Selling in Advance to Loss Averse Consumers*

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Abstract

This paper examines the influence of information on market performance in an advance purchase setting. Information reduces the risk that an advance purchase results in a mismatch between consumer preferences and product characteristics. However, information may also raise the number of advance purchases by increasing firms' incentive to offer advance purchase discounts. Accounting for consumers' aversion towards losses/risks turns out to be crucial as it changes our assessment of policies aiming to improve consumers' information: Under monopoly information can be detrimental both for efficiency and consumer surplus whereas under competition information is doubly beneficial because it mitigates inter-temporal business stealing.

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1 Introduction

In a variety of markets, consumers purchase products in advance, that is, with imperfect knowledge of their preferences. Advance selling is standard in ticketing markets (e.g. transportation, entertainment, accommodation) and has become an important marketing tool for new to-be-released products in the form of pre-orders (e.g. music albums, video games, "en primeur" wine). Recent trends towards a customer-based financing of entrepreneurial activity (crowdfunding) have widened the scope for advance selling by allowing products to be sold even prior to their development.

Advance selling benefits firms in various ways as it allows them to obtain more accurate demand forecasts (Moe and Fader, 2002), manage their capacity (Liu and van Ryzin, 2008), screen their customers inter-temporally (Nocke et al., 2011), or steal future business from their rivals (Möller and Watanabe, 2016). However, due to the risk of a mismatch between consumer preferences and product characteristics, advance selling can be detrimental for a market's allocative efficiency. For example, travel-industry experts have claimed that in 2018 approximately seven million flight-tickets within Europe and \$2.6 billion worth of non-refundable hotel reservations globally were purchased without being consumed.¹ In pre-order and crowdfunding settings, the gap between "what consumers pay for" and "what consumers get" is more difficult to assess but advertising standards authorities have repeatedly upheld customer complaints about product-descriptions failing to reflect actual product-characteristics.

Regulators have taken measures to limit the risk of consumer-product mismatches in advance purchase markets. For example, in 2011 the U.S. Department of Transportation has introduced a "24-hour reservation requirement" allowing consumers to redeem airline tickets

¹See interview with Co-founders of SpareFare, a platform for resale of non-refundable airline tickets and hotel reservations, available at https://www.phocuswire.com/Startup-Stage-SpareFare.

at full refund within one day of purchase. Similar policies are called for in the hotel industry where non-refundable room reservations have increased drastically with the emergence of online-booking platforms.² While in some European countries, state-owned train-operators allow tickets to be pre-purchased up to 12 months in advance (e.g. Switzerland), other countries have limited their advance booking horizon to a mere 30 days (e.g. Spain). In an attempt to improve consumers' information in pre-order markets the 2015 amendment to the U.K. Consumer Rights Act (Part I, Chapter 3, Section 36) stipulates that "digital content will match any description of it given by the trader to the consumer". Finally, the European Commission's guideline that "complicated concepts or products are less suitable for rewards crowdfunding" can be interpreted as an attempt to reduce mismatch in markets for yet-to-be-developed products.³

Regulatory policies, such as the ones described above, improve a consumer's information at the advance-purchase stage, either directly, by requiring firms to provide more accurate product descriptions, or indirectly by limiting the time-interval between purchase and consumption. Unclear is what effect such policies have on firms' incentive to sell in advance. In particular, improving consumers' information could have the adverse effect of increasing the number of advance sales. This paper therefore aims to shed light on the following question: What is the overall effect on market performance of policies that improve consumers' information in advance purchase markets?

In the existing literature, an answer to the above question has proven evasive for two reasons. First, for simplicity most models assume that consumers are completely uninformed about their preferences during the advance purchase stage. Hence, a policy variable measuring the consumers' quality of information is missing. Second, although risk attitudes

²See USA today: "Non-refundable room rules leave some hot about hotels" available at https://eu.usatoday.com/story/travel/hotels/2014/03/31/non-refundable-hotel-room/7127665.

