Incentives versus Competitive Balance*

Marc Möller†
Department of Economics
Universität Bern

Abstract

When players compete repeatedly, prizes won in earlier contests may improve the players' abilities in later contests. This paper determines the allocation of prizes within and across contests that maximizes the (weighted) sum of aggregate efforts.

Keywords: Contests; Incentives; Prize allocation.
JEL *classification*: D02; J31.

1. Introduction

In many social and economic settings, e.g. procurement or R&D contests, labor tournaments, or sports leagues, agents compete with each other repeatedly and resources obtained in an earlier contest may be used to improve the agents' abilities in later contests. For example, between 1985 and 1990, the US Air Force conducted a series of annual procurement contests between Pratt & Whitney and General Electric for a total of 2000 fighter jet engines. The first contest awarded the procurement of 160 engines to be purchased in 1985 and both companies used the resulting revenues of $800 million to strengthen their R&D departments.

This paper considers the optimal allocation of prizes within and across contests for an organizer interested in the maximization of the (weighted) sum of aggregate efforts. It highlights a trade–off between incentives and competitive balance. The provision of incentives in earlier contests via the implementation of large prize spreads leads to competitive imbalances and hence a reduction in incentives in later contests. This trade–off seems to be recognized in practice. In 1984 the US Air Force carried out extensive engine life–cycle cost analyses concluding that General Electric's costs were lower than Pratt and Whitney's. Nevertheless, 25% of the engines purchased in 1985 were awarded to Pratt

*I thank Mikhail Drugov for valuable suggestions.
†Mail: Department of Economics, University of Bern, Schanzeneckstrasse 1, 3001 Bern, Switzerland. Email: marc.moeller@vwi.unibe.ch, Tel.: +41-(0)31631-8078, Fax: +41-(0)31631-3783.

and Whitney's. The implementation of a split award secured a level of competition for which this procurement contest became known as the "The Great Engine War".

The model provides a new rationale for the wide spread occurrence of multiple prizes. It complements earlier work which has explained the use of multiple prizes by the contestants' risk aversion (Krishna and Morgan, 1998), the convexity of effort costs (Moldovanu and Sela, 2001), or the need to attract participation (Azmat and Möller, 2009). In addition, the model offers comparative statics results that are readily testable.

While contest design has attracted considerable interest in the economic literature, most papers focus on static settings and only a few consider dynamic aspects.[1] Some papers study the design of elimination tournaments (Rosen 1986, Gradstein and Konrad, 1999, Moldovanu and Sela, 2006). Others consider settings in which contestants exert effort various times and prizes are allocated either in dependence of aggregate efforts (Yildirim, 2005) or as a function of the contest outcomes at each stage (Gershkov and Perry, 2009, Konrad and Kovenock, 2009). The present paper differs from both by focusing on a setting where the set of contestants remains constant and the winner of each contest/stage is determined solely by the contestants' efforts in that stage. It adds to the literature on dynamic contest design by allowing for a dependence of players' abilities on past performance.

2. The model

Two risk neutral players $i \in \{1, 2\}$ compete in two successive contests $n \in \{1, 2\}$. In each contest, players choose individual efforts non–cooperatively and simultaneously and the winner is determined by Tullock's (1980) parameterized contest success function. In particular, if in contest n, player i exerts effort e_{ni} and player j exerts effort e_{nj}, then player i wins contest n with probability $p_{ni}(e_{ni}, e_{nj}) = \frac{e^s_{ni}}{e^s_{ni}+e^s_{nj}}$. The parameter $s \in (0, 1]$ measures the influence of the players' efforts on the contest's outcome.[2] Both contests are organized by a single contest designer endowed with a total prize budget normalized to 1. Denote by v_n the sum of the prizes awarded in contest n, and let $w_n \in [\frac{1}{2}, 1]$ be the share that is given to the winner. The players' aggregate effort in contest n is $E_n = e_{n1} + e_{n2}$. The contest designer maximizes the (weighted) sum of efforts $E = \gamma E_1 + (1-\gamma)E_2$ subject to his budget constraint $v_1 + v_2 \leq 1$.

