-\gamma)} & -\frac{\gamma^2+(1-\gamma)^2}{t(1-\gamma)} \end{pmatrix}. \tag{56}$$

The i-th leading principal minor of this Hessian, denoted by $H(i)$, is given by $H(1) = \frac{\partial^2 \Pi_A}{\partial p_1^2} = -\frac{\gamma}{t(2\gamma-1)(1-\gamma)} < 0$ and $H(2) = \frac{\partial^2 \Pi_A}{\partial p_1^2}\frac{\partial^2 \Pi_A}{\partial p_2^2} - \left(\frac{\partial^2 \Pi_A}{\partial p_1 \partial p_2}\right)^2 = \frac{\gamma}{t^2(1-\gamma)(2\gamma-1)} > 0$. Hence, H is negative definite and so the profit function Π_A is strictly concave in $\mathbf{P}_a$.

<u>Step 2:</u> Consider the set of deviations

$$\mathbf{P}_b \equiv \left\{(p_1, p_2) \in \mathbf{R}_+^2 \mid p_1 \in [\underline{p}_1', \underline{p}_1], p_2 \geq p_1\right\}. \tag{57}$$

By definition of the threshold $\underline{p}_1$, in $\mathbf{P}_b$ it no longer holds that $\sigma_W(B) > \bar{\sigma}$. In this case all advance consumers buy A independently of whether their favorite is A or B. Those whose favorite is B compare the expected utility from buying A early, $U(\sigma, B \mid 1, A) = s - \gamma\frac{t}{2}\sigma + (1-\gamma)\frac{t}{2}\sigma - p_1$, with the expected utility from buying late, $U(\sigma, B \mid 2) = s + \frac{t}{2}\sigma - \gamma p_2^* - (1-\gamma)p_2$:

$$U(\sigma, A \mid 1, B) > U(\sigma, B \mid 2) \Leftrightarrow \sigma < \sigma_W'(B) \equiv \frac{\gamma p_2^* + (1-\gamma)p_2 - p_1}{\gamma t}. \tag{58}$$

Consumers in $[0, \sigma_W(A))$ with favorite A and consumers in $[0, \sigma_W'(B))$ with favorite B buy product A in period 1 while the remaining consumers wait. Hence in $\mathbf{P}_b$ the profit of firm A is given by

$$\Pi_A = \frac{p_1}{2}(\sigma_W(A) + \sigma_W'(B)) + \frac{p_2}{2}[\gamma(1-\sigma_W(A)) + (1-\gamma)(1-\sigma_W'(B))]. \tag{59}$$

We now show that in $\mathbf{P}_b$, Π_A is strictly increasing in p_1 and $\Pi_A|_{p_1=p_2}$ is strictly increasing in p_2. By continuity, this implies that all deviations in $\mathbf{P}_b$ are dominated by a deviation in $\mathbf{P}_a$ (see Figure 4 For any $(p_1, p_2) \in \mathbf{P}_b$, we have

$$\begin{aligned}\gamma(1-\gamma)t\frac{\partial \Pi_A}{\partial p_1} &= -p_1 + \gamma(1-\gamma)p_2^* + ((1-\gamma)^2 + \gamma^2)p_2 \\ &\geq -\min\{\underline{p}_1, p_2\} + \gamma(1-\gamma)p_2^* + ((1-\gamma)^2 + \gamma^2)p_2 \\ &\geq \gamma\left(\gamma p_2^* - p_1^*\right) = \frac{(1-\gamma)^2\gamma t}{-4\gamma^2 + 7\gamma - 1} > 0\end{aligned} \tag{60}$$

where the second inequality follows from the fact that $\underline{p}_1 \leq p_2 \Leftrightarrow p_2 \geq \frac{p_1^* - (2\gamma-1)p_2^*}{2(1-\gamma)}$. Moreover, for any $(p_1, p_2) \in \mathbf{P}_b$ such that $p_1 = p_2$ it holds that

$$\frac{\partial \Pi_A}{\partial p_2}|_{p_1=p_2} = \frac{-2p_2 + p_2^* + t}{2t} \geq \frac{-p_1^* + \gamma p_2^* + (1-\gamma)t}{2t(1-\gamma)} = \frac{\gamma(3-2\gamma)}{-4\gamma^2 + 7\gamma - 1} > 0 \tag{61}$$

where the first inequality follows from $p_2 \leq \frac{p_1^* - (2\gamma-1)p_2^*}{2(1-\gamma)}$. Hence there cannot exist a profitable deviation in $\mathbf{P}_b$.