³See European Commission's Guide on Crowdfunding available at https://ec.europa.eu/growth/toolsdatabases/crowdfunding-guide/types/rewards_en.

form a viable element for the determination of a consumer's timing of purchase, most articles assume consumers to be risk neutral.⁴ In this paper, we propose a model in which the consumers' quality of information constitutes an exogenous parameter, as in Möller and Watanabe (2016), and risk attitudes are incorporated via the concept of expectation-based loss aversion à la Kőzegi and Rabin (2006, 2007). Accounting for consumer loss aversion turns out to be crucial as it changes the assessment of policies aiming to reduce advance purchase mismatch not only quantitatively but also qualitatively. Importantly, loss aversion alters our assessment in opposite directions depending on whether advance selling serves to screen customers (monopoly) or to steal a rival's future business (competition).

In our model, two differentiated products are offered either by a single monopolistic supplier (monopoly) or by two rival firms (competition). Firms commit to a pricing policy consisting of a regular price and an advance purchase discount. Consumers may purchase in advance at a discount but with imperfect information about their preferences. Consumers differ in their (privately known) "choosiness" measuring the extra utility a consumer derives from consuming his preferred product. The quality of information at the advance purchase stage is assumed identical across consumers and subject to regulatory policy.

We incorporate risk attitudes by assuming that consumers are expectation-based loss averse. Loss aversion entails consumers to evaluate outcomes in relation to a reference point and to weigh losses more heavily than gains (Kahneman and Tversky, 1979). Reference points arise endogenously through expectations as proposed by Kőzegi and Rabin (2006, 2007). Expectation-based loss aversion is well documented, with evidence both from the field (Card and Dahl, 2011; Crawford and Meng, 2011; Pope and Schweitzer, 2011; Allen et al., 2017) and the lab (Abeler et al., 2011; Ericson and Fuster, 2011; Gill and Prowse, 2012;

⁴Exceptions are the monopoly models by Xie and Shugan (2001) and Tang et al. (2004) and the more recent attempts to model consumer risk attitudes using non-standard preferences by Zhao and Stecke (2010) and Nasiry and Popescu (2012). See our review below. Yet, to the best of our knowledge we are the first to provide an advance selling model with competition incorporating risk preferences.

Karle et al., 2015) and has been applied to a large range of economic settings.⁵ We focus on loss aversion because for risky choices involving relatively small stakes, such as the choice between alternative (differentiated) products, loss aversion seems to offer a more adequate description of individual decision making than risk aversion (see the calibration theorem by Rabin).⁶

In the first part of the paper we determine the effect of information on the prevalence of advance selling and the market's allocative efficiency. In the absence of loss aversion, the fraction of consumers a monopolist serves in advance is independent of the consumers' information which implies that improving information has a positive effect on allocative efficiency. Accounting for loss aversion reveals that this conclusion might be flawed. We show that, in the presence of loss aversion, monopolistic advance selling is more frequent in an informed market than in an uninformed market. Policies aiming at a reduction of advance purchase mismatch in monopolistic markets via the improvement of consumers' information must therefore be assessed as less effective and, as we show, can even be detrimental when loss aversion is taken into account.

Surprisingly, under competition, loss aversion affects the way in which advance selling depends on information in the exact opposite direction. For the case of a uniform distribution of consumer types, Möller and Watanabe (2016) have shown that, in the absence of loss aversion, advance selling is monotonically increasing with information. We extend this insight by showing that, under competition, advance selling is more frequent in an informed market than in an uninformed market as long as the distribution of consumer types has an increasing hazard rate. However, we also show that this conclusion becomes reversed when consumers are sufficiently loss averse. Accounting for loss aversion offers the insight that improving

⁵See Grubb (2015) and Heidhues and Kőszegi (2018) for a recent overview.