Players have linear costs of effort. In contest 1 the players' marginal cost of effort is normalized to 1.[3] Players do not derive direct utility from the prizes awarded in contest 1. Instead, these prizes lower the players' marginal costs of effort in contest 2. A player who obtains the prize v in contest 1 has (constant) marginal cost of effort $c(v) \leq 1$ in contest 2. Following the contest literature, we interpret $a(v) = \frac{1}{c(v)}$ as the player's ability. Note that if $a(.)$ was concave (convex) the players' average ability in contest 2 would

[1] For an extensive overview of this literature see Konrad (2009).

[2] The assumption that s is restricted from above is standard and guarantees the existence of a unique equilibrium (Szidarovszky and Okuguchi, 1997).

[3] Players are assumed to be identical ex ante in order to rule out heterogeneity as the reason to award multiple prizes (Szymanski and Valletti, 2005).

be maximized by awarding two equal prizes (a single prize) in contest 1. In order to focus on the trade–off between incentives and competitive balance and to make the model tractable, we assume that $a(.)$ is linearly increasing, i.e. $a(v) = 1+\alpha v$ with $\alpha > 0$. Players discount second period payoffs with discount factor $\delta \in (0, 1)$.

3. Competition

The solution of a Tullock contest can be derived from first order conditions and is standard in the literature. In the unique equilibrium of a contest with first prize V_1 and second prize V_2, two players with marginal effort costs $c_i, c_j > 0$ exert efforts $e_i^* = s\frac{V_1-V_2}{c_i}\frac{(\frac{c_i}{c_j})^s}{[1+(\frac{c_i}{c_j})^s]^2}$ and obtain the payoffs $\pi_i^* = V_2 + (V_1 - V_2)\frac{1+(1-s)(\frac{c_i}{c_j})^s}{[1+(\frac{c_i}{c_j})^s]^2}$. In contest 2 the marginal cost of effort of the winner and the loser of contest 1 are $c_i = \frac{1}{a(w_1 v_1)}$ and $c_j = \frac{1}{a((1-w_1)v_1)}$ respectively and the prize spread is given by $V_1 - V_2 = v_2(2w_2 - 1)$. Aggregate effort in contest 2 therefore becomes:

$$E_2 = 2sv_2(2w_2 - 1)\frac{\bar{a}h^s}{(1 + h^s)^2}. \tag{1}$$

It depends on the players' level of heterogeneity, denoted as $h \equiv \frac{a(w_1 v_1)}{a((1-w_1)v_1)} \geq 1$, and their average ability, given by $\bar{a} \equiv \frac{1}{2}[a(w_1 v_1) + a((1 - w_1)v_1)] = 1 + \frac{\alpha}{2}v_1$.

In contest 1, $c_1 = c_2 = 1$ and V_1 and V_2 are given by the discounted (expected) payoffs that the winner and the loser of contest 1 will obtain in contest 2:

$$V_1 - V_2 = \delta v_2(2w_2 - 1)\left[\frac{1 + (1 - s)(\frac{1}{h})^s}{[1 + (\frac{1}{h})^s]^2} - \frac{1 + (1 - s)h^s}{[1 + h^s]^2}\right] = \delta v_2(2w_2 - 1)\frac{h^s - 1}{h^s + 1}. \tag{2}$$

Aggregate effort in contest 1 becomes:

$$E_1 = \frac{s}{2}\delta v_2(2w_2 - 1)\frac{h^s - 1}{h^s + 1}. \tag{3}$$

In accordance with the intuition mentioned in the Introduction, E_1 is increasing in the players' heterogeneity h whereas E_2 is decreasing. Prizes have a positive, although indirect effect on *present* incentives. Contestants exert effort in contest 1 to become stronger competitors in contest 2. However, prizes also have a negative effect on *future* incentives. The loser of contest 1 becomes demotivated by the comparative advantage of his rival in contest 2. The winner of contest 1 anticipates this and provides less effort himself.[4]

