Competition amongst Contests *

Ghazala Azmat †
and
Marc Möller ‡

Abstract

When several contests compete for the participation of a common set of players, a contest's allocation of prizes not only induces incentive effects but also participation effects. Our model predicts that an increase in the sensitivity with which contest outcomes depend on players' efforts makes flatter prize structures more attractive to participants. In equilibrium, contests that aim to maximize the number of participants will award multiple prizes if and only if this sensitivity is sufficiently high. Moreover, the prize awarded to the winner is decreasing in the contests' sensitivity. We provide empirical evidence from professional road running using race–distance as a measure of sensitivity. We show that steeper prize structures are more attractive to top ranked runners in longer, i.e. less sensitive, races. In line with our theory, longer races do in fact offer steeper prize structures.

*We are especially grateful to Michele Piccione and Alan Manning for their guidance and support. We also thank Jean–Pierre Benoit, Michael Bognanno, Jordi Blanes i Vidal, Pablo Casas–Arce, Vicente Cuñat, Maria–Ángeles de Frutos, Christopher Harris, Nagore Iriberri, Belen Jerez, Benny Moldovanu, Michael Peters and participants at various seminars and conferences for valuable discussions and suggestions. Additionally, the comments of two anonymous referees were highly appreciated. We thank Zeeshan Azmat for his helpful research assistance. The first author acknowledges financial support from Spanish Ministry of Education grant ECO2008-06395-C05-01 and the support of the Barcelona GSE Research Network and of the Government of Catalonia. The second author acknowledges financial support from the Spanish Ministry of Education grants RYC-2008-03371 and ECO-2008-02738. Finally, a special thanks to Julio d'Escriván for sharing his insights on long distance running.

†Department of Economics and Business, University Pompeu Fabra; ghazala.azmat@upf.edu.
‡Department of Economics, University Carlos III Madrid; mmoller@eco.uc3m.es.

JEL *classification*: D44, J31, D82.
Keywords: Contests, allocation of prizes, participation, incentives

1 Introduction

In 2008 the three major providers of game consoles, Microsoft, Nintendo, and Sony, each announced a contest to develop video game applications that could be operated on their platforms. In order to attract individual developers, the companies offered cash and retail prizes as well as royalties.[1] Due to technical and legal constraints, individual game developers were restricted to present their ideas to only one of the three contests. There are many other examples where contests compete for participants. Architectural competitions contend for design proposals. Auctions compete for bidders. Publishing houses offer promotional contests and sweepstakes in order to increase magazine subscriptions. Big city Marathons compete for runners. More generally, firms aim to attract workers by offering labor tournaments, as well as wages.[2]

In this article, we focus on one dimension of this competition; the competition in prize structures. In particular, we are interested in the dependence of participants' contest choice on the contests' allocation of prizes and, in turn, its implications for the contests' optimal prize structure. Although the incentive effects of a prize structure for a given set of players have been well understood in the existing literature, the participation effects, when the set of players is endogenous have not been considered so far.

We study and empirically test a complete information model in which two identical contests compete for the participation of a given set of homogeneous, risk neutral players. Players make their contest choice simultaneously and contingent on the contests' allocation of prizes. Once the set of participants has been determined in each contest,

[1]For example, with the aim to double its game library by the end of the year, Microsoft announced its"Dream-Build-Play Contest" which awarded a total of $75000 and a potential publishing contract to the developers of the best games playable on its Xbox 360.

[2]Whereas in some of these examples, e.g. labor tournaments, players can participate in only one of the available contests, in others, e.g. sweepstakes, players can participate in many contests simultaneously. For our analysis it is only important that players cannot take part in *all* available contests.

players exert effort in order to win prizes. A player's likelihood to win a contest is increasing in his own effort but decreasing in the efforts of his rivals.

The relationship between the contests' outcome and players' efforts is key in our model. Contests can be characterized by how sensitive the outcome is with respect to efforts. For example, in an auction, the bidder with the highest bid wins with certainty; in a R&D contest, results are more uncertain, but developers who exert high efforts are more likely to win; and in sweepstakes, winning probabilities are approximately independent of efforts and are the same for every agent.

We show that the players' contest choice crucially depends on how sensitive a contest's outcome is with respect to individual efforts. Our results show that an increase in sensitivity will decrease the relative attractiveness of steep prize structures as compared to flat prize structures. When outcomes are sufficiently sensitive, contests that award multiple prizes will attract more participants than contests that implement the winner–takes-all principle. The explanation for this result is that the award of multiple prizes mitigates competition amongst players and competition is stronger the more sensitive outcomes are with respect to players' efforts. For the same reason, multiple prizes become more attractive as the number of potential participants increases.

The implications for the contests' optimal allocation of prizes depend on the organizers' objectives. In reality, contests often differ with respect to their objectives. For example, although R&D contests and auctions typically aim to maximize players' efforts/bids, promotional contests like the Reader's Digest sweepstakes and student competitions like the International Science Olympiads seek to maximize the number of participants.

When contests aim to maximize participation, the implications of the above results for the contests' equilibrium prize structure are straight forward. In particular, multiple prizes will be awarded if and only if the sensitivity of outcomes with respect to efforts is sufficiently large. Moreover, the stronger the influence of a player's effort on his likelihood of winning, the smaller is the fraction of prize money awarded to the winner.

When turning our attention to the maximization of aggregate effort, we find that incentive effects outweigh the above participation effects. When contest organizers aim to maximize aggregate effort, in equilibrium contests will therefore award their entire prize budget to the winner.

Overall, our results are fairly general. They hold for an arbitrary number of players, any number of prizes and easily generalize to models with more than two contests. Our findings help to explain why in reality some contests implement the winner–takes–all principle whereas others choose to award multiple prices. For example, the Progressive Insurance Automotive X PRIZE awards a single prize of $10 million for the development of a super fuel–efficient vehicle. Similarly Reader's Digest promotional contests usually award one "Super Grand Prize". In contrast, the New York Marathon offers as much as twenty different prizes. According to our theory, the variations in the allocation of prizes are caused by differences in the contest organizers' objectives and in the sensitivity with which outcomes depend on players' efforts.

We provide empirical evidence in support of our theory using data from professional road running. We argue that as the race distance increases, outcomes become more random and hence less sensitive to runners' efforts.[3] Our empirical results indicate that the participation of top ranked runners is indeed influenced by the contests' choice of prize structure. In line with our theory, in long races steeper prize structures are relatively more attractive to top runners than in short races. For example, when moving from a completely flat prize structure to a winner–takes–all contest, a representative Marathon from our sample would gain an additional five top runners. In contrast, for shorter races we find no significant relationship between prize structures and participation. Hence if race organizers care about attracting top runners, longer races are more likely to exhibit steeper prize structures than shorter races. Using various measures of prize structure and controlling for several important factors, we do indeed find that as the race distance increases, there is a monotonic increase in the prize structure's concentration towards the first prize. For example, the Herfindahl concentration index, based on the first three prizes, increases by almost 4% when the race moves from 5km to 42km which is consistent with as much as a 8% increase in the prize awarded to the winner or a 48% increase in the difference between first and second prize.

We also consider the relationship between prize structures and winning perfor-

[3]Although in a 5km race the prediction of the winner based on past performance turns out to be correct in 43% of the cases, this number reduces to 20% for a marathon. For details see Section 5.

mance. Steeper prize structures lead to higher efforts irrespective of the race–distance. However, their effects on the participation of top runners depends on race distance, in the way described above. Hence in long races, steeper prize structures should have a more positive effect on winning performance than in short races. Although we do find some suggestive evidence for this relationship, these last results fail to be statistically significant.

The article is organized as follows. In the remainder of this section we review the related theoretical literature. A discussion of the empirical literature is referred to Section 5. Section 2 introduces the theoretical model. Section 3 contains our main result about the dependence of players' contest choice on the contests' prize structures. In Section 4 we consider the implications for the contests' optimal prize structure while empirical evidence is presented in Section 5. Section 6 concludes. All proofs are contained in the Appendix.

Related literature

The existing literature on contest design has focused on single contests with an exogenously given set of participants. In their seminal article, Moldovanu and Sela (2001) show that the optimal allocation of prizes depends critically on the shape of players' cost of effort functions. Multiple prizes become optimal when the costs of effort are sufficiently convex. Multiple prizes have also been justified by players' risk aversion (Krishna and Morgan (1998)) and players' heterogeneity (Szymanski and Valletti (2005)) but under the restrictive assumption that the number of players is small ($N \leq 4$). Most articles provide arguments for the use of a single (Clark and Riis (1998b), Glazer and Hassin (1988)) or large (Rosen (2001)) first prize or few prizes (Barut and Kovenock (1998)).[4] Endogenizing participation by allowing for the existence of several contests provides us with a new argument for the wide spread emergence of multiple prizes.

Although some articles endogenize the set of participants, they maintain the focus

[4]Other issues considered by this literature include optimal effort dependent rewards (Cohen et al. (2008)), simultaneous versus sequential designs (Clark and Riis (1998a)), the splitting of a contest into sub–contests (Moldovanu and Sela (2007)), and optimal seeding in elimination tournaments (Groh et al. (2008)).

on a single contest. Taylor (1995) and Fullerton and McAfee (1999), for example, study how the set of participants, and hence the expected winning performance, in a research tournament varies with its entry fee.

Competition for participants has attracted some attention in the literature on auction and mechanism design.[5] McAfee (1993), Peters and Severinov (1997) and Burguet and Sakovics (1999) for example, consider models in which auctions compete for bidders. However, while in our model contests compete via their prize allocation, in these articles, prizes, i.e. the object(s) on sale, are fixed and auctions compete by using their reservation price. More related, Moldovanu et al. (2008) consider quantity competition between two auction sites. Although their model is different in its setup it shares a common feature with ours. In the same way in which in our model contests increase participation by awarding multiple prizes (at the cost of undermining incentives), in their model auctions increase the number of bidders by raising their supply (at the cost of lowering prices).

Finally, the literature on labor tournaments is also relevant. Lazear and Rosen (1981), Green and Stokey (1983), Nalebuff and Stiglitz (1983), and Mookherjee (1984) have shown that the introduction of some form of contest among workers could provide optimal incentives to exert effort inside a firm. While Green and Stokey (1983) and Mookherjee (1984) take the set of workers as exogenously given, Lazear and Rosen (1981) and Nalebuff and Stiglitz (1983) assume a competitive labor market in which each firm hires a fixed number of workers. While in these articles each worker faces a fixed number of opponents, our results are driven by the fact that a player's number of opponents itself depends on the contest design.

[5] An auction can be understood as one particular example for a contest. Bidders' bids are the analog to players' efforts and the auction format determines the players' costs of effort. For example, an all–pay auction is often called a "perfectly discriminating contest" because the bidder (player) with the highest bid (effort) wins with certainty.

2 The model

We consider two contests, $i \in \{1, 2\}$, and $N \geq 3$ players.[6] Contests are identical and face the same prize budget V. Each contest i chooses a *prize structure*, i.e. a vector of non–negative real numbers $v_i = (v_i^1, v_i^2, \ldots, v_i^N)$ such that v_i^m is (weakly) decreasing in m and $\sum_{m=1}^{N} v_i^m = 1$.[7] The m'th prize awarded by contest i has the value $v_i^m V$. Note that in order to focus on the participation effects implied by a contest's prize structure we rule out the possibility that contests pay participants for attendance. Our results remain valid when we allow for attendance pay (see discussion at the end of Section 4). It will become clear that the contests' competition in prize structures resembles price competition a la Bertrand. As a consequence our results generalize to an arbitrary number of contests.

In order to identify competition for participants as the reason for the emergence of multiple prizes, we assume that players are identical, risk neutral, and have linear costs of effort. Under these assumptions, a contest with an exogenously given set of participants would award its entire prize budget to the winner. Each player can participate in, at most, one of the two contests because of time or other resource constraints. In each contest, participants exert effort in order to win a prize. A player who enters contest i, exerts effort $e_n \geq 0$, and wins the m'th prize, receives the payoff $U_n^i = v_i^m V - Ce_n$.

The parameter $C > 0$ denotes the players' constant marginal cost of effort. Assuming that players have a zero outside option we can normalize, without a loss of generality, by setting $V = C = 1$.