⁶Yet, as we show in our discussion section (Section 6.3), an analysis with CARA-risk preferences, when tractable, leads to qualitatively similar results. We also clarify that in our competitive model, loss aversion has a similar effect as some form of anticipated regret (Bell, 1985; Loomes and Sugden, 1986, 1987).

consumer information can be doubly beneficial; it reduces the likelihood that an advance purchase by any particular consumer results in a mismatch and *decreases* the number of consumers that competing firms convince (by way of a discount) to purchase in advance.

While the above results hold very generally, i.e. for a large class of distributions of consumer types, our analysis of pricing and consumer surplus contained in the second part of the paper assumes types to be distributed uniformly. This assumption is standard in models investigating the influence of loss aversion on (static) pricing (e.g. Heidhues and Kőszegi, 2008; Karle and Peitz, 2014) and makes our results comparable to this literature (reviewed below). In the absence of loss aversion, prices turn out to be monotonically decreasing with information under monopoly whereas under competition prices are monotonically increasing. Hence, without loss aversion, we would conclude that information benefits consumers under monopoly but harms consumers under competition. Accounting for consumer loss aversion reveals that the effect of information can be rather the opposite. In particular, with loss aversion, prices can increase with information under monopoly whereas under competition (spot) prices can be decreasing, leaving at least some of the (choosy) consumers better off.

In summary, accounting for consumer loss aversion in advance purchase markets allows for two novel insights: (1) Under monopoly, improving consumers' information can be harmful both for (allocative) efficiency and consumer surplus. (2) Under competition, improving information can be doubly beneficial for welfare and may benefit late purchasers through a decrease in spot-prices. Hence, one way to summarize the key message of our paper is that, from a regulatory viewpoint, policies which induce competition in advance purchase markets should go hand in hand with policies that improve consumers' information.

The plan of the paper is as follows. After reviewing related literature we introduce the setup and derive consumer demand in Sections 2 and 3, respectively. In Section 4 we start

our analysis with a focus on the effect of information on advance selling and allocative efficiency before turning our attention to pricing and consumer surplus in Section 5. In Section 6 we comment on firms' disclosure incentives by considering how profits vary with information and discuss alternative consumer preferences such as risk aversion and anticipated regret. While in our setup, firms are assumed to be symmetric, in Section 6 we also show how asymmetries can be easily included into our model. Section 7 concludes. All proofs are contained in the Appendix.

Related Literature. This paper contributes to our understanding of advance purchase markets. Originating from the marketing and operations research literature (Xie and Shugan, 2001; McCardle et al., 2004; Tang et al., 2004) with its focus on capacity planning and revenue management, the economic literature has emphasized inter-temporal price discrimination and business stealing as potential explanations for the prevalence of advance selling in markets characterized by demand uncertainty.⁷ None of the existing models parameterizes the quality of consumer information at the advance purchase stage, with the exception of Möller and Watanabe (2016), whose focus lies on the effects of market structure. The main contribution of our article is a comprehensive investigation of the role of advance-purchase information, for which the accommodation of risk attitudes (in the form of consumer loss aversion) turns out to be crucial.

Although risk attitudes constitute an important determinant of the consumers' timing of purchase, the existing literature on advance purchase markets has mostly abstracted from this issue by focusing on risk-neutral consumers with standard preferences. There are two notable exceptions. Zhao and Stecke's (2010) model of a monopolistic pre-order market

⁷See Gale and Holmes (1993), DeGraba (1995), Courty (2003), Nocke et al. (2011), and Möller and Watanabe (2010) for monopoly, Gale (1993) and Möller and Watanabe (2016) for duopoly, and Dana Jr. (1998) for perfect competition.

assumes that a certain group of consumers incur a disutility of constant, exogenous size when their surplus from placing a pre-order turns out to be negative. They show that the firm can benefit from segmenting the market by inducing this group of "loss-averse" consumers to postpone their purchase. We differ from Zhao and Stecke (2010) in allowing reference points to be idiosyncratic (given by expectations) and (dis-) utilities to depend on the size of the deviations from the reference point. Moreover, our focus on the consumers' risk of buying the *wrong* product enables a comparison of the case of a (multi-product) monopoly with the case of competition. An important insight of our analysis is that consumer loss aversion can have very different effects when pre-orders are used to steal business rather than to segment a market.