Step 3: Consider the set of deviations

$$\mathbf{P}_c \equiv \left\{(p_1, p_2) \in \mathbf{R}_+^2 \mid p_1 \leq \min\{\underline{p}_1', \underline{p}_1\}, p_2 \geq p_2^* - t\right\}. \tag{62}$$

The only difference to the previous case is that it no longer holds that $\sigma_W(A) \leq 1$. The profit of firm A is thus given by

$$\Pi_A = \frac{p_1}{2}(1 + \sigma_W'(B)) + \frac{p_2}{2}(1-\gamma)(1 - \sigma_W'(B)). \tag{63}$$

We now show that in $\mathbf{P}_c$, Π_A is strictly increasing in p_1. By continuity, this implies that all deviations in $\mathbf{P}_c$ are dominated by a deviation in $\mathbf{P}_b \cup \mathbf{P}_d$ (see Figure 4). For any $(p_1, p_2) \in \mathbf{P}_c$, we have

$$\frac{\partial \Pi_A}{\partial p_1} = \frac{\gamma t - 2p_1 + \gamma p_2^* + 2(1-\gamma)p_2}{2\gamma t}. \tag{64}$$

As $\frac{\partial \Pi_A}{\partial p_1}$ is decreasing in p_1 and increasing in p_2 it becomes minimal on the boundary $\mathbf{P}_b \cap \mathbf{P}_c$. We have

$$\frac{\partial \Pi_A}{\partial p_1}|_{p_1=\underline{p}_1'} = \frac{(2-\gamma)t + 2(1-2\gamma)p_2 - (2-3\gamma)p_2^*}{2\gamma t}. \tag{65}$$

As $\frac{\partial \Pi_A}{\partial p_1}|_{p_1=\underline{p}'_1}$ is decreasing in p_2 it becomes minimal at the point $\mathbf{P}_a \cap \mathbf{P}_b \cap \mathbf{P}_c$ where

$$\frac{\partial \Pi_A}{\partial p_1} = \frac{-2\gamma^3+\gamma^2+4\gamma-1}{(3\gamma-1)(-4\gamma^2+7\gamma-1)} > 0. \tag{66}$$

This shows that $\frac{\partial \Pi_A}{\partial p_1} > 0$ in the entire set $\mathbf{P}_c$. Hence there cannot exist a profitable deviation in $\mathbf{P}_c$

Step 4: Consider the set of deviations

$$\mathbf{P}_d \equiv \left\{(p_1,p_2)\in \mathbf{R}^2_+ \mid p_1 \in [\min\{0,\underline{p}_1\},\underline{p}'_1], p_2 \le \bar{p}_2\right\}. \tag{67}$$

In comparison to the set $\mathbf{P}_a$ the only difference is that in $\mathbf{P}_d$ it no longer holds that $\sigma_W(A) \le 1$. Firm A's profit is thus given by

$$\Pi_A = \frac{p_1}{2}[1-\frac{p_1-p_1^*}{t(2\gamma-1)}] + \frac{p_2}{2}(1-\gamma)(1-\sigma_W(B)). \tag{68}$$

For any $(p_1,p_2)\in \mathbf{P}_d$, we have

$$\frac{\partial \Pi_A}{\partial p_1} = \frac{(2\gamma-1)t-2p_1+p_1^*}{2(2\gamma-1)t} > \frac{(2\gamma-1)t-2\underline{p}'_1+p_1^*}{2(2\gamma-1)t}. \tag{69}$$