4. Contest design

In both contests aggregate effort is strictly increasing in v_2 and w_2. It is therefore optimal for the organizer to set $w_2 = 1$ and $v_2 = 1 - v_1$, i.e. contest 2 should be a winner–takes–all

[4]The insight that ability differences are detrimental for incentives can be traced back to Lazear and Rosen (1981).

contest and award all remaining prize money. The organizer's problem simplifies to:

$$\max_{v_1 \in [0,1], w_1 \in [\frac{1}{2},1]} s(1-v_1)\left\{\frac{\gamma\delta}{2}\frac{h^s-1}{h^s+1}+2(1-\gamma)\frac{\bar{a}h^s}{(1+h^s)^2}\right\}. \tag{4}$$

We consider this problem in two steps. We first focus on the allocation of prizes *within* contest 1 by deriving the optimal winner's share $w_1^*(v_1)$ for a given v_1. We then consider the optimal allocation of prizes *across* the two contests by deriving the optimal v_1^*.

Proposition 1 *If $v_1 \leq \bar{v}_1$, contest 1 should be winner–takes–all, i.e. $w_1^*(v_1) = 1$. If $v_1 > \bar{v}_1$, awarding two prizes is optimal and the winner's share $w_1^*(v_1) \in (\frac{1}{2}, 1)$ is strictly increasing in γ and δ but strictly decreasing in v_1, α, and s. $\bar{v}_1$ is strictly increasing in δ and γ, but strictly decreasing in s and α. $\bar{v}_1 \in (0,1)$ if $\alpha > \bar{\alpha}(\gamma, \delta, s)$ and $\bar{v}_1 = 1$ otherwise.*

Proposition 1 is interesting in its own right since it applies to the case where the organizer is able to determine the allocation of prizes within each contest but the prize budget of each contest is fixed for exogenous reasons. In order to gain intuition for the comparative statics results it is useful to note from the proof of Proposition 1 that the organizer chooses $w_1^*(v_1)$ to induce an optimal degree of heterogeneity given by

$$h^* \equiv \left(\frac{\bar{a}+\frac{\gamma\delta}{2(1-\gamma)}}{\bar{a}-\frac{\gamma\delta}{2(1-\gamma)}}\right)^{\frac{1}{s}}. \tag{5}$$

An increase in the winner's share, w_1, increases the players' heterogeneity. This strengthens incentives in contest 1 at the cost of reducing incentives in contest 2. As the weight attached to first period efforts, γ, becomes larger, the organizer should therefore award a larger share to the winner. The effect of an increase in the players' discount factor, δ, is as follows. In contest 1 players exert effort in order to obtain a comparative advantage in contest 2. An increase in δ makes such a comparative advantage more valuable and players more responsive to first period incentives. As a consequence, the organizer should implement a larger prize spread in contest 1.

An increase in the total prize, v_1, awarded in contest 1 has two effects. First, it increases the players' average ability, $\bar{a}$, and hence the level of effort in contest 2. Inducing heterogeneity becomes more costly for the organizer since it leads to a stronger reduction in second period efforts. As a consequence, the optimal degree of heterogeneity h^* decreases. Second, as v_1 increases, a lower w_1 is required to achieve the required degree of heterogeneity h^*. A similar reasoning applies to changes in the parameter α measuring the effectiveness with which prizes translate into ability improvements.

Finally, an increase in the Tullock parameter s makes contest outcomes more responsive to changes in efforts. Competitive imbalances weigh more heavily and heterogeneity has a more detrimental effect on second period incentives. As a result, the optimal degree of heterogeneity and hence the share awarded to the winner decreases.