The timing is as follows. First, contests simultaneously choose their prize structures. We denote the subgame, which starts after contests have announced the prize structures v_1 and v_2, as the (v_1, v_2) entry game. Second, players simultaneously decide which contest to enter.[8] Third, upon observing the number of opponents, players

[6]For $N = 2$ our results remain valid when formulated in a weak rather than a strict way.

[7]The assumption that prizes are (weakly) decreasing is standard in the literature on contest design. Increasing prizes lead to sub–optimal levels of effort.

[8]While our results remain unchanged when contests are allowed to choose their prize structure sequentially, the assumption that entry takes place simultaneously is important as it rules out coordination. Note however that when entry is sequential, contests have an incentive to conceal the entry

simultaneously choose their effort levels.[9]

In order to determine the contests' outcome we employ Tullock's (1980) widely used contest success function (see Skaderpas (1996) for axiomatization and Nti (1997) for properties). In particular, letting $\mathcal{N}_i$ denote contest i's set of participants and N_i its cardinality, prizes in contest i are distributed as follows. The probability that player $n \in \mathcal{N}_i$ wins the first prize v_i^1 is given by

$$p_n^1 = \frac{(e_n)^s}{\sum_{k\in\mathcal{N}_i}(e_k)^s}. \tag{1}$$

Conditional on player m winning the first prize, player n wins the second prize v_i^2 with probability

$$p_{n|m}^2 = \frac{(e_n)^s}{\sum_{k\in\mathcal{N}_i-\{m\}}(e_k)^s}. \tag{2}$$

Hence the (unconditional) probability that player n wins the second prize is given by

$$p_n^2 = \sum_{m\in\mathcal{N}_i-\{n\}} p_m^1 p_{n|m}^2. \tag{3}$$

The probabilities to win higher prizes can be defined accordingly. Note that each player wins the contest with positive probability and that this probability is increasing in his own effort and decreasing in the efforts of his rivals. The parameter $s \geq 0$ is the same for both contests. It measures the sensitivity of the contests' outcome with respect to players' efforts. For $s \to 0$ the contests' outcome is independent of players' efforts and every player is equally likely to win. The sensitivity of outcomes with respect to players' efforts is increasing in s. For $s \to \infty$ the contests' outcome is determined

of earlier players from later players. Hence our results remain valid under sequential entry as long as players cannot communicate with each other.

[9]While in sports contests and labor tournaments players observe the number of opponents directly, in architectural contests this information is often supplied by the organizers. Myerson and Wärneryd (2006) show that contest organizers have an incentive to do so because expected aggregate effort in a contest with a commonly known number of participants is higher than in a contest with the same expected participation but uncertainty about the number of players. In other contests, e.g. procurement and promotional contests, the exact number of participants might be unknown but players may have fairly accurate expectations due to the repeated nature of these contests. So far only a few articles consider contests in which the number of participants is uncertain (see Münster (2006) and references therein).

entirely by players' efforts. In this case, the player with the highest effort wins the first prize, the player with the second highest effort wins the second prize, and so on. In the next section we will show that the players' contest choice depends crucially on the parameter s.

As participation is assumed to be costless, players prefer to participate in *some* contest rather than to not participate at all. Player n will therefore enter contest 1 with probability $q_n(v_1, v_2) \in [0, 1]$ and contest 2 with probability $1 - q_n(v_1, v_2)$. As players are identical we restrict our attention to the symmetric equilibria of the entry game, where $q_n(v_1, v_2) = q^*(v_1, v_2)$ for all players.[10]

Although players always choose contests and effort in order to maximize their expected payoff, with respect to the contest organizers we will distinguish between different objectives. However, before we turn to the optimal contest design, in the next section we consider the players' contest choice for a given choice of prize structures (v_1, v_2).

3 Contest choice

In this section we consider players' contest choice given the prize structures v_1 and v_2. The probability $q^*(v_1, v_2)$ with which players enter contest 1 in equilibrium can be derived as follows. We first consider the effort choice for all players $n \in \mathcal{N}_i$ participating in contest i given the prize structure v_i. This allows us to determine a player's expected payoff in contest i conditional on contest i having N_i participants, $E[U_n^i|N_i]$. Next, assuming that all players enter contest 1 with the same probability q, we can then obtain a player's expected payoffs from entering contest 1 or contest 2, respectively:

$$E[U_n^1] = \sum_{m=1}^{N} \tbinom{N-1}{m-1} q^{m-1}(1-q)^{N-m} E[U_n^1|m] \tag{4}$$

$$E[U_n^2] = \sum_{m=1}^{N} \tbinom{N-1}{m-1} (1-q)^{m-1} q^{N-m} E[U_n^2|m]. \tag{5}$$

[10] We discuss the possibility of asymmetric equilibria at the end of Section 3.

The equilibrium probability $q^*(v_1, v_2)$ will be the unique solution of the equation

$$\Delta(q) \equiv E[U_n^1] - E[U_n^2] = 0. \tag{6}$$

Below we consider the players' contest choice for different values of s. The main insight of this section is that an increase in the sensitivity with which outcomes depend on players' efforts makes flatter prize structures relatively more attractive to participants than steeper prize structures. Before we derive this result more generally we consider the following three–player example in order to build some intuition.

An example

Suppose that $N = 3$, and contest 1 chooses to implement the winner–takes–all principle, i.e. $v_1 = (1, 0, 0)$, whereas contest 2 awards two prizes, i.e. $v_2 = (k, 1-k, 0)$ where $k \in [\frac{1}{2}, 1)$.[11] Consider first the extreme case where $s = 0$. In this case the contests' outcome is completely random and players will exert zero effort. In contest i a player wins the first prize when he happens to be the only participant. Otherwise his expected payoff is $\frac{1}{N_i}$ with N_i being the number of participants of contest i. Hence if every player participates in contest 1 with probability q then each player expects the payoff

$$E[U_n^1] = (1-q)^2 \cdot 1 + 2q(1-q) \cdot \frac{1}{2} + q^2 \cdot \frac{1}{3} \tag{7}$$

from entering contest 1 whereas the expected payoff from entering contest 2 is given by

$$E[U_n^2] = q^2 \cdot k + 2q(1-q) \cdot \frac{1}{2} + (1-q)^2 \cdot \frac{1}{3}. \tag{8}$$

Suppose that each contest expects the same number of participants, i.e. $q = \frac{1}{2}$. Then $E[U_n^1] > E[U_n^2]$ and all players strictly prefer to enter contest 1. Even though players are risk neutral and the likelihood to meet an opponent is identical in both contests, players prefer the winner–takes–all contest as it offers a higher award in the case where a player turns out to be the only participant. Hence in equilibrium contest 1 has to expect a higher number of participants than contest 2. Indeed, in equilibrium $q^* = \frac{2-\sqrt{6k-2}}{3(1-k)} > \frac{1}{2}$

[11] We are indebted to one of the anonymous referees for suggesting this example.

and the expected number of participants in contest 2 is decreasing in $1-k$, i.e. the fraction of prize money contest 2 awards as a second prize.

Next consider the other extreme case where $s \to \infty$. As before, in each contest a player will exert zero effort and win the first prize if he happens to be the only participant. If $N_i > 1$ players have entered the same contest i then in every equilibrium of the simultaneous effort choice game each player expects the payoff $v_i^{N_i}$ (see Barut and Kovenock (1998)). For $s \to \infty$ competition is so strong that players expect zero payoffs when the number of participants is strictly larger than the number of prizes. Hence a player's expected payoff from entering the winner–takes all contest is given by

$$E[U_n^1] = (1-q)^2 \cdot 1 \tag{9}$$

whereas the expected payoff from entering the two prize contest is given by

$$E[U_n^2] = q^2 \cdot k + 2q(1-q) \cdot (1-k). \tag{10}$$

Note that for $q = \frac{1}{2}$ it holds that $E[U_n^1] = \frac{1}{4} < \frac{2-k}{4} = E[U_n^2]$. Even though the winner–takes–all contest offers a higher award in the case where a player turns out to be the only participant, all players strictly prefer the two–prize contest. The reason for this is the fact that in the two–prize contest competition is mitigated. While in the winner–takes–all contest, players expect zero payoffs when they meet one opponent, in the two–prize contest their expected payoff is strictly positive and given by the second prize, $1-k$. Because for $q = \frac{1}{2}$ the likelihood to meet one opponent is twice as high as the likelihood to meet no opponent at all, this advantage of the two–prize contest outweighs its disadvantage in the case where a player turns out to be the only participant. Indeed, in equilibrium $q^* = \frac{2-k-\sqrt{1-k+k^2}}{3(1-k)} < \frac{1}{2}$ and the expected number of participants in contest 2 is strictly increasing in $1-k$, i.e. the fraction of prize money contest 2 awards as a second prize.

General results

The above example suggests that players prefer to enter single–prize contests when the sensitivity s of outcomes with respect to effort is sufficiently low, whereas multiple–prize contests are more attractive when s is sufficiently high. In the remainder of this section we generalize this insight to the case of an arbitrary number of players and prizes.

We start our analysis by considering low values of s. As payoffs are continuous with respect to s we can focus on the limiting case where $s \to 0$ while our results remain valid as long as s is sufficiently small. For $s \to 0$, each participant $n \in \mathcal{N}_i$ of contest i is equally likely to win any of the prizes $v_i^1, \ldots, v_i^{N_i}$, irrespective of the players' efforts. Hence in the limit all players will exert zero effort and player n's expected payoff in contest i conditional on contest i having N_i participants is

$$\lim_{s \to 0} E[U_n^i | N_i] = \bar{v}_i(N_i) \tag{11}$$

for all $n \in \mathcal{N}_i$, where

$$\bar{v}_i(m) = \frac{1}{m} \sum_{m'=1}^{m} v_i^{m'} \tag{12}$$

denotes the average of the m highest prizes in contest i.[12] In the Appendix we prove the following:

Proposition 1 *Suppose that s is sufficiently small. If $v_i \neq (\frac{1}{N}, \frac{1}{N}, \ldots, \frac{1}{N})$ for some $i \in \{1, 2\}$ then the (v_1, v_2) entry game has a unique symmetric equilibrium $q^*(v_1, v_2)$. The expected number of participants in contest i is strictly larger than in contest j if and only if $P^0(v_i) > P^0(v_j)$ where $P^0(v) \equiv \sum_{m=1}^{N} \binom{N-1}{m-1} \bar{v}(m)$.*

Note that when players enter both contests with equal probability $q = \frac{1}{2}$ then a player's expected payoff in contest i is given by $\frac{1}{2^{N-1}} P^0(v_i)$. Hence if $q = \frac{1}{2}$ and $P^0(v_1) > P^0(v_2)$ then players expect a higher payoff in contest 1 than in contest 2. As in the three–player example in equilibrium it therefore has to hold that $q^*(v_1, v_2) > \frac{1}{2}$.

$P^0(v_i)$ is a weighted sum over the terms $\bar{v}_i(m)$. As $\bar{v}_i(m)$ denotes the average of the m highest prizes in contest i, a transfer of prize money from a smaller prize v_i^{m+1} to a larger prize v_i^m increases $\bar{v}_i(m)$ without decreasing $\bar{v}_i(m+1)$. In particular, $P^0(v_i)$

[12] Note that in the case where the number of participants falls short of the number of strictly positive prizes, the remaining prize money is retained by the organizer. Proposition 1 remains valid as long as the organizer retains a positive fraction of this money.

is maximized by setting $v_i = (1, 0, \dots, 0)$. Hence Proposition 1 implies that when the sensitivity of outcomes with respect to efforts is sufficiently small, a winner–takes–all contest attracts more participation than any other contest. Player's prefer a winner–takes–all contest because it minimizes the chance that some share of the prize money is retained by the organizer which happens whenever the number of participants falls short of the number of strictly positive prizes.

Note that the existence of *several* contests is crucial for this result. In a monopolistic model with a single contest, risk neutral players would be indifferent with respect to different prize structures when $s \to 0$. The result is driven by the fact that players face uncertainty about the number of rivals they will encounter. In a model with a single contest and an exogenously given set of players this uncertainty is absent.