Nasiry and Popescu (2012) consider advance selling when consumers perceive regret. In their model, a monopolist sells a single product to ex-ante identical consumers with uncertain valuations who may experience two sorts of regret. Early buyers incur a disutility when their valuation turns out to be below the price they paid in advance (*action-regret*). In contrast, late buyers incur a disutility when they should have bought the seller's product at a lower price in advance (*inaction-regret*). Hence, a consumer's reference point consists of his optimal rather than his expected outcome. In Nasiry and Popescu (2012), the consumers' inter-temporal preferences and their influence on firm conduct depend on the relative weight of action-regret versus inaction-regret. They coincide with the preferences of a consumer with standard preferences when both types of regret are equally important. In our setting, the effects of loss aversion in money and loss aversion in taste on the consumers' propensity to purchase in advance are similarly opposed. However, in our model the effect of consumers' non-standard preference on firm behavior is unambiguous. This enables us to derive predictions for market performance that depend exclusively on observable market characteristics, like the number of firms in the industry, or the quality of the consumers' information.

Loss aversion and its effect on competition has been considered for markets *without* an advance purchase option.⁸ Heidhues and Kőszegi (2008) show that in markets for inspection goods, loss aversion can explain the frequently observed tendency for (not necessarily identical) firms to charge a common "focal" price, leading to a reduction in price-variation. In contrast, the effect of loss aversion on price *levels* has been found to be indeterminate. Karle and Peitz (2014) show that whether loss aversion leads to an increase or a decrease in prices depends on the relative strength of loss aversion in the taste- and in the money-dimension. While, in these articles, consumers form expectations based on a consumption plan (confirmed in equilibrium), at the (unique and exogenous) time of purchase each product's price and match value are well known. In an advance purchase setting, the consumers' timing of purchase and its influence on the types of gains or losses experienced adds an important new element. Although seemingly more complicated, our setting allows for the strong conclusion that prices are increasing in loss aversion, independently of whether loss aversion acts in the money- or the taste-dimension, and that loss aversion increases (inter-temporal) price variation.

2 Model

Setup. There are two differentiated products, *A* and *B*, and a continuum of consumers with mass two, each having unit demand. Products are offered during two periods; an advance purchase period (1) and a consumption period (2).⁹ We distinguish between two market structures. Under *competition* there are two firms $j \in \{A, B\}$ each offering one product.

⁸For the effects of loss aversion in monopolistic settings see Spiegler (2012), Herweg and Mierendorff (2013), Heidhues and Kőszegi (2014), Rosato (2016), Carbajal and Ely (2016), Hahn et al. (2017), and Karle and Schumacher (2017).

⁹Although simplistic, a two-period approach resonates well with the temporal pattern of sales in pre-order markets, where most transactions take place right before and right after the release date, as found by Hui et al. (2008) for the case of DVDs.

Under *monopoly*, both products are offered by a single firm. For simplicity we abstract from production costs. Firms can commit to a pricing policy (p_j, z_j) , consisting of a *price level* p_j and an *advance purchase discount* z_j .¹⁰ While a consumer is required to pay p_j for product *j* during the consumption period, in the advance purchase period he pays only $p_j - z_j$.

Information. In the advance purchase period each consumer receives a private signal about the identity of his preferred product. The product indicated by the signal will be denoted as the consumer's *favorite* product. In the consumption period, all consumers learn the identity of their preferred product. The quality of information is the same for all consumers.¹¹ It is given by the signal's precision, $\gamma \in (\frac{1}{2}, 1)$, measuring the probability with which a consumer's favorite turns out to be his preferred product. The parameter γ constitutes a key element in our comparative statics analysis of the effects of information in advance purchase markets. Preferences constitute the consumers' private information and are introduced next.