To identify the sign of the R.H.S. of the last inequality, observe that

$$\begin{aligned}(2\gamma-1)t-2\underline{p}'_1+p_1^* &= (2\gamma-1)t-2\left(\gamma p_2+(1-\gamma)(p_2^*-t)\right)+p_1^* > 0 \\ \Leftrightarrow p_2 &< \frac{p_1^*+t-2(1-\gamma)p_2^*}{2\gamma} \equiv \tilde{p}_2 \in (p_2^*,\bar{p}_2).\end{aligned} \tag{70}$$

Hence, for $p_2 < \tilde{p}_2$, profit is strictly increasing in p_1 so that a deviation $(p_1,p_2)\in \mathbf{P}_d$ is dominated by a deviation in $\mathbf{P}_a \cap \mathbf{P}_d$. For $p_2 \ge \tilde{p}_2$, observe that

$$\frac{\partial \Pi_A}{\partial p_2} = \frac{(1-\gamma)t-\gamma p_2^*-2(1-\gamma)p_2+p_1^*}{2t} \le \frac{(1-\gamma)t-\gamma p_2^*-2(1-\gamma)\tilde{p}_2+p_1^*}{2t}, \tag{71}$$

with the last term being negative if and only if $2(1-\gamma)(2\gamma^2+1) > 0$. Hence, profit is strictly decreasing in $p_2 \ge \tilde{p}_2$. As

$$\frac{\partial \Pi_A}{\partial p_1}|_{p_1=\underline{p}_1} > 0 \Leftrightarrow 4\gamma(1-\gamma)^2 > 0, \tag{72}$$

this implies that for $p_2 > \tilde{p}_2$, profit is smaller than the one evaluated at the intersection of $\tilde{p}_2$ and $\underline{p}_1$. It thus remains to show that this profit is smaller than the equilibrium profit. This is indeed the case because

$$\Pi_A(p_1^*, p_2^*) - \Pi_A(\underline{p}_1, \tilde{p}_2) = \frac{t(1-\gamma)^2\left[4\gamma(2\gamma-1)(1-\gamma)+1\right]}{2(-4\gamma^2+7\gamma-1)^2} > 0. \tag{73}$$

Hence there cannot exist a profitable deviation in $\mathbf{P}_d$

Step 5: Consider the set of deviations

$$\mathbf{P}_e \equiv \left\{(p_1, p_2) \in \mathbf{R}_+^2 \mid p_1 \in [\underline{p}_1', \bar{p}_1], p_2 \geq \bar{p}_2\right\}, \tag{74}$$

In comparison to the set $\mathbf{P}_a$ the only difference is that in $\mathbf{P}_e$ it no longer holds that $\sigma_W(B) \leq 1$. Firm A's profit is thus given by

$$\Pi_A = \frac{p_1}{2}[\sigma_W(A) - \frac{p_1 - p_1^*}{t(2\gamma-1)}] + \frac{p_2}{2}\gamma(1-\sigma_W(A)). \tag{75}$$

Differentiation yields

$$\frac{\partial \Pi_A}{\partial p_2} = \frac{\gamma\left[t(1-\gamma) - 2\gamma p_2 - (1-\gamma)p_2^* + 2p_1\right]}{2t(1-\gamma)}. \tag{76}$$

As $\frac{\partial \Pi_A}{\partial p_2}$ is increasing in p_1 and decreasing in p_2 it becomes maximal at the intersection of $\bar{p}_1$ and $\bar{p}_2$. We have

$$\begin{aligned} \frac{\partial \Pi_A}{\partial p_2}\Big|_{p_1=\bar{p}_1,\ p_2=\bar{p}_2} < 0 \quad &\Leftrightarrow \quad 0 < \gamma(1-\gamma)t + (\gamma^2 - 5\gamma + 2)p_2^* + 2(2\gamma-1)p_1^* \\ &\Leftrightarrow \quad 4\gamma(1-\gamma)(-\gamma^2+\gamma+1) > 0. \end{aligned} \tag{77}$$

Hence, any deviation in $\mathbf{P}_e$ is dominated by a deviation in $\mathbf{P}_a \cap \mathbf{P}_e$, i.e. no deviation in $\mathbf{P}_e$ can be profitable.