We now turn our attention to the opposite case where outcomes are extremely sensitive with respect to players' efforts. When s is large, small increases in a player's effort lead to large increases in his likelihood of winning. In the extreme case, $s \to \infty$, the contests' outcome is determined entirely by the players' efforts. In particular, the player with the highest effort wins the first prize (with certainty), the player with the second highest effort wins the second prize and so on. An important example for this type of contest is an all–pay auction.[13] Barut and Kovenock (1998) have characterized the equilibria of an all–pay auction with identical risk–neutral players and several not necessarily identical prizes. For $s \to \infty$ their results apply here. Barut and Kovenock (1998) show that in a contest with N_i participants in every equilibrium each player $n \in \mathcal{N}_i$ obtains the expected payoff

$$\lim_{s \to \infty} E[U_n^i | N_i] = v_i^{N_i}. \tag{13}$$

Due to the high sensitivity of outcomes with respect to efforts, the N_i participants compete so fiercely for the first N_i prizes that none of them is able to obtain a higher (expected) payoff than $v_i^{N_i}$. All of the potential gains in prize money $v_i^1 - v_i^{N_i}$ are spent in form of effort costs. Using this result, in the Appendix we prove the following:

[13] In an all–pay auction all bidders pay their bids and then the goods are allocated according to the ranking of bids. In the literature on contest design, all–pay auctions have been frequently used as a modeling device (see for example Moldovanu and Sela (2001 and 2006)).

Proposition 2 *Suppose that s is sufficiently large. If $v_i \neq (\frac{1}{N}, \frac{1}{N}, \ldots, \frac{1}{N})$ for some $i \in \{1, 2\}$ then the (v_1, v_2) entry game has a unique symmetric equilibrium $q^*(v_1, v_2) \in (0, 1)$. The expected number of participants in contest i is strictly larger than in contest j if and only if $P^\infty(v_i) > P^\infty(v_j)$ where $P^\infty(v) \equiv \sum_{m=1}^{N} \binom{N-1}{m-1} v^m$.*

Analog to the case where $s = 0$, when all players enter both contests with equal probability ($q = \frac{1}{2}$), a player's expected payoff in contest i is given by $\frac{1}{2^{N-1}} P^\infty(v_i)$. Hence when $P^\infty(v_1) < P^\infty(v_2)$ then in equilibrium it has to hold that $q^*(v_1, v_2) < \frac{1}{2}$.

$P^\infty(v_i)$ is the sum over contest i's prizes v_i^m weighted by the binomial coefficients $\binom{N-1}{m-1}$. As $N \geq 3$, the binomial coefficient for $m = 1$ is strictly smaller than the coefficients for $m \in \{2, N-1\}$. Hence when contest 1 sets $v_1^1 = 1$, it holds that $P^\infty(v_1) < P^\infty(v_2)$ for all $v_2 \neq (1, 0, \ldots, 0)$. Proposition 2 therefore implies that when the sensitivity of outcomes with respect to efforts is sufficiently large, a winner–takes–all contest attracts less participation than any other contest.

Players prefer a single first prize when s is small but multiple prizes when s is large. The reason for this is that for large values of s, competition amongst players is very strong. In order to win a contest a player has to exert more effort than his rivals. As players are relatively unlikely to be the only participant in the contest of their choice they prefer contests that mitigate competition by awarding multiple prizes.

We now derive the threshold $\bar{s}$ that determines whether single prize contests attract more or less participants than multiple prize contests. We want to understand how this threshold depends on the total number of potential participants N. To keep the analysis as simple as possible for the remainder of this section we assume that contests are restricted to award at most two prizes, i.e. $v_i = (v_i^1, 1 - v_i^1, 0, \ldots, 0)$ for $i = 1, 2$. In the Appendix we show that our results remain valid when this assumption is relaxed.

For $0 < s < \infty$ and $N_i \geq 2$ each player $n \in \mathcal{N}_i$ who participates in contest i chooses effort e_n in order to solve

$$\max_{e_n \geq 0} \left[p_n^1(e_n, e_{-n}) v_i^1 + p_n^2(e_n, e_{-n})(1 - v_i^1) - e_n \right]. \tag{14}$$

A symmetric pure strategy equilibrium can be derived by calculating the first order condition and substituting $e_n = e^*$ for all $n \in \mathcal{N}_i$. We find that

$$e^* = \frac{s}{N_i}\left(\frac{N_i - 1}{N_i} - \frac{1 - v_i^1}{N_i - 1}\right) \tag{15}$$

and in equilibrium each player $n \in \mathcal{N}_i$ expects the payoff

$$E[U_n^i | N_i] = \frac{1}{N_i}\left(1 - s\left(\frac{N_i - 1}{N_i} - \frac{1 - v_i^1}{N_i - 1}\right)\right). \tag{16}$$

Note that this equilibrium is unique and it exists if and only if $s \leq \frac{N}{N-1}$. A player's equilibrium effort strictly increases in s whereas his expected payoff decreases. The loss in payoff is higher the larger is the first prize v_i^1. As a consequence in the Appendix we are able to prove the following:

Proposition 3 *Suppose that* $s \in (0, \frac{N}{N-1}]$. *If* $v_i = (v_i^1, 1 - v_i^1, 0, \ldots, 0)$ *with* $v_1^1 > v_2^1$ *then the* (v_1, v_2) *entry game has a unique symmetric equilibrium* $q^*(v_1, v_2) \in (0, 1)$. *The expected number of participants in contest* 1 *is strictly smaller (larger) than in contest* 2 *if and only if* $s > (<)\ \bar{s}$ *where*

$$\bar{s} \equiv \left(\sum_{m=1}^{N-1} \frac{\binom{N-1}{m}}{m(m+1)}\right)^{-1} \in (0, 1) \tag{17}$$

is strictly decreasing in N *with* $\lim_{N\to\infty} \bar{s} = 0$.

Proposition 3 shows that a single prize contest attracts more participation than a multiple prize contest if and only if $s < \bar{s}$. As the number of potential participants increases, the threshold $\bar{s}$ decreases and multiple prize contests become more attractive. This is because the higher the number of potential opponents the more important it becomes for the players to choose a contest in which competition is mitigated. As the number of potential participants N grows infinitely large the threshold $\bar{s}$ converges towards zero. Hence as long as $s > 0$ so that the contests' outcome may be influenced at least to some extent by the players' efforts, a contest that awards multiple prizes will attract more players than a winner–takes–all contest if N is sufficiently large.

Note that for $N \to \infty$ players expect to meet an infinite number of opponents in each contest. This implies that for $N \to \infty$ players are indifferent with respect to the contests' prize structures and $q^*(v_1, v_2) \to \frac{1}{2}$. Hence for $N \to \infty$ participation effects become negligible. Participation effects are strongest when the number of potential participants is small.

Finally let us comment on the possibility of asymmetric equilibria. In our analysis we have assumed that in equilibrium all players behave symmetrically by entering contest 1 with the same probability $q^*(v_1, v_2)$. This assumption is reasonable as players are identical. However, there do also exist asymmetric equilibria. For instance, some players may enter contest 1 with certainty ($q = 1$) while others enter contest 2 ($q = 0$). To see this consider again the three–player example above. For $s = 0$, two players choosing contest 1 and one player choosing contest 2 constitutes an equilibrium. As participation is higher in contest 1, the prediction of Proposition 1 extends to this asymmetric equilibrium. However, there also exists an asymmetric equilibrium in which one player enters contest 1 and two players enter contest 2 so that Proposition 1 fails to hold. For $s = \infty$ only the latter equilibrium exists and Proposition 2 remains valid. Hence our results in this section depend at least to some extent on the assumption that in equilibrium identical players will behave symmetrically. Nevertheless, allowing for asymmetric equilibria, we expect the implications for the contests' choice of prize structure to remain valid if contests contemplate that each of these equilibria will occur with equal probability.

In this section we have shown that players' contest choice depends on the contests' allocation of prizes while empirical evidence for this dependence is contained in Section 5. In the presence of several contests, organizers therefore have to account for the fact that prize structures not only influence players' incentives to exert effort but also their incentives to participate. In the following section we will consider the implications of these participation effects for a contest's optimal allocation of prizes.

4 Equilibrium prize structure

Contests need to attract participants, without participants there is no contest. Referring to the recent contests for European 3G telecom licenses, Paul Klemperer (2002) notes that "a key determinant of success [...] was how well their designs attracted entry

[...]". An indication of the importance of participation is the fact that the ordering of these contests with respect to revenue per population coincides with their ordering with respect to the number of participants. For example while the UK contest had 13 participants and lead a revenue of 642 Euro per capita, those numbers are reduced to 6 participants and 173 Euro for the Netherlands and 3 participants and 44 Euro for Belgium.

In general, contest organizers differ with respect to their objectives. In the above example organizers were interested in the maximization of total revenue. Similarly, science and engineering contests often aim to maximize participants' aggregate effort. On the other hand, in promotional contests, student competitions and sports contests, organizers might have the objective to maximize the number of participants. In this section we derive the contests' equilibrium prize structure for both of these objectives.

To start with, suppose that contest organizers award prizes in order to maximize the (expected) number of participants. In particular, contest 1 chooses v_1 to maximize $Nq^*(v_1, v_2)$, whereas contest 2 chooses v_2 to maximize $N(1-q^*(v_1, v_2))$. In this case the implications of our results in Section 3 are immediate. For example, for s sufficiently small, Proposition 1 implies that awarding a single first prize is a strictly dominant strategy for each contest. Hence in equilibrium both contests will award their entire prize budget to the winner. In contrast, for s sufficiently large, Proposition 2 implies that in equilibrium each contest will award N^* identical prizes, where $N^* = \frac{N+1}{2}$ for N odd and $N^* = \frac{N+2}{2}$ for N even, because this prize structure is the unique maximizer of $P^\infty(v)$.

To understand the intuition for this result consider for example the case where $N = 7$. It is immediate from Proposition 2 that in equilibrium both contests will choose the same prize structure and players will enter both contests with equal probability. The likelihood that a player who enters contest i finds himself in a contest with m participants expecting the payoff v_i^m is therefore given by $\frac{1}{2^6}\binom{6}{m-1}$. This likelihood is maximized for $m = N^* = 4$ and players therefore prefer the contest which awards the highest 4th prize. The prize structure that maximizes v_i^4 is given by $v_i = (\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, 0, 0, 0)$. In equilibrium both contests will therefore award four identical prizes.

While contests award a single first prize when s is small, they choose N^* identical prizes when s is large. By restricting the number of prizes that contests are allowed to award, in the Appendix we are able to show that the number of prizes awarded in equilibrium increases monotonically in s.

Finally, a direct consequence of Proposition 3 is that for $s > 0$ in equilibrium contests will award multiple prizes if the number of potential participants is sufficiently high. We summarize these results in the following:

Proposition 4 *Suppose that contest organizers aim to maximize the (expected) number of participants. The prize structure that contests choose in equilibrium depends on the sensitivity of outcomes with respect to players' efforts, s, and the number of potential participants, N:*

1. *If s is sufficiently small, each contest will award a single first prize, i.e. $v_1^* = v_2^* = (1, 0, \ldots, 0)$.*
2. *If s is sufficiently large, each contest will award N^* identical prizes, i.e. $v_1^* = v_2^* = (\frac{1}{N^*}, \ldots, \frac{1}{N^*}, 0, \ldots, 0)$, where $N^* = \frac{N+1}{2}$ for N odd and $N^* = \frac{N+2}{2}$ for N even.*
3. *If $N \geq 6$ and contests are restricted to award at most three prizes, then the equilibrium prize structure is $v_1^* = v_2^* = (1, 0, \ldots, 0)$ for $s \in (0, \bar{s})$, $v_1^* = v_2^* = (\frac{1}{2}, \frac{1}{2}, 0, \ldots, 0)$ for $s \in (\bar{s}, \bar{\bar{s}})$, and $v_1^* = v_2^* = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, 0, \ldots, 0)$ for $s \in (\bar{\bar{s}}, \frac{N}{N-1})$ where $\bar{s}$ and $\bar{\bar{s}}$ are as defined in (17) and (37) respectively and $0 < \bar{s} < \bar{\bar{s}} < 1$.*
4. *If $s > 0$ and N is sufficiently large then contests will award multiple prizes.*

Proposition 4 shows that when organizers aim to maximize participation, and the sensitivity of outcomes with respect to players' efforts is small, contests will award their entire prize budget to the winner. One example that fits these assumptions are promotional contests. In a promotional contest organizers typically aim to maximize the number of participating households and participants may influence their chance of winning only marginally. Indeed one can find many examples for promotional contests that award a single grand prize. For instance, the Home and Garden Television "Dream

Home Giveaway" awards a single prize valued in excess of one million dollars once every year.