Note that it is never optimal to award two equal prizes in contest 1, i.e. $w_1^* \neq \frac{1}{2}$. For $w_1 = \frac{1}{2}$, increasing w_1 has a first order (positive) effect on incentives in contest 1, but only a second order (negative) effect on incentives in contest 2.[5]

The following proposition shows that these results remain unchanged when the organizer is able to determine the allocation of his prize budget across the two contests:

Proposition 2 *If α is sufficiently large, the optimal prize allocation $(v_1^*, w_1^*, v_2^*, w_2^*)$ is such that $v_1^* \in (0,1)$, $w_1^* \in (\frac{1}{2}, 1)$, $v_2^* = 1 - v_1^*$, and $w_2^* = 1$. v_1^* is independent of s, strictly increasing in α and strictly decreasing in γ and δ. w_1^* is strictly increasing in γ and δ but strictly decreasing in s and α.*

Proposition 2 focuses on the more interesting case where it is optimal for the organizer to award multiple prizes in contest 1. When α is small, the organizer will choose a $v_1^* < \bar{v}_1$ and implement a winner–takes–all contest by setting $w_1^* = 1$. For large enough α, the organizer is able to resolve the trade–off between first period incentives and second period competitive imbalance completely, by choosing the optimal degree of heterogeneity h^* via the selection of the optimal winner's share $w_1^*(v_1)$. What remains for the choice of v_1, is a trade–off between ability improvement and incentives. A higher v_1 increases the players' (average) ability in contest 2 but reduces the amount $1 - v_1$ that is left for the provision of incentives.

The intuition for the comparative statics with respect to v_1^* are as follows. As γ decreases and first period efforts become less important, the organizer will implement a more balanced prize allocation in contest 1, i.e. $w_1^*(v_1)$ decreases for a given v_1. A balanced prize allocation allows the organizer to improve the players' abilities without the introduction of competitive imbalances. As a consequence, a larger share of the organizer's prize budget will be allocated to the first contest. A similar effect results from a reduction in the players' discount factor. It is interesting to consider the limiting case $\gamma \to 0$ in which the organizer is only concerned about the players' efforts in contest 2. In this case, the organizer avoids the introduction of any comparative advantage by letting $w_1^*(v_1) \to \frac{1}{2}$ for any v_1. He chooses v_1 to maximize $E = \frac{s}{2}(1 - v_1)(1 + \frac{1}{2}\alpha v_1)$. The solution is given by $\lim_{\gamma \to 0} v_1^* = \frac{1}{2} - \frac{1}{2\alpha}$. The same solution is obtained for $\delta \to 0$, i.e. when players are myopic. It serves as an upper bound on the solution v_1^* for the case where $\gamma, \delta > 0$.

While the positive dependence of v_1^* on α is straight forward, its independence of s deserves some discussion. An increase in s has two effects. It increases the value of ability improvements since efforts become more dependent on ability. It also makes players react more strongly to the provision of incentives. In the present setting it does not make a difference whether players are motivated via the reduction of their effort costs, or through the award of prize money. As a consequence, v_1^* is independent of s.

[5] This is similar to a result obtained by Meyer (1992) in the context of a repeated labor tournament where heterogeneity is induced by biasing the second contest in favor of the winner of the first.

5. Concluding remark

The present analysis assumes that prizes improve the players' abilities in future contests but offer no immediate benefits. For example, scientists winning a research grant obtain better professional capabilities but often derive no direct income from such an award. In an extension, contests could offer monetary rewards and players could determine the amount to be invested into future abilities. The analysis of investment decisions in a contest setting requires a clear distinction between efforts and investments as in Münster (2007). It is an interesting topic that is left for future research.