Preferences. Consumers differ in their *choosiness* $\sigma \in [0, 1]$, measuring the intensity with which they prefer one product over the other. A consumer with choosiness σ obtains the value $s + \frac{1}{2}\sigma$ from consuming his preferred product and $s - \frac{1}{2}\sigma$ from consuming his nonpreferred product. The parameter s > 0 measures a consumer's average consumption value. It is assumed to be the same for all consumers. We make the standard assumption that s is sufficiently large for the market to be covered. Consumer choosiness σ is distributed with continuous and strictly positive density f and cumulative distribution function F. We assume that f has an increasing hazard rate, i.e. $\frac{f}{1-F}$ is non-decreasing.¹² To keep the model sym-

¹⁰In pre-order settings, firms commonly announce their product's release price in advance. In ticketing markets, the repeated nature of transactions provides firms with an incentive not to lower their prices close to the consumption date. For a discussion of the non-commitment case, see Möller and Watanabe (2016).

¹¹Note that we could introduce heterogeneity with respect to consumer information by allowing for a share of ex-ante informed consumers as in Karle and Peitz (2014). This would not change our qualitative results.

¹²This requirement is relatively mild. Every log-concave distribution has an increasing hazard rate and logconcavity is satisfied by most commonly used densities. For examples see Bagnoli and Bergstrom (2005). Trivially, the hazard rate of f is increasing when more choosy consumers are (weakly) more frequent than less choosy ones, i.e. when f is non-decreasing.

metric, we assume that for each level of choosiness, the mass of consumers whose preferred product is *A* is identical to the mass of consumers whose preferred product is *B*.

Loss aversion. A consumer's overall utility consists of his *intrinsic utility*, given by the difference between his product's consumption value and its price, and a *gain-loss utility*. Consumers are loss averse in that losses loom larger than gains. In particular, we make the standard assumption that gains and losses are weighted by η and $\lambda \eta$, respectively, with $\eta > 0$ denoting the diversion from standard preferences and $\lambda > 1$ measuring the consumers' degree of loss aversion. To abbreviate notation we define $l \equiv \eta(\lambda - 1) > 0$. We make the standard assumption of no-dominance of gain-loss utility by requiring that l < 2 (see for example Herweg et al., 2010).¹³ Following Kőzegi and Rabin (2006) we assume that gains and losses are evaluated separately in the taste and the money dimension, with the consumer's expectations serving as the reference point.

Equilibrium concept: After observing prices (p_A, z_A) , (p_B, z_B) and the identity of his favorite product $i \in \{A, B\}$ a consumer decides when and which product to purchase. A consumer's *consumption plan* (i.e. his strategy) induces expectations about the payments he will make and the consumption value he will receive. If a consumer buys in period 1, his payment is deterministic but match value is random. Hence, the consumer's gain-loss utility originates from potential differences between expected and realized consumption values. In contrast, a consumer who waits until period 2 (in order to guarantee the purchase of his preferred product) faces uncertainty with respect to his payment (when products are priced differently). Hence, in this case, the consumer's gain-loss utility stems from the difference between the price the consumer expects to pay (for the product that he turns out to prefer) and the price he actually pays. According to Kőzegi and Rabin (2006), consumption plans have to be credible in the sense that given the expectations (i.e. reference points) they induce,

¹³This assumption guarantees uniqueness of equilibrium in the limiting case of a completely uninformed market but has no qualitative effect on our results. For details see proof of Proposition 2.

it must be optimal for a consumer to follow them through. Formally, a credible consumption plan constitutes a *personal equilibrium* (PE). In our setting, there exist three possibilities for a personal equilibrium: "buy favorite product in period 1", "buy non-favorite (but cheaper) product in period 1", and "buy preferred product in period 2".¹⁴ We assume that consumers always play their preferred personal equilibrium (PPE), i.e. the PE that maximizes their expected overall utility. Hence our solution concept is subgame-perfect Nash equilibrium with each consumer following his PPE strategy.