Proposition 4 also suggests that equilibrium prize structures depend in a monotone way on the parameter s. As s increases, the number of prizes awarded in equilibrium increases whereas the fraction of prize money awarded to the winner decreases. As an application of our theory, in Section 5 we provide empirical evidence for this relationship using data from professional road running.

Before we do so however, let us now follow most of the literature on contest design by considering contests that aim to maximize players' aggregate effort. Let $s \to \infty$ and suppose that contest i happens to have N_i participants. The expected payoff of player $n \in \mathcal{N}_i$ is given by (13) and in equilibrium each player is equally likely to win any of the first N_i prizes. Hence in equilibrium player n is expected to exert the effort $E[e_n^i|N_i] = \bar{v}_i(N_i) - v_i^{N_i}$ and expected aggregate effort in contest i conditional on contest i having N_i participants is

$$E[\Sigma e^i|N_i] = N_i[\bar{v}_i(N_i) - v_i^{N_i}] = \sum_{m=1}^{N_i}(v_i^m - v_i^{N_i}) \tag{18}$$

Note that for a given number of participants N_i, steeper prize structures lead to higher levels of aggregate effort and aggregate effort is maximized when $v_i = (1, 0, \ldots, 0)$. Ex post, once entry has taken place, aggregate effort would therefore be maximized by awarding a single first prize.

Also note however, that aggregate effort increases in N_i. That is, for a given prize structure, aggregate effort increases in the number of participants. If players enter contest 1 with probability $q(v_1, v_2) \in [0, 1]$ then expected aggregate efforts in contest 1 and contest 2 are given by

$$\begin{aligned} E[\Sigma e^1] &= \sum_{m=2}^{N}[q(v_1, v_2)]^m[1 - q(v_1, v_2)]^{N-m}\tbinom{N}{m}m[\bar{v}_1(m) - v_1^m] && (19)\\ E[\Sigma e^2] &= \sum_{m=2}^{N}[1 - q(v_1, v_2)]^m[q(v_1, v_2)]^{N-m}\tbinom{N}{m}m[\bar{v}_2(m) - v_2^m]. && (20) \end{aligned}$$

If contests aim to maximize (expected) aggregate effort then in equilibrium, contest i will choose v_i to maximize $E[\Sigma e^i]$. As $q(v_1, v_2)$ depends on the contests' prize structures, awarding multiple prizes rather than a single prize has a direct and an indirect effect on aggregate effort. On the one hand, multiple prizes decrease aggregate effort directly through their detrimental effect on incentives to exert effort for a given set of participants. On the other hand multiple prizes increase expected participation and thereby raise aggregate effort indirectly. In the Appendix we show that for $s \to \infty$ the incentive effect outweighs the participation effect, making a single first prize optimal. Because participation effects are strongest for $s \to \infty$ this finding extends to all $s < \infty$. In particular we have the following:

Proposition 5 *Suppose that contest organizers aim to maximize (expected) aggregate effort. The prize structure that contests choose in equilibrium is independent of s and N. Each contest will award a single first prize, i.e. $v_1^* = v_2^* = (1, 0, \dots, 0)$.*

Proposition 5 shows that second and higher order prizes cannot be used to increase expected aggregate effort. The negative incentive effect outweighs the possibly positive participation effect and the overall effect is a reduction in expected aggregate effort. Expected aggregate effort is highest in a single prize contest. If expected aggregate effort was higher in a multiple prize contest then all players would strictly prefer the single prize contest where they expect to win higher prizes at lower levels of effort.

Proposition 5 is important as it provides justification for the literature's focus on an exogenously given set of participants. It shows that when players are homogeneous, risk neutral, and have linear costs of effort, winner–takes–all contests maximize aggregate incentives even when the set of participants is endogenous.

One example where organizers care about aggregate efforts are science and engineering contests. In line with Proposition 5 many of these contests, as for example the Progressive Insurance Automotive X PRIZE or the Defense Advanced Research Projects Agency's Grand Challenge driverless car competition, award their entire prize budget to the winner.

Note that the results of this section remain valid when contests are allowed to pay players for their attendance. To see this suppose that in an initial stage contests can approach individual players and offer attendance pay which players can either accept or reject. After this initial stage the timing is as specified before. In the subgame that starts after each contest has signed up $N^s \leq \frac{N}{2}$ players for a total attendance payment of $A \leq V$ competition in prize structures will take place as described in Propositions 4 and 5 if we substitute N by $N - 2N^s$ and the contests' prize budget is reduced to $V - A$.

5 Empirical Framework

In this section we will test our theory using data from professional road running. In order to do so we have collected a dataset that contains almost 400 road running contests. Running is an internationally important industry. In the US alone, the industry grossed \$55.7 billion in 2005, with a higher than average annual growth rate of 6.8 percent.

There are various reasons for using road running data rather than data from other types of contests. First, in contrast to labor tournament data, the outcomes of road running races are invariably rank ordered and the measurement of individual performance is straight forward. Second, running contests are organized at a disaggregate or "firm" level instead of being governed by a federation, as it is the case for many other sports such as tennis and golf. Third, although different in their race course, road running contests are almost identical with regard to their organizational set–up.[14] Finally, as media interest and sponsor support are crucial determinants of a race's success, participation is of utmost importance. One may therefore expect that the organizers of a road running race care for both, runners' efforts *and* participation.[15] In fact, the provision of incentives to exert effort might be less of an issue than it is in most other applications.

As our theoretical results are concerned with the sensitivity s with which outcomes depend on players' effort, we need to consider contests that exhibit variation in s. We do so by allowing for races of different distances. Below we argue that longer races

[14]Race directors, who are typically running clubs, event management companies, charities or recreation departments, tend to fulfill a common set of objectives. Some of the issues that they must consider are sponsorship, course design, supplies and equipment, and timing and scoring.

[15]The prestigious Race Director of the Year Award (http://www.rrm.com/rdm/rdy.htm) is based on several factors, including the participation of top runners.

are more likely to be affected by exogenous factors and are therefore less sensitive with respect to changes in runners' effort than shorter races. As a consequence, race–distance serves as a measure of s.

Using the relationship between race distance and sensitivity, we can empirically test three predictions of our model.[16] If contest organizers care about participation (at least to some extent) and their objectives are independent of the race–distance, then our results in Section 4 imply that, (1) longer races will choose steeper prize structures. Indeed, our empirical analysis below confirms this finding. (2) In Section 3 we showed that for low values of sensitivity, players prefer to enter contests with steeper prize structures whereas for high values of sensitivity, flatter prize structures are preferred. This result suggests that in long races, steep price structures will be relatively more attractive to participants than in short races. In fact, our empirical results below indicate that by offering steeper price structures, long races are more successful in attracting the top ranked runners, whereas for short races there seems to be no significant relationship between prize structures and participation. (3) A contest's expected winning performance depends on two factors; the set of participants and their efforts. Our theory has shown that both of these factors are influenced by a contest's prize structure. Irrespective of the contest's sensitivity s, a steeper prize structure induces higher efforts, thereby improving the expected winning performance. However, as argued above, participation effects depend on s. In particular, a steeper prize structure increases the participation of top runners in a long race but has no effect on participation in a short race. Hence our model predicts that an increase in the steepness of the prize structure will improve the winning times in long races, whereas the effect will be weaker in short races. Our empirical results offer suggestive evidence in this direction.

The empirical literature on contest design is scarce and the few articles that do exist test whether prize levels and prize differentials have incentive effects.[17] For example, Ehrenberg and Bognanno (1990a, b) use individual player and aggregate event data

[16]We thank one of the anonymous referees for his suggestions with respect to the empirical testing of our model.

[17]For a recent review of the literature that uses sports data to test contest theory, see Frick (2003).

from US and European Professional Golf Associations to test whether prizes affect players' performance. There are two articles that share our focus on professional road running. Both articles seek to test the hypothesis that prize structures affect finishing times. Maloney and McCormick (2000) use 115 foot races in the US and find that the average prize and prize spread have negative effects on finishing times. Lynch and Zax (2000) use 135 races and also find that finishing times are faster in races offering higher prize money. While Lynch and Zax (2000) fail to distinguish between incentive effects and participation effects, Maloney and McCormick (2000) argue that most of the improvement in finishing times is due to the fact that larger prize purses attract a higher number of top runners. Although we share this finding with Maloney and McCormick (2000) our results regarding the differential effect of prize structures on participation in races of differing distances goes beyond the scope of their analysis.

Race distance as a measure of sensitivity

There are two strands of support for the assertion that in longer races winning probabilities are less sensitive to changes in runners' efforts. Firstly, the longer the race, the stronger is the influence of external factors, like weather conditions, race course profile, or nutrition, on the runners' performance. This was evident during the 2004 Olympic Games in Athens. In the women's marathon the highly acclaimed world recorder holder, Paula Radcliffe, was predicted to win. However, after a consistent lead, at the 23rd mile mark, Paula stopped and sat crying on the side path suffering the symptoms of heat exhaustion.

Secondly, there exists statistical evidence showing that longer races exhibit a higher level of randomness than shorter races. This evidence has been kindly provided to us by Ken Young, a statistician at the "Association of Road Racing Statisticians" (www.arrs.net). Using a data set containing more than 500,000 performances, Ken Young has predicted the outcome of several hundred road running contests of varying distances between 1999 and 2003. As an example, Table 1 reports his results for the Men's races in 1999.[18]

[18]The complete set of results is available on http://www.econ.upf.edu/azmat/.

Two distinct methods were used to predict the winner of a given race. A regression based handicapping (HA) evaluation attempts to predict each runner's finishing time based on past performance. The predicted time was assumed to be normally distributed for each runner and the numerical integration yielded the probability that each runner would win the race. The second method was a Point Level (PL) evaluation based on a rating system similar to the Elo system in chess or the ATP ranking in tennis, in which runners take points from runners they beat and lose points to runners they are beaten by.

Averaging over 274 Men's races with distances between 5km and 42km, the PL prediction of the winner was correct in 43% of the "Short" races (distance $\leq$ 10km), 41% of the "Medium" races (10km $<$ distance $<$ 42km) and 20% of the Long races (distance $\geq$ 42km). For the HA prediction the numbers are 45%, 46%, and 21% respectively. Hence, while Short and Medium distance races are similar, Long distance races appear to be much more random.

Data Description

The empirical investigation is done using data on professional road running from the Road Race Management Directory (2004). This Directory provides a detailed account of the prize structures, summaries, invitation guidelines, and contacts for almost 500 races. It is a major source of information for elite athletes planning their race season. With the exception of a few, most of the races took place in the United States. The event listings are arranged in chronological order beginning in April 2004 and extend through to April 30th 2005. In our analysis, we only include races that have a race distance of at least 5km, leaving us with 368 races. Excluding races with shorter distances leaves us with a more homogeneous set of contests as these races are often charity events, fun runs, or track events.

The Directory provides us with information on the event name, event date, as well as its location. The prize money information includes the total amount of prize money, as well as the prize money breakdown. We focus on the Men's races by including only the Men's prize money distributions. The Directory contains further information that may influence runners' race selection. In particular, it includes data on whether a race was a championship, took place on a cross country or mountain course, and the race's winning performance in the previous year.[19] Finally, the Directory also provides us with information about the size of the event by stating last year's total number of participants. This number typically consists of a large share of amateur runners who rarely win prize money and whose race participation is unlikely to be affected by the race's prize structure. This information is therefore only used to control for the popularity of the event in the world of running.

In order to test how prize structures influence participation, we have collected additional data on the participation of top runners. As race organizers benefit from media coverage and sponsor support, they aim to attract as many runners as possible from a given set of top runners. In order for us to define the set of top runners over a given distance, we use the 2003 ranking of best athletic performances. Because the performance in the 2004 races may be contained in the 2004 rankings we use the previous year ranking (2003) to avoid endogeneity problems. For long races, we employ the ranking provided by the International Association of Athletics Federations (IAAF).[20] As shorter races have a more local character and are less likely to attract participants at an international level, we use the US–ranking provided by the internet site Active.com for race distances smaller than 42km.[21] In addition, for 332 out of the 368 races in our data set, we are able to identify the names and finishing times of the top 100 male finishers using the races' individual websites. Using the rankings of best performance, we count how many of these runners were amongst the top finishers in the current years' races. This provides us with a measure for how many top runners

[19] In order to make finishing times in races over different distances comparable with each other, we use the Riegel formula (see Riegel (1981)) to calculate 10km equivalent finishing times. This formula predicts an athlete's finishing time t in a race of distance d on the basis of his finishing time T in a race of distance D as $t = T(\frac{d}{D})^{1.06}$. It is used by the IAAF to construct scoring tables of equivalent athletic performances.