Step 6: For all remaining areas it is easy to see that any deviation is dominated by a deviation already contained in $\bigcup_{i=a}^{e} \mathbf{P}_i$. If $p_2 < p_2^* - t$ then all consumers prefer

A over B in period 2. Firm A can achieve a higher profit by deviating instead to $(p_1, p_2) = (p_2^* - t, p_2^* - t) \in \mathbf{P}_b \cap \mathbf{P}_c$. If (p_1, p_2) is such that $p_1 > \bar{p}_1$ then it no longer holds that $\bar{\sigma} = \frac{p_1 - p_1^*}{t(2\gamma-1)} < \sigma_W(A)$, i.e. firm A makes no sales in period 1. Moreover, the thresholds determining second period demands are independent of p_1 as they are based on a comparison of p_2 and p_2^* with firm B's first period price p_1^*. Hence firm A can achieve the same profit by deviating to $(\bar{p}_1, p_2) \in \mathbf{P}_a \cup \mathbf{P}_d$. Finally, if (p_1, p_2) is such that $p_1 < \underline{p}_1'$ and $p_2 > \bar{p}_2$ then all consumers purchase in period 1. Hence profit in this region is independent of p_2 and needs to be considered only on the boundary shared with $\mathbf{P}_d \cup \mathbf{P}_e$.

In summary, we have shown that there does not exist a profitable deviation from the price schedule (p_1^*, p_2^*). Hence (p_1^*, p_2^*) constitutes an equilibrium. QED.

Quantity effects

In the following we abandon the assumption of unitary demand. We suppose instead that each consumer has a downward sloping demand schedule $q(p)$. For simplicity, we let demand be linear, $q(p) = 1 - p$, and, as in the first part of this Appendix, restrict attention to a uniform distribution of consumer types. A consumer's indirect utility function associated with $q(p)$ is denoted as $v(p)$ and is given by the usual triangular area between the demand curve and the price line, i.e. $v(p) = \frac{(1-p)^2}{2}$. We maintain our assumption that consumers can purchase only one of the two products. When purchasing at a price p, a consumer's payoff is assumed to be given by $v(p) + \frac{t}{2}\sigma$ if he consumes his preferred product and by $v(p) - \frac{t}{2}\sigma$ if he consumes his non-preferred product. Hence, with respect to our previous analysis, the only change to the consumers' payoff is that the term $s - p$ becomes substituted by $v(p)$. This has the immediate implication that the critical consumer types σ_W, $\sigma_W(A)$, $\sigma_W(B)$, and $\bar{\sigma}$ can be obtained from (3), (8), (9), and (10) simply by substituting $v(p)$ for $-p$.

Facing a price schedule (p_1, p_2) for both products, early consumers purchase the quantity $q_1 = 1 - p_1$ whereas late consumers buy the quantity $q_2 = 1 - p_2$. A monopolist's profit becomes

$$\Pi^M = p_1 q_1 \sigma_W + p_2 q_2 (1 - \sigma_W). \tag{78}$$

Note that monopoly profit is a weighted average of the per-period revenues $p_t q_t = p_t(1-p_t)$ and in each period $t = 1, 2$ revenue is maximized by setting $p_t = \frac{1}{2}$. Hence a monopolist maximizes profit by choosing $p_1^M = p_2^M = \frac{1}{2}$ inducing all consumers to purchase their preferred product in period 2. The fact that a monopolist refrains from price-discrimination is an immediate consequence of the fact that demand schedules are the same for all consumer types.[12]

Under competition a time-invariant pricing equilibrium can be ruled out by the same argument as in the case of unitary demand. A small APD secures advance demand from consumers who may become the rival's customers in the future. Hence in equilibrium, competing firms must offer an APD by setting $p_1^* < p_2^*$. In a price discrimination equilibrium, firm A's profit for a small deviation $(p_{1,A}, p_{2,A})$ from a price discrimination equilibrium schedule (p_1^*, p_2^*) is given by

$$\Pi_A = \frac{1}{2} p_{1,A} q_{1,A} [\sigma_W(A) - \bar{\sigma}] + \frac{1}{2} p_{2,A} q_{2,A} \{\gamma [1 - \sigma_W(A)] + (1 - \gamma)[1 - \sigma_W(B)]\}. \tag{79}$$