3 Demand

In this section we consider the consumers' purchase decision, for given pricing policies (p_A, z_A) and (p_B, z_B) . Consumers need to determine whether to buy in advance or whether to postpone their purchase until the consumption period. They also need to choose between products *A* and *B*. Besides the potential differences in their choosiness, consumers differ with respect to the realization of their first period signal. Half of the consumers favor product *A* whereas the other half favor product *B*. We will denote these two types of consumers as *A*-types and *B*-types respectively. In order to gain a better understanding of the separate effects of loss aversion in money and loss aversion in taste, in this section we denote the loss aversion parameter *l* in the money and taste dimensions as $l_m > 0$ and $l_t > 0$, respectively.

3.1 Buying favorite versus buying non-favorite

Consider a consumer's product choice in the advance purchase market. If a consumer with choosiness σ buys his favorite product in period 1 he expects a match value of size $s + \gamma \frac{\sigma}{2}$ –

 $^{^{14}}$ Any consumption plan which involves buying a particular product in period 2 (e.g. product *A* whether preferred or not) is (weakly) dominated by the plan to buy the same product in period 1 because of the existence of an advance purchase discount. In other words, paying for additional information by forfeit of a discount makes sense only when the information is actually employed.

 $(1 - \gamma)\frac{\sigma}{2} = s + (\gamma - \frac{1}{2})\sigma$. With probability γ he will be lucky obtaining the actual match value $s + \frac{\sigma}{2}$ leading to a taste-gain with respect to the reference point of size $\eta\{s + \frac{\sigma}{2} - [s + (\gamma - \frac{1}{2})\sigma]\} = \eta(1 - \gamma)\sigma$. With probability $(1 - \gamma)$ the consumer will be unlucky obtaining the actual match value $s - \frac{\sigma}{2}$ leading to a taste-loss with respect to the reference point of size $\eta\lambda[s + (\gamma - \frac{1}{2})\sigma - (s - \frac{\sigma}{2})] = \eta\lambda\gamma\sigma$. Hence an *i*-type consumer's expected utility from the consumption plan of buying his favorite product *i* in period 1 is

$$U_{1,i}(\sigma, i) = s + (\gamma - \frac{1}{2})\sigma - (p_i - z_i) - l_t \gamma (1 - \gamma)\sigma.$$
(1)

An *i*-type consumer's expected utility from the consumption plan of buying his non-favorite product $j \neq i$ in period 1 can be calculated analogously and is given by

$$U_{1,j}(\sigma, i) = s - (\gamma - \frac{1}{2})\sigma - (p_j - z_j) - l_t \gamma (1 - \gamma)\sigma.$$
 (2)

A trade-off arises in period 1, when a consumer's favorite product is more expensive than his non-favorite product. In that case, buying the less expensive, non-favorite product yields a higher expected utility if and only if the consumer's choosiness is sufficiently low, i.e. if and only if $U_{1,j}(\sigma, i) > U_{1,i}(\sigma, i) \Leftrightarrow \sigma \leq \overline{\sigma}$ where

$$\bar{\sigma} \equiv \frac{|p_i - z_i - p_j + z_j|}{2\gamma - 1}.$$
(3)

Note that loss aversion has no effect on the relative comparison of the consumption plans described above because the consumer's product-choice and the formation of his expectations are based on the same information. However, as we will see in the following subsection, loss aversion does have an effect on the consumer's timing of purchase, i.e. his choice between an uninformed and an informed purchase.