[20] The IAAF is the international governing body for the sport of athletics. It provides an annual international "top list" of race finishing times (by gender and race category). In some cases, the same runner has achieved several of the ranked top performances in different races. We restrict our attention to the top 200 *runners*.

[21] Active.com is a comprehensive website for outdoor sports. It provides runner rankings based on races that took place in the US. As with the IAAF, we restrict our attention to the top 200 runners in each race category.

were attracted by any given race.

Finally, given that the weather conditions play a role in the outcome of an outdoor race, a runner's decision on whether to participate in a given race may also be affected by the weather that he expects on the race day. For example, if a runner is accustomed to running in warm climates, he may derive an advantage from entering races that take place in the summer. In order to control for this, we have collected information on the weather using an internet site called Weatherbase (www.weatherbase.com). The information includes, the average temperature and average rainfall in the month and the place where the race takes place. We use this information when we estimate the participation of top runners.

Table 2 presents the summary statistics for three race distance categories: "Short" (distance $\leq$ 10km); "Medium" (10km $<$ distance $<$ 42km) and "Long" (distance $\geq$ 42km). In general, races tend to be clustered into certain distance categories. The most common categories being 5km, 10km, 16km, 21km and 42km. Most runners specialize and run either Short or Long distance races, while Medium distance races are run by both types.

From the summary statistics in Table 2 we see that there are some obvious differences between the three distance categories. In particular, the mean total prize money (in US$) increases as the distance increases ($2,990, $5,664 and $23,207, respectively).[22] The average size of the event also increases with distance (3,359, 5,268 and 5,324, respectively). These differences reflect the fact that shorter races have a more regional appeal and are therefore smaller than longer races. There are, however, many similarities in variables that are important when comparing across race categories. In particular, the average number of top runners is similar across distance categories. There is consistency in the weather variables when we look across the race types, although longer races tend to exhibit slightly colder climates. In addition, there is a similar probability that the race has a championship status and the average Riegel

[22]We use the sum of the top 10 Men's prizes as the "total prize money". This variable is more important for the race choice of male runners than the race's total prize budget which also includes prize money distributed to female and junior runners. For comparison of prize money across countries, we convert all prizes into US dollars using monthly historical exchange rates for 2004–2005 (www.gocurrency.com).

measure of last year's winning performance is almost identical.

Analysis

Prediction 1: Longer races exhibit steeper price structures.

The positive relation between a race's distance and the steepness of its prize structure is strikingly exemplified by the 2004 Boston Marathon that awarded 10000$ to the winner and 4000$ to the runner–up, compared with the Boston Half Marathon which paid 5000$ and 2500$, respectively. To obtain estimates for the differences in prize structure, we estimate the following compensation equations using 368 men's races:

$$Y_i = \alpha + \beta D_i + \varepsilon_i. \tag{21}$$

Y_i represents the steepness of the prize structure and D_i denotes the distance and acts as our measure of sensitivity for race i. We use various measures of steepness Y: (1) a concentration index (C. I.), similar to the Herfindahl-Hirschman index, calculated from the top three prizes, i.e. $Y = \frac{(1st)^2+(2nd)^2+(3rd)^2}{(1st+2nd+3rd)^2}$, (2) the ratio between first and second prize, (3) the ratio between first and third prize and (4) the ratio between first prize and total prize money. We expect these measures to increase with the race distance.

We use a continuous measure of distance D, as well as a discrete comparison between Short, Medium and Long distance races, i.e., indicator variables. As mentioned earlier, races tend to be clustered and so it is more informative to look at how the prize structure changes when we compare each group. In doing so, we can estimate the percentage point change in the prize structures' steepness when going from Short to Medium or to Long races.

We report the results for all four measures of steepness, using the two different distance measures in columns (1) to (4) Table 3, Panels A and B. Overall, the results support the hypothesis that as the distance increases, the prize structure becomes steeper. In particular, using our concentration index we observe that as the distance of a race increases by 1km, there is a 0.1% increase in steepness. This implies, for example, that the prize structure of a Marathon is 3.7% more concentrated toward the first prize than the prize structure of a 5km race. Similarly, we find that when the race changes from being Short to Long, there is a 3.2% increase in the concentration index. To understand the economic significance of these effects, consider a typical 5km race from our sample that awards three prizes of size 1000$, 750$, and 500$. If the distance was increased to 42km, the results imply that the race's prize structure would become 1080$, 710$, and 460$.[23] Hence, when moving from 5km to a Marathon, the prize awarded to the winner increases by about 8% while the difference between first and second prize increases by 48%. The coefficient of moving from Short to Medium is positive but insignificant. This is reassuring, as with Ken Young's analysis these races had a similar degree of randomness.

When we look at the other measures of steepness, we observe very similar patterns. In particular, we find that as the distance increases, the gap between the first prize and the second or third prize widens. When the distance increases by 1km, there is a 0.1% rise in the ratio between the first and the second or third prize. When we look across different race types, we see that the ratio between the first and second prize increases by 3.0%, whereas the ratio between the first and the third prize increases by 2.5% when moving from Short to Long. The proportion of total prize money that goes to the winner also increases with the distance but results are not significant.

Next, we extend the analysis of looking at the simple correlation to account for various factors that may affect runners' race selection and hence the prize structure. We control for these factors by estimating the following equation:

$$Y_i = \alpha + \beta D_i + \delta X_i + \varepsilon_i. \tag{22}$$

X includes an indicator identifying whether the race was a championship, its size in 2003, total prize money, the 10km Riegel equivalent of the previous edition's winning time, and an indicator for whether the race is a cross country or a mountain race. It is reassuring to see that the results remain very similar to the results without controls. In fact, as we can see in columns (5) to (8) of Table 3, Panels A and B, the coefficients for all of the prize structure measures and both measures of distance are almost identical with and without controls. In addition, it is important to note that the total prize budget has no effect on the spread of prizes.

[23]The concentration indices for these prize structures are 0.3580 and 0.3717 respectively which is consistent with a 3.7% increase.

Finally, as we observe a large degree of heterogeneity across races, in particular with respect to the total prize budget, we want to check that our results are not being driven by this variation. We repeat the analysis above with restriction at different points of the distribution of total prize money to ensure that our results remain robust. In particular, we exclude races that are within the top 10% of the total prize money (i.e., above 21,500$); races within the bottom 10% (i.e., below 350$); and both, the top and bottom 10%. In Table 4 we look at the relationship between the prize spread and distance when we allow for these restrictions. It can be seen that neither the point estimate of distance, nor the level of significance change when we impose these restrictions.

Prediction 2: In longer races steep prize structures are more attractive to participants than in shorter races.

In Section 3 we have shown that in contests whose outcome is sufficiently (in)sensitive to players' efforts, flatter (steeper) prize structures attract a higher number of participants. Moreover, the proof of Proposition 4 provides evidence for the monotonicity of this relationship. In particular, the steepness of the players' preferred prize structure is monotonically decreasing in the sensitivity parameter s. Because s depends negatively on race distance these results imply that in long races, steep prize structures should be relatively more attractive to participants than in short races.

In the following we will therefore consider the influence of the prize structure on runners' participation decisions separately for each of the three distance categories. As prize structures are important for the race choice of elite runners but negligible for amateurs we will address the above issue by considering the number of top ranked runners participating in each of the races, rather than their total number of participants. We estimate:

$$Part_i^D = \alpha^D + \beta Y_i^D + \delta X_i^D + \varepsilon_i^D \tag{23}$$

where $Part_i^D$ represents the participation of top ranked runners in race i, within distance group $D \in \{S, M, L\}$. Y denotes the steepness of the prize structure and X is the vector of race event controls used in the previous section but now also includes controls for the weather. By focusing on the variation within race categories, we can observe how many participants a race is able to attract for a given level of sensitivity.

In Table 5 we estimate the above equations using the concentration index, C.I., as the measure of steepness.[24] For long races there exists a positive and significant relationship between concentration and participation. In particular, if a long race moved from a flat prize structure (C.I. = 0) to a winner-takes-all structure (C.I. = 1), it would attract an additional five ($\beta = 4.6$) top–ranked runners. For medium and short races the corresponding coefficients are smaller as predicted by our theory. In fact, the coefficients are negative but the effects fail to be statistically significant. This insignificance may be a consequence of the small sample size coupled with a higher variance in short and medium races, which might be explained by the fact that top runners participate in only a few Marathons per year whereas shorter races are run more frequently. To see this suppose that, as in our theoretical model, runners make their race choice solely based on prize structures. If a long distance runner participates in three events per year he will choose the races that offer the three best prize structures. Similarly a short or medium distance runner participating in ten races will choose the races with the ten best prize structures. As a consequence, the number of events that are selected, despite awarding a sub–optimal prize structure is larger for short and medium distance races than for long distance races. Hence the optimal race choice of short and medium distance runners is subject to greater noise than the optimal race choice of long distance runners. Note that in line with our earlier findings, there is no statistical difference in β between medium and short races.

The only other important control variable is total prize budget. This, however, seems intuitive. In our theoretical setup, the total prize budget is the same for all races, such that participation decisions are based solely on the distribution of prizes. If there is variation, it is reasonable to expect that the races offering more money will attract more top ranked runners.

[24] We have replicated the analysis using all other measures of steepness and find very similar results.

Prediction 3: In longer races steep prize structures have a more positive effect on winning performance than in shorter races.

The winning time of a race will depend on the set of runners that participate and the effort they exert. Maloney and McCormick (2000) find that an increase in the total prize budget, as well as an increase in prize spreads leads to faster winning times. Maloney and McCormick attribute the improvement in response to a higher prize budget to participation effects. In contrast, they attribute improvements associated with a greater prize spread to be the result of an incentive effect. However, our theoretical as well as our empirical results so far, suggest that prize spreads not only have incentive but also participation effects. In particular, we have seen that steeper prize structures increase the number of top runners in long races but seem to have no significant effect in medium and short distance races. As steeper prize structures should imply higher efforts in all races, regardless of distance, we would therefore expect that an increase in the steepness of the prize structure will improve the winning times in long races, whereas the effect will be weaker in medium and short distance races. Using the variation (across distance) in the relationship between prize spread and participation may therefore allow us to improve our understanding of the relative importance of participation and incentive effects.

In line with Maloney and McCormick (2000) we first test whether total prize money and prize structures affect winning times:

$$W_i = \beta_0 + \beta_1 Y_i + \beta_2 P_i + \beta_3 D_i + \beta_4 X + \varepsilon_i. \tag{24}$$

W_i is the winning time (seconds) for race i. Recall that to make finishing times in races over different distances comparable with each other, we use the Riegel formula. We control for the steepness of the prize structure, Y_i, and the total prize budget P_i, the distance categories D_i, as well as the characteristics of the race event, X. An important control variable in this regression is last year's winning time (Riegel 2003). As we do not have race fixed effects, this variable should capture some of the unobserved differences across the races. For example as for historic reasons some races may attract a stronger field than others, last year's winning time can serve as a control for the "competitiveness" of a given race. We estimate our equations with and without it.

Second, to test whether there are differential effects in the relationship between distance categories and the prize structures, we include interaction terms:

$$W_i = \beta_0 + \beta_1 Y_i + \beta_2 P_i + \beta_3 D_i + \beta_4 X + \sum_{D \in \{S,M,L\}} [\gamma_D (Y * D)_i + \delta_D (P * D)_i] + \varepsilon_i \quad (25)$$

where the parameters associated with the interaction terms γ_D and δ_D reflect the differential effects of the prize amount and prize structure, respectively, across distance categories.