Appendix

Proof of Proposition 1

Defining $x \equiv h^s \in [1, a(v_1)^s]$ and $\eta \equiv \frac{\gamma\delta}{2(1-\gamma)}$ we have

$$\frac{dE}{dx} = \frac{2s(1-\gamma)(1-v_1)}{(x+1)^2} g(x) \qquad \text{where} \qquad g(x) \equiv \eta - \bar{a}\frac{x-1}{x+1}. \tag{6}$$

Note that $g(1) = \eta > 0$ and $\frac{dg}{dx} = -\frac{2\bar{a}}{(x+1)^2} < 0$. If $g(a(v_1)^s) \geq 0$ then E is increasing in the entire range. Since x and $\bar{a}$ are strictly increasing in v_1 it holds that $\frac{dg}{dv_1} < 0$. Hence there exists a $\bar{v}_1 > 0$ such that $g(a(v_1)^s) \geq 0$ if and only if $v_1 \leq \bar{v}_1$. The threshold $\bar{v}_1$ is the unique solution to $g(a(v_1)^s) = 0$. Since $\frac{dx}{ds} > 0$, $\frac{dg}{d\eta} > 0$, and $\frac{dg}{d\alpha} < 0$, the Implicit Function Theorem implies that $\bar{v}_1$ is strictly increasing in δ and γ, but strictly decreasing in s and α. $\bar{v}_1 < 1$ if $g(a(1)^s) < 0$ which is equivalent to $\alpha > \bar{\alpha}(\gamma, \delta, s)$ since $g(a(1)^s)$ is strictly decreasing in α. For $v_1 \leq \bar{v}_1$, E is maximized by setting $w_1^* = 1$. For $v_1 > \bar{v}_1$ there exists a unique $x^* = \frac{\bar{a}+\eta}{\bar{a}-\eta} \in (1, a(v_1)^s)$ such that $g(x^*) = 0$ and E is maximized by choosing $w_1^* \in (\frac{1}{2}, 1)$. The comparative statics with respect to w_1^* follow from $\frac{dh^s}{dw_1} > 0$, $\frac{dh^s}{dv_1} > 0$, $\frac{dh^s}{ds} > 0$, $\frac{dh^s}{d\alpha} = \frac{sh^{s-1}v_1(2w_1-1)}{(1+\alpha v_1(1-w_1))^2} > 0$, $\frac{d}{d\eta}[\frac{\bar{a}+\eta}{\bar{a}-\eta}] = \frac{2\bar{a}}{(\bar{a}-\eta)^2} > 0$, $\frac{d}{d\alpha}[\frac{\bar{a}+\eta}{\bar{a}-\eta}] = -\frac{\eta v_1}{(\bar{a}-\eta)^2} < 0$, and $\frac{d}{dv_1}[\frac{\bar{a}+\eta}{\bar{a}-\eta}] = -\frac{\eta\alpha}{(\bar{a}-\eta)^2} < 0$. ■

Proof of Proposition 2

For $v_1 < \bar{v}_1$, $w_1^*(v_1) = 1$ and the organizer's objective function becomes

$$E(v_1, w_1^*(v_1)) = s(1-v_1)\left\{\frac{\gamma\delta}{2}\frac{a(v_1)^s - 1}{a(v_1)^s + 1} + 2(1-\gamma)\frac{\bar{a}a(v_1)^s}{(1+a(v_1)^s)^2}\right\}. \tag{7}$$

Since the constraint $w_1^*(v_1) \leq 1$ is binding, it follows from the proof of Proposition 1, that the term in parenthesis is increasing in $a(v_1) = 1 + \alpha v_1$. It is also increasing in $\bar{a} = 1 + \frac{1}{2}\alpha v_1$. Both increases are stronger when α is large. Since $\bar{v}_1$ is decreasing in α, we can therefore choose α sufficiently large, so that $E(v_1, w_1^*(v_1))$ is increasing in v_1 for all