Taking derivatives and evaluating at the equilibrium gives the following two necessary conditions which have to be satisfied by any price discrimination equilibrium:

$$0 = \gamma \left[p_1^* q_1^* - p_2^* q_2^* (2\gamma - 1)\right] - \frac{1 - 2p_1^*}{1 - p_1^*} \sigma_W^* t (1 - \gamma)(2\gamma - 1) \tag{80}$$

$$0 = p_1^* q_1^* \gamma - p_2^* q_2^* [\gamma^2 + (1 - \gamma)^2] - \frac{2p_2^* - 1}{1 - p_2^*} (1 - \sigma_W^*) t (1 - \gamma). \tag{81}$$

Determining a closed form solution to this system of equations is not possible in general. For our welfare comparison we therefore focus on the limiting case where $\gamma \to \frac{1}{2}$. In

[12]If the monopolist was able to offer two-part tariffs price discrimination would emerge in the fixed-fee dimension.

this case products appear homogeneous in period 1 and it follows from (80) that first period prices must tend to marginal cost, i.e. $p_1^* \to 0$. We can then use (81) to uniquely determine $p_2^*(t)$ as an implicit function of the degree of product differentiation t.[13] This allows us to compare welfare under the two market structures. Welfare under monopoly is given by

$$W^M = p_2^M(1-p_2^M) + \int_0^1 [v(p_2^M) + \frac{t}{2}\sigma]d\sigma = \frac{3}{8} + \frac{t}{4}. \tag{82}$$

It is independent of γ because no consumer purchases in advance. In a price discrimination equilibirum (p_1^*, p_2^*) welfare is

$$W^* = p_1^* q_1^* \sigma_W^* + p_2^* q_2^* (1-\sigma_W^*) + \int_0^{\sigma_W^*} [v(p_1^*) + t(\gamma - \frac{1}{2})\sigma]d\sigma + \int_{\sigma_W^*}^1 [v(p_2^*) + \frac{t}{2}\sigma]d\sigma \tag{83}$$

The prices (p_1^*, p_2^*) must satisfy the equilibrium conditions (80) and (81). For $\gamma \to \frac{1}{2}$, it follows from (80) that $p_1^* \to 0$, and (81) implies

$$3(p_2^*)^3 - 7(p_2^*)^2 + (3+2t)p_2^* - t = 0. \tag{84}$$

The welfare difference becomes

$$\Delta W \equiv W^M - W^* \to \frac{(p_2^*)^2 - (\frac{1}{2})^2}{2} - \frac{1-(1-p_2^*)^2}{4t} p_2^* \left(3p_2^* - 2\right). \tag{85}$$

As $t \to \infty$, (84) implies $p_2^* \to \frac{1}{2}$, and (85) implies $\Delta W \to 0$. In the following we show that ΔW converges to zero from above for $t \to \infty$. Applying the Implicit Function Theorem to (84) gives

$$\frac{dp_2^*}{dt} = -\frac{2p_2^* - 1}{9(p_2^*)^2 - 14p_2^* + 3 + 2t} = \frac{3(p_2^*)^3 - 7(p_2^*)^2 + 3p_2^*}{[9(p_2^*)^2 - 14p_2^* + 3]t + 2t^2} \tag{86}$$

[13] The above analysis implicitly assumed that $v(p_1) - \frac{t}{2}\sigma \geq 0$ for all consumers who purchase in advance. One might be concerned that, in analogy to Assumption (A1) for the case with unitary demands, this condition can only be satisfied when $v(0)$ is sufficiently large. Note however that for $\gamma \to \frac{1}{2}$ it follows from $\sigma_W \to \frac{2}{t}[v(p_1) - v(p_2)]$ that for all $\sigma \leq \sigma_W$, $v(p_1) - \frac{t}{2}\sigma \geq v(p_1) - \frac{t}{2}\sigma_W = v(p_2) \geq 0$. This shows that when preferences are sufficiently uncertain, all consumers obtain positive utility from participating in a price discrimination equilibrium, independently of the degree of product differentiation t.