Using the concentration index, C. I., as the measure of steepness in Table 6, we can see that as the prize structure becomes steeper, the winning time improves although the effect is not statistically significant.[25] The winning time does improve significantly as the total prize budget increases. In particular, from column (1), if the total prize doubles, the winning time will fall by almost 0.2%. In column (2) when we control for last years' winning time (Riegel 2003), the coefficients remain negative but both become insignificant. We find that there is no significant difference in the coefficients on (adjusted) winning times across the different distance categories.[26]

In columns (3) and (4) we control for the interactions between distance categories and prizes, with and without last year's winning time, respectively. We find that for all race categories, increases in the total prize budget improve the winning time significantly. There is, however, a stronger effect on short and medium distance races. When we look at the effects of the prize structure, we find that there is no statistically significant effect. However, the differences across the interaction coefficients are significant. For example, the coefficients for short and long races are statistically different at the 10% level. As the interaction coefficient for short races is larger than the interaction coefficient for long races this suggests that the effect of a steeper prize structure on winning times is more positive in long races than in short races. This last result is consistent with our earlier finding that in long races steep prize structures increase the number of participating top runners whereas in short races they seem to have no effect.

Although these last results are not very strong, they are in line with the predictions of our theory. They provide at least suggestive evidence for the fact that the alloca-

[25] We have replicated the analysis using other measures of steepness and find very similar results.
[26] The excluded category is the constant.

tion of prizes influences a contest's set of participants and hence its expected winning performance.

6 Conclusion

In this article we have considered a model in which several contests compete for the participation of a common set of players. We have studied how the players' contest choice is influenced by the contests' prize structures. We find that if the players' winning probabilities are sufficiently sensitive with respect to their efforts, contests awarding multiple prizes will attract more participants than contests offering their entire prize budget to the winner.

The implications for the contests' (equilibrium) allocation of prizes depend on the organizers' objective. When organizers aim to maximize the number of participants, multiple prizes will be awarded if and only if the sensitivity s of outcomes with respect to players' efforts is sufficiently high. Moreover, as this sensitivity increases, the number of prizes awarded in equilibrium increases and the share awarded to the winner decreases. In contrast, when organizers aim to maximize expected aggregate effort, contests will implement the winner–takes–all principle.

We have provided empirical support for the above results using data from professional road running. In particular, using race distance as a measure of sensitivity we find evidence indicating that in long (less sensitive) races, steeper prize structures increase the participation of top runners significantly whereas in medium and short races there seems to be no significant relationship. This has implications for the contests' allocation of prizes because race organizers care for the participation of top runners. In particular, in our dataset longer races offer (significantly) steeper prize structures than shorter races.

There are several interesting issues that go beyond the scope of our analysis. Because we use Tullock's (1980) contest success function rather than deriving players' winning probabilities from some stochastic mapping between efforts and performance, our theory cannot predict the players' expected winning performance. In reality, some contests aim to maximize the players' winning performance. For example, the organizer of an architectural design competition might be interested in maximizing the quality of the winning design proposal. According to the theory of order statistics, a contest's winning performance depends in a very strong way on the number of participants. Hence in such a setting participation effects might be strong enough to outweigh incentive effects and multiple prizes might become optimal.

Another extension one might want to consider is to allow players to differ in ability, i.e. their marginal cost of effort. Intuitively one might expect that contests with steeper prize structures attract more able players. If this intuition is correct then contests might use their prize structure in order to screen players thereby providing us with a new argument for the use of a single first prize.

Finally, one of the assumptions of our model is that players can participate in only one of the available contests. We expect our results to remain valid in a more general model where players can simultaneously participate in several but not all available contests. In such a model one could analyze how the equilibrium prize structure depends on the number of contests players can participate in. We leave these issues for future research.

Appendix

Proof of Proposition 1

Consider the limit as $s \to 0$. In this case (11) holds and $\Delta(\frac{1}{2}) = \frac{1}{2^{N-1}}(P^0(v_1) - P^0(v_2))$. If $v_1 \neq (\frac{1}{N}, \frac{1}{N}, \ldots, \frac{1}{N}) \neq v_2$ then Δ is strictly decreasing in q with $\Delta(0) = v_1^1 - \frac{1}{N} > 0$ and $\Delta(1) = \frac{1}{N} - v_2^1 < 0$. Hence $\Delta(q^*) = 0$ defines a unique symmetric equilibrium $q^* \in (0, 1)$. Moreover, $q^* > (<) \frac{1}{2}$ if and only if $\Delta(\frac{1}{2}) > (<) 0$. If $v_1 = (\frac{1}{N}, \frac{1}{N}, \ldots, \frac{1}{N}) \neq v_2$ then $\Delta(\frac{1}{2}) < 0$ and $q^* = 0$ while for $v_1 \neq (\frac{1}{N}, \frac{1}{N}, \ldots, \frac{1}{N}) = v_2$ it holds that $\Delta(\frac{1}{2}) > 0$ and $q^* = 1$. As payoffs are continuous in s and the above inequalities are strict, the result holds as long as s is sufficiently small. ■

Proof of Proposition 2

Consider the limit as $s \to \infty$. In this case (13) holds and $\Delta(\frac{1}{2}) = \frac{1}{2^{N-1}}(P^\infty(v_1) - P^\infty(v_2))$. If $v_i \neq (\frac{1}{N}, \frac{1}{N}, \ldots, \frac{1}{N})$ for some $i \in \{1, 2\}$ then Δ is strictly decreasing in q with $\Delta(0) =$

$v_1^1 - v_2^N > 0$ and $\Delta(1) = v_1^N - v_2^1 < 0$. Hence $\Delta(q^*) = 0$ defines a unique symmetric equilibrium $q^* \in (0,1)$. Moreover, $q^* > (<)\ \frac{1}{2}$ if and only if $\Delta(\frac{1}{2}) > (<)\ 0$. As the above inequalities are strict and payoffs are continuous in s the result holds as long as s is sufficiently large. ■

Proof of Proposition 3

For $s > 0$, in a contest with $N_i \geq 2$ participants, players maximize their expected payoff by choosing e^* as defined in (15) and the maximized payoff $E[U_n^i|N_i]$ is given by (16). Note that $E[U_n^i|N_i] \geq 0$ for all v_i^1 if and only if $s \leq \frac{N_i}{N_i-1}$. Hence the equilibrium defined in (15) and (16) is unique and it exists for all v_i and all $N_i \geq 2$ if and only if $s \leq \frac{N}{N-1}$. We therefore have

$$E[U_n^1] = (1-q)^{N-1}v_1^1 + \sum_{m=2}^{N} \tbinom{N-1}{m-1} q^{m-1}(1-q)^{N-m}\frac{1}{m}\left(1 - s(\frac{m-1}{m} - \frac{1-v_1^1}{m-1})\right) \quad (26)$$

$$E[U_n^2] = q^{N-1}v_2^1 + \sum_{m=2}^{N} \tbinom{N-1}{m-1}(1-q)^{m-1}q^{N-m}\frac{1}{m}\left(1 - s(\frac{m-1}{m} - \frac{1-v_2^1}{m-1})\right). \quad (27)$$

$\Delta(q) = E[U_n^1] - E[U_n^2]$ is strictly decreasing in q with $\Delta(0) = v_1^1 - E[U_n^2|N] > 0$ and $\Delta(1) = E[U_n^1|N] - v_2^1 = \frac{1}{N}(1 - s(\frac{N-1}{N} - \frac{1-v_1^1}{N-1})) - v_2^1 < \frac{1}{N}(1 - s(\frac{N-1}{N} - \frac{1}{2(N-1)})) - v_2^1 < \frac{1}{N} - v_2^1 < 0$. Hence $\Delta(q^*) = 0$ defines a unique symmetric equilibrium and $q^* \in (0,1)$. Note that $\Delta(\frac{1}{2}) = 0$ if and only if $s = \bar{s}$ where $\bar{s}$ is as defined in (17). Also note that

$$\frac{\partial \Delta}{\partial s}\Big|_{q=\frac{1}{2}} = \frac{v_2^1 - v_1^1}{2^{N-1}} \sum_{m=1}^{N-1} \frac{\tbinom{N-1}{m}}{m(m+1)} < 0. \quad (28)$$

As payoffs are continuous in q and in s it follows that $q^* < (>)\ \frac{1}{2}$ if and only if $s > (<)\ \bar{s}$. It is immediate from its definition that $\bar{s}$ is strictly decreasing in N. ■

Proof of Proposition 4

In equilibrium contest 1 chooses v_1 to maximize $q^*(v_1, v_2)$ and contest 2 chooses v_2 to maximize $1 - q^*(v_1, v_2)$. In what follows we will proof the three parts of Proposition 4:

Part 1: The prize structure v^* that maximizes $P^0(v)$ is unique and $v^* = (1, 0, \ldots, 0)$. Hence it follows from Proposition 1 that for each contest, v^* is a strictly dominant strategy. Therefore in equilibrium $v_1 = v_2 = v^*$.

Part 2: For N odd the binomial coefficient $\tbinom{N-1}{m-1}$ increases in m for all $m < N^* = \frac{N+1}{2}$, is maximized at $m = N^*$, and decreases for all $m > N^*$. Hence $v^* = (\frac{1}{N^*}, \frac{1}{N^*}, \ldots, \frac{1}{N^*}, 0, \ldots, 0)$ is the (unique) prize structure that maximizes $P^\infty(v)$. Hence it follows from Proposition 2 that for each contest, v^* is a strictly dominant strategy. Therefore in equilibrium $v_1 = v_2 = v^*$. The argument for N even is similar.

Part 3: Consider the case where contests can distribute their prize budget between three prizes. Although more tedious, for a higher number of prizes the following proof would work analogously and we therefore expect our result to hold more generally. Suppose that contest i has chosen the prize structure $v_i = (v_i^1, v_i^2, v_i^3, 0, \ldots, 0)$. For $N_i = 1$ it is immediate that $E[U_n^i | N_i] = v_i^1$. For $N_i = 2$ we can derive the equilibrium effort as before and for $s \leq 2$ we find that there exists a unique symmetric equilibrium given by $e^* = \frac{s}{4}(v_i^1 - v_i^2)$ and $E[U_n^i | N_i] = (\frac{1}{2} - \frac{s}{4})v_i^1 + (\frac{1}{2} + \frac{s}{4})v_i^2$. When $N_i \geq 3$ players participate then we need to derive a player's likelihood to win the third prize. Conditional on player l winning the first prize and player m winning the second prize, player $n \in \mathcal{N}_i$ wins the third prize v_i^3 with probability

$$p_{n|lm}^3 = \frac{(e_n)^s}{\sum_{k \in \mathcal{N}_i - \{l,m\}} (e_k)^s}. \tag{29}$$

Hence the (unconditional) probability that player n wins the third prize is given by

$$p_n^3 = \sum_{l,m \in \mathcal{N}_i - \{n\}, l \neq m} p_l^1 p_{m|l}^2 p_{n|lm}^3. \tag{30}$$

where p_l^1 and $p_{m|l}^2$ are as defined in (1) and (2) respectively. Each player $n \in \mathcal{N}_i$ chooses effort e_n in order to solve

$$\max_{e_n \geq 0} \left[p_n^1(e_n, e_{-n})v_i^1 + p_n^2(e_n, e_{-n})v_i^2 + p_n^3(e_n, e_{-n})v_i^3 - e_n \right]. \tag{31}$$

A symmetric pure strategy equilibrium can be derived by calculating the first order condition and substituting $e_n = e^*$ for all $n \in \mathcal{N}_i$. We find that

$$e^* = \frac{s}{N_i} \left[\frac{N_i - 1}{N_i} v_i^1 + \left(\frac{N_i - 1}{N_i} - \frac{1}{N_i - 1} \right) v_i^2 + \left(\frac{N_i - 3}{N_i - 2} - \frac{1}{N_i - 1} - \frac{1}{N_i} \right) v_i^3 \right] \tag{32}$$

and in equilibrium each player $n \in \mathcal{N}_i$ expects the payoff

$$E[U_n^i | N_i] = \frac{1}{N_i} - e^*. \tag{33}$$

Note that this equilibrium is unique. As $\frac{N_i-1}{N_i} > \frac{N_i-1}{N_i} - \frac{1}{N_i-1} > \frac{N_i-3}{N_i-2} - \frac{1}{N_i-1} - \frac{1}{N_i}$ the equilibrium exists for all v_i if and only if $s \leq \frac{N_i}{N_i-1}$. In equilibrium contest 1 expects a strictly higher (lower) number of participants than contest 2 if and only if $\Delta(\frac{1}{2}) > (<)\ 0$ where $\Delta(q)$ is as defined in (6). Hence in equilibrium contests will choose v^1, v^2 and v^3 to maximize