$v_1 \leq \bar{v}_1$. It then becomes optimal to choose a v_1^* such that $v_1^* > \bar{v}_1$. For $v_1 > \bar{v}_1$, $w_1^*(v_1)$ solves $h^s = \frac{\bar{a}+\eta}{\bar{a}-\eta}$. Substitution into (4) gives

$$E(v_1, w_1^*(v_1)) = (1-\gamma)s\frac{1-v_1}{2+\alpha v_1}[\eta^2 + (1+\frac{1}{2}\alpha v_1)^2]. \tag{8}$$

We have $\frac{dE}{dv_1} = s\frac{1-\gamma}{(2+\alpha v_1)^2}q(v_1)$ where

$$q(v_1) = 2\alpha(1+\frac{1}{2}\alpha v_1)^2(1-v_1) - (2+\alpha)[\eta^2 + (1+\eta)^2]. \tag{9}$$

Note that $q(1) < 0$. Moreover, when α is sufficiently large, $q(\bar{v}_1) > 0$. Since

$$\frac{\partial q}{\partial v_1} = 2\alpha(1+\frac{1}{2}\alpha v_1)(\alpha - 1 - \frac{3}{2}\alpha v_1) \tag{10}$$

the function $q(.)$ is either strictly decreasing in $(\bar{v}_1, 1)$ or it is first increasing and then decreasing. Hence $q(.)$ crosses zero exactly once and from above, i.e. there exists a unique $v_1^* \in (\bar{v}_1, 1)$ at which E is maximized. The comparative statics for v_1^* follow from $\frac{\partial q}{\partial \eta} < 0$, $\frac{\partial q}{\partial s} = 0$, and $\frac{\partial q}{\partial \alpha} > 0$. The comparative statics for w_1^* follow from the comparative statics of v_1^* and Proposition 1. ■

References

Azmat, G., Möller, M. (2009), Competition Amongst Contests. *RAND Journal of Economics*, **40**, 743-768.

Gershkov, A., Perry, M. (2009), Tournaments with Midterm Reviews. *Games and Economic Behavior*, **66**, 162-190.

Gradstein, M., Konrad, K., A. (1999), Orchestrating rent seeking contests. *Economic Journal*, **109**, 536-545.

Konrad, K., A., Kovenock, D. (2009), Multi–battle contests. *Games and Economic Behavior*, **66**, 256-274.

Konrad, K., A. (2009), *Strategy and Dynamics in Contests*, Oxford University Press.

Krishna, V., Morgan, J. (1998), The Winner–Take–All Principle in Small Tournaments. *Advances in Applied Microeconomics*, **7**, 61-74.

Lazear, E., Rosen, S. (1981), Rank order tournaments as optimal labor contracts. *Journal of Political Economy*, **89**, 841-864.

Meyer, M. A. (1992), Biased Contests and Moral Hazard. *Annales d'Economie et de Statistique*, **25**, 165-187.

Moldovanu, B., Sela, A. (2001), The Optimal Allocation of Prizes in Contests. *American Economic Review*, **91**, 542-558.

Moldovanu, B., Sela, A. (2006), Contest Architecture. *Journal of Economic Theory*, **126**, 70-97.

Münster, J. (2007), Contests with Investment. *Managerial and Decision Economics*, **28**, 849-862.

Rosen, S. (1986), Prizes and Incentives in Elimination Tournaments. *American Economic*

Review, **76**, 701-715.
Szidarovszki, F., Okugushi, K. (1997), On the Existence and Uniqueness of Pure Nash Equilibrium in Rent–Seeking Games. *Games and Economic Behavior*, **18**, 135-140.
Szymanski, S., Valletti, T. (2005), Incentive Effects of Second Prizes. *European Journal of Political Economy*, **21**, 467-481.
Tullock, G. (1980), Efficient Rent Seeking. In J. Buchanan et al., eds., *Towards a Theory of the Rent–Seeking Society* College Station: Texas A&M University Press.
Yildirim, H. (2005), Contests with Multiple Rounds. *Games and Economic Behavior*, **51**, 213-227.