where the last equality used (84). Hence from $\lim_{t\to\infty} p_2^* = \frac{1}{2}$ it follows that $\lim_{t\to\infty} t\frac{dp_2^*}{dt} = 0$ and $\lim_{t\to\infty} t^2\frac{dp_2^*}{dt} = \frac{1}{16}$. From (85) we get

$$\begin{aligned} t^2\frac{d\Delta W}{dt} &= p_2^* t^2 \frac{dp_2^*}{dt} - \frac{t}{4}\frac{dp_2^*}{dt}\left[2(1-p_2^*)p_2^*(3p_2^*-2) + (1-(1-p_2^*)^2)(6p_2^*-2)\right] \\ &\quad + \frac{1-(1-p_2^*)^2}{4} p_2^*(3p_2^*-2) \end{aligned} \tag{87}$$

and $\lim_{t\to\infty} t^2\frac{d\Delta W}{dt} = \frac{1}{32} - \frac{3}{64} < 0$. Hence for $t \to \infty$, ΔW converges to zero from above which, by continuity, implies that for γ sufficiently close to $\frac{1}{2}$ and t sufficiently large it must hold that $W^M > W^*$.

We can therefore conclude that if demand is sufficiently uncertain and products are sufficiently differentiated, then in any price discrimination equilibrium, competition has a negative effect on welfare although it increases the total quantity sold. This extends our main result (Corollary 1) to the case where consumers have non-unitary demands. It shows that the negative effect of competition on the intertemporal allocation of sales can outweigh its positive effect on the total quantity sold. This happens when demand uncertainty is strong and products are sufficiently differentiated because in this case competition in the advance market is intensified whereas competition in the spot market is mitigated.

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Figures

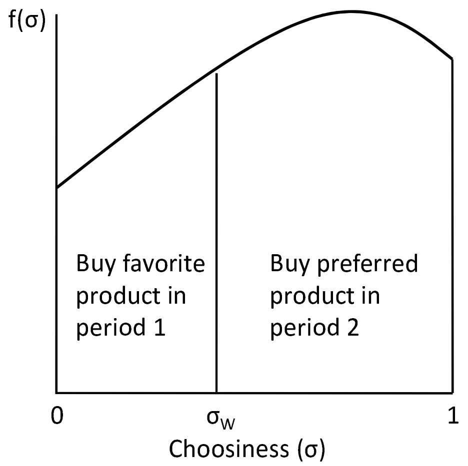

Figure 1: Consumer behavior under monopoly: Consumers with low choosiness $\sigma < \sigma_W$ buy their favorite product in period 1, whereas consumers with high choosiness $\sigma \geq \sigma_W$ postpone their purchase until period 2 in order to buy their preferred product.

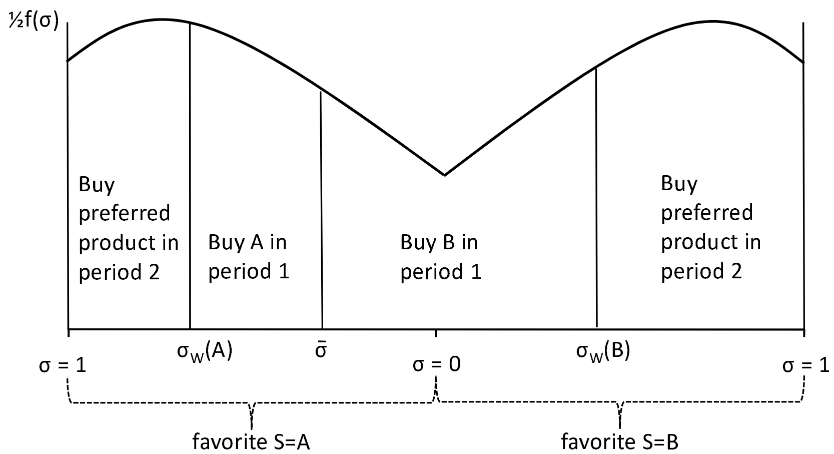

Figure 2: Consumer behavior under competition: The most choosy consumers postpone their purchase until period 2. Less choosy consumers select between their favorite and the cheaper product in period 1. In the case depicted, $p_{1,A} > p_{1,B}$. Firm A's first period demand consists of the consumers whose favorite product is A and who are sufficiently choosy not to be attracted by the less expensive product B.