$P^s(v^1, v^2, v^3) = \alpha(s)v^1 + \beta(s)v^2 + \gamma(s)v^3$ where

$$\alpha(s) = 1 + (N-1)(\frac{1}{2} - \frac{s}{4}) - s\sum_{m=2}^{N-1}\binom{N-1}{m}\frac{m}{(m+1)^2} \tag{34}$$

$$\beta(s) = (N-1)(\frac{1}{2} + \frac{s}{4}) - s\sum_{m=2}^{N-1}\binom{N-1}{m}(\frac{m}{(m+1)^2} - \frac{1}{m(m+1)}) \tag{35}$$

$$\gamma(s) = -s\sum_{m=2}^{N-1}\binom{N-1}{m}\frac{1}{m+1}(\frac{m-2}{m-1} - \frac{1}{m} - \frac{1}{m+1}). \tag{36}$$

Note that $\beta(s) > \alpha(s)$ if and only if $s > \bar{s}$ where $\bar{s} \in (0,1)$ is as defined in (17). Moreover $\gamma(s) > \beta(s)$ if and only if $N \geq 4$ and $s > \bar{\bar{s}}$ where

$$\bar{\bar{s}} = \frac{N-1}{2}\left(\sum_{m=2}^{N-1}\frac{\binom{N-1}{m}}{(m-1)(m+1)} - \frac{N-1}{4}\right)^{-1} > \bar{s}. \tag{37}$$

For $N \geq 6$ it holds that $\bar{\bar{s}} < 1$. As P^s is linear in its arguments it follows that the equilibrium prize structure is $(1, 0, \ldots, 0)$ for $s \in (0, \bar{s})$, $(\frac{1}{2}, \frac{1}{2}, 0, \ldots, 0)$ for $s \in (\bar{s}, \bar{\bar{s}})$, and $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, 0, \ldots, 0)$ for $s \in (\bar{\bar{s}}, \frac{N}{N-1})$. ■

Part 4: As $\bar{s}$ is strictly decreasing in N with $\lim_{N\to\infty} \bar{s} = 0$, for any given $s > 0$, there exists a $\bar{N}$ such that $s > \bar{s}$ for all $N \geq \bar{N}$. Hence according to Proposition 3 for all $N \geq \bar{N}$ a winner–takes–all contest will attract strictly less participation than a contest that awards two prizes. In equilibrium contests therefore have to award at least two prizes if $N \geq \bar{N}$.

Proof of Proposition 5

Consider the case where $s \to \infty$. Suppose that $v_1 = (1, 0, \ldots, 0) \neq v_2$. Define $\delta(q) \equiv E[\Sigma e^1] - E[\Sigma e^2]$ where $E[\Sigma e^1]$ and $E[\Sigma e^s]$ are given by (19) and (20) respectively. As $m(\bar{v}_i(m) - v_i^m)$ is increasing in m, δ is strictly increasing in q. Moreover

$$\delta(0) = -N(\frac{1}{N} - v_2^N) \leq 0 \tag{38}$$

and

$$\delta(\frac{1}{2}) = \frac{1}{2^N}\sum_{m=2}^{N}\binom{N}{m}(1 - m[\bar{v}_2(m) - v_2^m]) > 0. \tag{39}$$

Hence there exists a unique $q^e < \frac{1}{2}$ such that $\delta(q^e) = 0$. Suppose that all players enter contest 1 with probability q^e so that expected aggregate effort is the same in each contest. Because the prize money players expect to win in contest 1 is strictly larger than in contest 2, players must strictly prefer contest 1 to contest 2, i.e. $\Delta(q^e) > 0$. As $\Delta(q)$ is strictly decreasing this implies that in equilibrium $q^*(v_1, v_2) > q^e$. Hence $\delta(q^*(v_1, v_2)) > 0$ which means that in equilibrium expected aggregate effort has to be higher in contest 1. Awarding a single first prize is therefore a strictly dominant strategy when organizers aim to maximize aggregate effort and in equilibrium contests will choose $v_1^* = v_2^* = (1, 0, \ldots, 0)$. As the increase in participation and the resulting positive effect on aggregate effort caused by the award of multiple prizes is strongest for $s \to \infty$, multiple prizes are even less desirable for $s < \infty$. Our result therefore holds for all s. ■

References

[1] Barut, Y., Kovenock, D. "The Symmetric Multiple Prize All–Pay Auction with Complete Information." *European Journal of Political Economy*, Vol. 14 (1998), pp. 627-644.

[2] Burguet, R., Sákovics, J. "Imperfect Competition in Auction Designs." *International Economic Review*, Vol. 40 (1999), pp. 231-247.

[3] Clark, D., J., Riis, C. "Competition over more than one Prize." *American Economic Review*, Vol. 88 (1998a), pp. 276-289.

[4] Clark, D., J., Riis, C. "Influence and the Discretionary Allocation of Several Prizes." *European Journal of Political Economy*, Vol. 14 (1998b), pp. 605-625.

[5] Cohen, C., Kaplan, T., R., Sela, A. "Optimal Rewards in Contests." *RAND Journal of Economics*, Vol. 39 (2008), pp. 434-451.

[6] Ehrenberg, R., G, Bognanno, M., L. "Do Tounaments have Incentive Effects?" *Journal of Political Economy*, Vol. 98 (1990a), pp. 1307-1324.

[7] Ehrenberg, R., G, Bognanno, M., L. "The Incentive Effects of Tournaments Revisted: Evidence from the European PGA Tour." *Industrial and Labor Relations Review*, Vol. 43 (1990b), pp. 74-88.

[8] Frick, B. "Contest Theory and Sport?" *Oxford Review of Economic Policy* , Vol. 19 (2003), pp. 512-529.

[9] Fullerton, R., L., McAfee, R., P. "Auctioning Entry into Tournaments." *Journal of Political Economy*, Vol. 107 (1999), pp. 573-605.

[10] Glazer, A., Hassin, R. "Optimal Contests." *Economic Inquiry*, Vol. 26 (1988), pp. 133-143.

[11] Green, J., Stokey, N. A "Comparison of Tournaments and Contracts." *Journal of Political Economy*, Vol. 91 (1983), pp. 349-364.

[12] Groh, C., Moldovanu, B., Sela, A., Sunde, U. "Optimal Seedings in Elimination Tournaments." *Economic Theory*, forthcoming.

[13] Klemperer, P. "How (not) to run auctions: The European 3G telecom auctions." *European Economic Review*, Vol. 46 (2002), pp. 829-845.

[14] Krishna, V., Morgan, J. "The Winner–Take–All Principle in Small Tournaments." *Advances in Applied Microeconomics*, Vol. 7 (1998), pp. 61-74.

[15] Lazear, E., P., Rosen, S. "Rank–Order Tournaments as Optimal Labor Contracts." *Journal of Political Economy*, Vol. 89 (1981), pp. 841-864.

[16] Lynch, J., Zax, J., S. "The Rewards to Running: Prize Structure and Performance in Professional Road Racing." *Journal of Sports Economics*, Vol. 1 (2000), pp. 323-340.

[17] McAfee, R., P. "Mechanism Design by Competing Sellers." *Econometrica*, Vol. 61 (1993), pp. 1281-1312.

[18] Maloney, M., T., McCormick, R., E. "The Response of Workers to Wages in Tornaments: Evidence from Foot Races." *Journal of Sports Economics*, Vol. 1 (2000), pp. 99-123.

[19] Moldovanu, B., Sela, A. "The Optimal Allocation of Prizes in Contests." *American Economic Review*, Vol. 91 (2001), pp. 542-558.

[20] Moldovanu, B., Sela, A. "Contest Architecture." *Journal of Economic Theory*, Vol. 126 (2006), pp. 70-97.

[21] Moldovanu, B., Sela, A., Shi, X. "Contests for Status." *Journal of Political Economy*, Vol. 115 (2007), pp. 338-363.

[22] Moldovanu, B., Sela, A., Shi, X. "Competing Auctions with Endogenous Quantities." *Journal of Economic Theory*, Vol. 141 (2008), pp. 1-27.

[23] Mookherjee, D. "Optimal Incentive Schemes with Many Agents." *Review of Economic Studies*, Vol. 51 (1984), pp. 433-446.

[24] Münster, J. (2006) Contests with an Unknown Number of Contestants. *Public Choice*, Vol. 129, pp. 353-368.

[25] Myerson, R.,B., Wärneryd "Population Uncertainty in Contests." *Economic Theory*, Vol. 27 (2006), pp. 469-474.

[26] Nalebuff, B., J., Stiglitz, J., E. "Prizes and Incentives: Towards a General Theory of Compensation and Competition." *Bell Journal of Economics*, Vol. 14 (1983), pp. 21-43.

[27] Nti, K., O. "Comparative Statics of Contests and Rent–Seeking Games." *International Economic Review*, Vol. 38 (1997), pp. 43-59.

[28] Peters, M., Severinov, S. "Competition among Sellers Who Offer Auctions Instead of Prices." *Journal of Economic Theory*, Vol. 75 (1997), pp. 114-179.

[29] Riegel, P., S. "Athletic Records and Human Endurance." *American Scientist*, Vol. 69 (1981), pp. 285-290.

[30] Rosen, S. "Prizes and Incentives in Elimination Tournaments." *American Economic Review*, Vol. 76 (2001), pp. 701-715.

[31] Skaderpas, S. "Contest Success Functions." *Economic Theory*, Vol. 7 (1996), pp. 283-290.

[32] Szymanski, S., Valletti, T. "Incentive Effects of Second Prizes." *European Journal of Political Economy*, Vol. 21 (2005), pp. 467-481.

[33] Taylor, C., R. "Digging for Goldon Carrots: An Analysis of Research Tournaments." *American Economic Review*, Vol. 85 (1995), pp. 872-890.

[34] Tullock, G. "Efficient Rent Seeking." In J. Buchanan et al., eds., *Towards a Theory of the Rent–Seeking Society* College Station: Texas A&M University Press, 1980.

Table 1: Ken Young's prediction of Men's winner (1999)

Date	Race Name	Distance (km)	HA Prob	HA WP	PL CI	PL WP
3/5/1999	IAAF World Indoor Champs (JPN)	3.0	80	1	796	1
3/5/1999	NCAA Indoor Champs (IN/USA)	5.0	70	2	398	1
3/6/1999	Gate River Run (FL/USA)	15.0	78	1	434	4
3/6/1999	NCAA Indoor Champs (IN/USA)	3.0	78	1	458	4
3/14/1999	Los Angeles (CA/USA)	42.2	54	2	650	4
3/27/1999	Azalea Trail (AL/USA)	10.0	97	1	432	1
4/11/1999	Cherry Blossom (DC/USA)	16.1	43	3	677	4
4/17/1999	Stramilano (ITA)	21.1	80	1	864	1
4/18/1999	Rotterdam (HOL)	42.2	71	2	709	1
4/19/1999	Boston (MA/USA)	42.2	37	2	727	4
4/25/1999	Sallie Mae (DC/USA)	10.0	66	2	728	1
5/2/1999	Pittsburgh (PA/USA)	42.2	47	1	355	4
5/16/1999	Volvo Midland Run (NJ/USA)	16.1	59	4	376	6
5/16/1999	Bay to Breakers (CA/USA)	12.0	50	3	676	1
5/31/1999	Bolder Boulder (CO/USA)	10.0	25	9	673	13
6/2/1999	NCAA Champs (ID/USA)	10.0	35	5	379	5
6/4/1999	NCAA Champs (ID/USA)	5.0	78	1	456	1
6/12/1999	Stockholm (SWE)	42.2	47	2	433	1
6/19/1999	Grandma's (MN/USA)	42.2	40	2	392	2
6/27/1999	Fairfield (CT/USA)	21.1	60	6	572	4
7/4/1999	Peachtree (GA/USA)	10.0	68	1	831	2
7/4/1999	Golden Gala (ITA) 5000m	5.0	52	1	1003	1
7/17/1999	Crazy 8's (TN/USA)	8.0	60	5	714	2
7/25/1999	Wharf to Wharf (CA/USA)	9.7	75	1	673	3
7/31/1999	Quad-Cities Bix (IA/USA)	11.3	88	1	767	1
8/15/1999	Falmouth (MA/USA)	11.3	72	2	845	2
8/21/1999	Parkersburg (WV/USA)	21.1	57	1	338	1
8/24/1999	IAAF World Champs (ESP)	10.0	74	2	960	1
8/28/1999	IAAF World Champs (ESP)	5.0	68	1	992	2
8/28/1999	IAAF World Champs (ESP)	42.2	7	5	699	18
9/3/1999	Ivo Van Damme (BEL)	10.0	42	13	843	12
9/26/1999	Berlin (GER)	42.2	57	1	586	1
10/24/1999	Chicago (IL/USA)	42.2	66	1	752	1
11/7/1999	New York City (NY/USA)	42.2	57	1	702	9
12/5/1999	California International (CA/USA)	42.2	12	8	378	11

Data kindly provided by Ken Young, Association of Road Racing Statisticians. For the handicapping (HA) evaluation, "HA Prob" denotes the probability with which the predicted winner was expected to win and "HA WP" reports the placing he actually obtained. Using a Point Level (PL) system the average rating for the five highest ranked runners in the race was compared to the average rating for the ten highest ranked runners in the world at the time of the race in order to construct the competition index (CI). The higher the index the better the quality of the field. The column "PL WP" reports the actual placing obtained by the highest ranked runner.

Table 2: Descriptive statistics

	Short (Distance $\leq$ 10km)				
Variable	Obs	Mean	Std. Dev.	Min	Max
Rain (cm)	175	7.29	3.19	0	17
Temperature (^{o}C)	175	21.44	6.65	3	33
Championship	175	0.07	0.26	0	1
Total Prize ('0000 US $)	175	0.30	0.64	0.01	6.00
Size ('0000)	175	0.34	0.73	0.00	5.50
Top Ranked Runners	154	0.60	1.82	0	15
Riegel 2003 (Sec)	175	1,821.77	96.91	1,647.00	2,276.75
Trail	175	0.01	0.11	0	1
	Medium (10km < Distance < 42km)				
Variable	Obs	Mean	Std. Dev.	Min	Max
Rain (cm)	97	7.06	2.92	0	16
Temperature (^{o}C)	97	20.27	6.00	0	32
Championship	97	0.12	0.33	0	1
Total Prize ('0000 US $)	97	0.57	1.00	0.02	7.00
Size ('0000)	97	0.53	1.07	0.01	8.00
Top Ranked Runners	90	1.15	2.94	0	17
Riegel 2003 (Sec)	97	1,849.14	203.44	1,653.62	3,056.75
Trail	97	0.05	0.22	0	1
	Long (Distance $\geq$ 42km)				
Variable	Obs	Mean	Std. Dev.	Min	Max
Rain (cm)	96	6.49	3.25	0	16
Temperature (^{o}C)	96	18.44	5.13	9	33
Championship	96	0.11	0.32	0	1
Total Prize ('0000 US $)	96	2.32	4.69	0.02	27.00
Size ('0000)	96	0.63	0.87	0.01	4.60
Top Ranked Runners	88	1.32	2.54	0	11
Riegel 2003 (sec)	96	1,883.97	180.92	1,629.07	2,628.47
Trail	96	0.03	0.17	0	1

Notes: Means and standard deviations for each race distance category, "Short", "Medium" and "Long", respectively. "Championship" refers to whether or not the race held a championship title. "Total Prize" is the total amount of the prize budget available to senior men (all values are expressed in 10,000 (real) US dollars evaluated at monthly historical exchange rate for 2004-2005). "Size" refers to the total number of contestants in the race measured in units of 10,000. "Top Ranked Runners" is the number of participants who were included in the ranking of the fastest performances in 2003. "Riegel 2003" calculates the 10km equivalent race finishing times. "Trail" refers to whether the race took place on a cross country or mountain course.

Table 3: Prize structures without/with controls

PANEL A	Measures of Prize Structure Steepness							
	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]
Dep. Var.	C. I.	1:2	1:3	1:Total	C. I.	1:2	1:3	1:Total
Distance(km)	0.0008	0.0008	0.0007	0.0006	0.0008	0.0009	0.0007	0.0006
	[0.0003]**	[0.0002]**	[0.0002]**	[0.0004]	[0.0003]*	[0.0003]**	[0.0003]**	[0.0004]
Champ.					-0.001	-0.0044	-0.0052	-0.0236
					[0.0159]	[0.0122]	[0.0132]	[0.0207]
Prize('0000$)					-0.0013	-0.0012	-0.0016	-0.0030
					[0.0012]	[0.0009]	[0.0010]	[0.0015]
Size('0000)					0.0024	0.0065	0.0031	-0.0051
					[0.0059]	[0.0046]	[0.0049]	[0.0077]
Riegel 2003					0.0001	0.0001	0.0001	0.0002
					[0.0000]	[0.0000]	[0.0000]†	[0.0001]**
Trail					-0.0022	0.0013	-0.0316	-0.0811
					[0.0356]	[0.0274]	[0.0296]	[0.0463]†
Constant	0.394	0.6241	0.7321	0.4511	0.2806	0.607	0.6248	0.0502
	[0.0075]**	[0.0058]**	[0.0063]**	[0.0102]**	[0.0715]**	[0.0550]**	[0.0594]**	[0.0930]**
Obs.	368	368	368	368	368	368	368	368

PANEL B	Measures of Prize Structure Steepness							
	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]
Dep. Var.	C. I.	1:2	1:3	1:Total	C. I.	1:2	1:3	1:Total
Medium	0.0119	0.0115	0.0112	0.0162	0.0103	0.0105	0.011	0.0164
	[0.0113]	[0.0086]	[0.0094]	[0.0153]	[0.0114]	[0.0088]	[0.0095]	[0.0148]
Long	0.0316	0.0299	0.025	0.0245	0.0305	0.0303	0.0255	0.0233
	[0.0113]**	[0.0087]**	[0.0094]**	[0.0153]	[0.0122]*	[0.0094]**	[0.0101]**	[0.0158]
Champ.					-0.0014	-0.0047	-0.0056	-0.0244
					[0.0159]	[0.0123]	[0.0132]	[0.0207]
Prize('0000$)					-0.0011	-0.0009	-0.0014	-0.0027
					[0.0011]	[0.0009]	[0.0009]	[0.0015]
Size('0000)					0.0019	0.0061	0.0026	-0.0058
					[0.0060]	[0.0046]	[0.0050]	[0.0078]
Riegel 2003					0.0001	0.0001	0.0001	0.0002
					[0.0000]†	[0.0000]	[0.0000]†	[0.0001]**
Trail					-0.0039	-0.0012	-0.0342	-0.0847
					[0.0357]	[0.0275]	[0.0297]	[0.0464]†
Constant	0.3989	0.6292	0.7358	0.4531	0.2789	0.6029	0.6209	0.0461
	[0.0067]**	[0.0052]**	[0.0056]**	[0.0091]**	[0.0716]**	[0.0551]**	[0.0595]**	[0.0930]
Obs.	368	368	368	368	368	368	368	368

Notes: Standard errors are in parentheses. (†), (*) and (**) represent significance at the 90, 95 and 99 percent level, respectively. The dependent variables refer to various measures of prize structure steepness (concentration index (C. I.); ratio between first and second prize (1:2); ratio between first and third prize (1:3); and ratio between first and total prize (1:Total)). For a description of the other variables see Table 2.

Table 4: Prize structure with restrictions on total prize budget

Dependent Variable	Concentration Index (C. I.)			
	[1]	[2]	[3]	[4]
Restrictions	none	TP<90%	TP>10%	10%<TP<90%
Distance	0.0008	0.0011	0.0010	0.0015
	[0.0003]*	[0.0004]**	[0.0003]**	[0.0004]**
Championship	-0.001	-0.0248	-0.0018	-0.024
	[0.0159]	[0.0183]	[0.0158]	[0.0180]
Total Prize('0000$)	-0.0013	-0.0184	-0.0017	-0.0259
	[0.0012]	[0.0139]	[0.0011]	0.0137
Size('0000)	0.0024	0.0076	0.0019	0.0082
	[0.0059]	[0.0075]	[0.0058]	[0.0072]
Riegel 2003	0.0001	0.0001	0.0001	0.0001
	[0.0000]	[0.0000]	[0.0000]	[0.0001]
Trail	-0.0022	0.0015	0.024	0.0429
	[0.0356]	[0.0387]	[0.0388]	[0.0432]
Constant	0.2806	0.2888	0.3421	0.3939
	[0.0715]**	[0.0837]**	[0.0786]**	[0.0937]**
Observations	368	331	332	295

Notes: Standard errors are in parentheses. (†), (*) and (**) represent significance at the 90, 95 and 99 percent level, respectively. Column (1) includes all races. In column (2) we exclude races with a total prize budget amongst the top 10%. In column (3) we exclude races with a total prize budget (TP) amongst the bottom 10%. In column (4) we exclude races with a total prize budget either amongst the bottom 10% or above the top 10%

Table 5: Participation of Top Ranked Runners

Dependent Variable	Top Ranked Runners		
	[1]	[2]	[3]
Distance	Long	Medium	Short
Concentration Index	4.61531	-0.86782	-0.76312
	[1.65795]**	[2.41828]	[2.43981]
Rain	0.04489	0.05635	-0.05065
	[0.06056]	[0.07855]	[0.04546]
Temperature	0.04256	0.00113	-0.03806
	[0.03716]	[0.03476]	[0.02239]†
Total Prize ('0000$)	0.3300	1.900	0.7600
	[0.1000]**	[0.3000]**	[0.3000]**
Size ('0000)	0.6300	0.3600	-0.0200
	[0.3000]†	[0.2000]†	[0.2000]
Riegel 2003	0.00001	-0.00153	-0.00162
	[0.00157]	[0.00199]	[0.00164]
Trail	-2.68802	1.86795	0.32307
	[1.94804]	[1.79707]	[1.54183]
Constant	-2.92514	2.51767	4.82139
	[3.00638]	[3.90509]	[2.89613]†
Observations	88	90	154

Notes: Standard errors are in parentheses. (†), (*) and (**) represent significance at the 90, 95 and 99 percent level, respectively. The dependent variable, "Top Ranked Runners", counts the number of participating runners who were amongst those with the best finishing performances in 2003.

Table 6: Winning Performance

Dependent Variable	Riegel 2004			
	[1]	[2]	[3]	[4]
Total Prize ('0000$)	-15.4810	-4.7410		
	[4.4200]**	[4.3600]		
Concentration Index	-0.4651	-85.7087		
	[114.5542]	[106.8533]		
Short	1,903.42	805.0429	1,727.53	921.2702
	[67.8566]**	[164.7526]**	[106.8274]**	[182.4072]**
Medium	1,912.44	806.5594	1,954.00	1,028.11
	[68.6082]**	[165.9799]**	[96.1924]**	[196.3595]**
Long	1,943.06	786.3585	1,932.21	999.972
	[68.9412]**	[172.6224]**	[81.6556]**	[191.3044]**
Riegel 2003		0.6214		0.5043
		[0.0861]**		[0.0944]**
Total Prize('0000$)*Short			-65.5810	-41.1420
			[22.2120]**	[21.7680]†
Total Prize('0000$)*Medium			-107.4940	-57.8030
			[20.7550]**	[21.9550]**
Total Prize('0000$)*Long			-12.1120	-4.8310
			[4.3120]**	[4.3500]
C.I.*Short			405.1186	140.3977
			[236.3836]†	[231.8450]
C.I.*Medium			-55.8521	-79.5659
			[205.3986]	[196.8460]
C.I.*Long			-61.2985	-110.4742
			[154.0820]	[147.9156]
Rain	-2.71	-4.1102	-2.6291	-3.7594
	[3.4507]	[3.2048]	[3.3307]	[3.1982]
Temperature	0.3737	0.6999	1.4284	1.2587
	[1.7493]	[1.6224]	[1.6935]	[1.6229]
Size('0000)	-27.2970	-17.2510	-14.2620	-11.2400
	[12.8240]*	[11.9700]	[12.6400]	[12.1240]
Trail	564.144	220.566	685.9003	353.7795
	[83.8881]**	[91.1957]*	[85.7670]**	[103.0575]**
Observations	332	332	332	332

Notes: Standard errors are in parentheses. (†), (*) and (**) represent significance at the 90, 95 and 99 percent level, respectively. The dependent variable, "Riegel 2004", is measured in seconds and calculates the 10km equivalent race finishing times. In order to compare all distance categories, the excluded category is the constant.