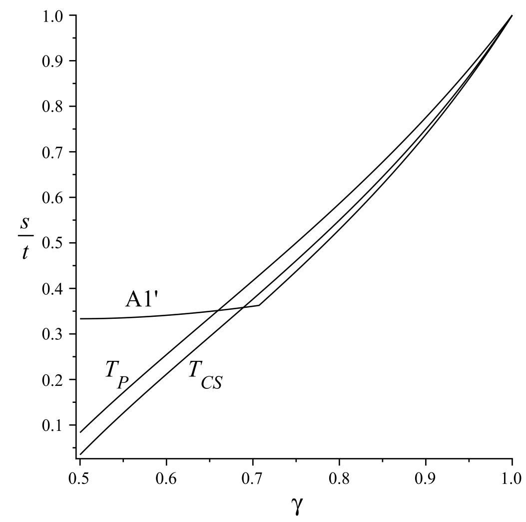

Figure 3: Price and surplus comparison for the uniform case: The parameters satisfying Assumption (A1') lie above the kinked curve. Competition increases spot prices (decreases aggregate consumer surplus) in the area below the threshold T_P (T_{CS}).

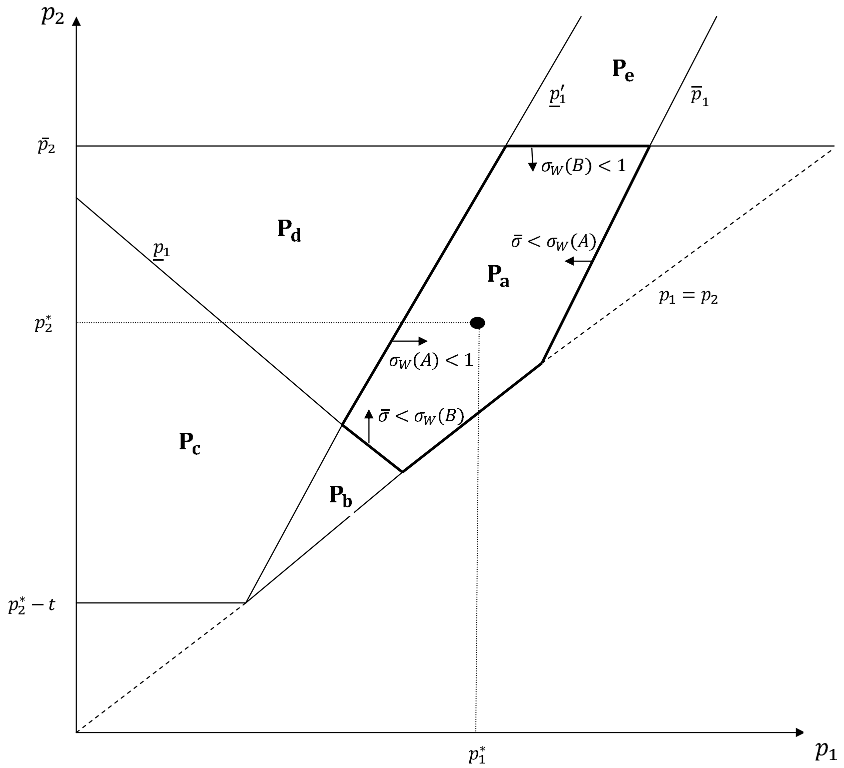

Figure 4: Appendix B: Proof of existence of equilibrium. The figure shows all possible deviations (p_1, p_2) from the equilibrium price schedule (p_1^*, p_2^*).