3.2 Buying early versus buying late

Consider a consumer's choice between the consumption plans of purchasing (his favorite product) in advance or postponing his purchase (to guarantee the consumption of his pre-

ferred product). If a consumer postpones his purchase until period 2, he will obtain a consumption value of size $s + \frac{\sigma}{2}$. However, when $p_A \neq p_B$, the consumer faces uncertainty about the price he will have to pay because ex ante the identity of his preferred product is uncertain. In particular, a consumer with favorite *i* expects to pay $\gamma p_i + (1 - \gamma)p_j$ and assuming for the sake of the argument that $p_i \leq p_j$, he will experience a money-gain of size $\eta[\gamma p_i + (1 - \gamma)p_j - p_i] = \eta(1 - \gamma)(p_j - p_i)$ with probability γ and a money-loss of size $\eta\lambda\{p_j - [\gamma p_i + (1 - \gamma)p_j]\} = \eta\lambda\gamma(p_j - p_i)$ with probability $1 - \gamma$. More generally, an *i*-type consumer's expected utility from the consumption plan of purchasing his preferred product in period 2 can be written as

$$U_{2}(\sigma, i) = s + \frac{\sigma}{2} - \gamma p_{i} - (1 - \gamma)p_{j} - l_{m}\gamma(1 - \gamma)|p_{j} - p_{i}|.$$
(4)

From (1), (2), and (4) it becomes obvious that, while loss aversion in taste decreases a consumer's propensity to buy early, loss aversion in money reduces the consumer's willingness to purchase late. While loss aversion in money affects all consumers equally, the effect of loss aversion in taste is stronger for consumers with higher degrees of choosiness. As a consequence, the consumption plan of postponing his purchase until period 2 dominates a consumer's plan of purchasing his favorite product *i* in period 1 if and only if his choosiness is sufficiently high, i.e. $U_2(\sigma, i) > U_{1,i}(\sigma, i) \Leftrightarrow \sigma > \sigma_{Wi}$ where

$$\sigma_{Wi} \equiv \frac{(1-\gamma)(p_j - p_i) + z_i}{(1-\gamma)(1+l_i\gamma)} + \frac{l_m\gamma}{1+l_i\gamma}|p_j - p_i|.$$
(5)

A comparison with the consumption plan of purchasing the consumer's non-favorite product $j \neq i$ in advance is not necessary because, in equilibrium, prices will be identical so that $\bar{\sigma} \approx 0$. When $(p_A, z_A) = (p_B, z_B) = (p, z)$ the fraction of consumers who purchase in advance is given by

$$\sigma_W \equiv \frac{z}{(1-\gamma)(1+l_t\gamma)}.$$
(6)

Note from this equation that only loss aversion in taste has a direct effect on advance selling. Loss aversion in taste reduces advance selling directly by lowering the consumers' willingness to make an uninformed purchase. However, loss aversion (in both dimensions) may also have an indirect effect on advance selling by changing the discount *z* that is being offered. In particular, loss aversion in money affects the profitability of deviations from equilibrium that induce regular prices to be different, $p_A \neq p_B$. It is the interaction of these effects that renders the analysis of the influence of loss aversion on advance selling non-trivial.

4 Advance selling and allocative efficiency

In advance purchase settings where consumers have unit demands and valuations that are large enough for the market to be covered, the fraction of advance sales $F(\sigma_W)$, provides us with a measure of market performance. More specifically, in our setting, allocative efficiency is given by

$$V = 2s + 2\int_0^{\sigma_W} [\gamma \sigma - (1 - \gamma)\sigma] f(\sigma) d\sigma + 2\int_{\sigma_W}^1 \sigma f(\sigma) d\sigma.$$
(7)

V measures the (expected) aggregate consumption value generated by the market's allocation of the products *A* and *B* when consumers with choosiness $\sigma \in [0, \sigma_W]$ are induced to purchase in advance while consumers with choosiness $\sigma \in (\sigma_W, 1]$ guarantee the consumption of their preferred product by postponing their purchase. Note that, in the presence of loss aversion, allocative efficiency differs from welfare, in that it fails to account for the gain-loss term $-l_t\gamma(1 - \gamma)\sigma$ in the consumers' (expected) utility from advance purchases. Independently of which measure of market performance is considered, market performance is strictly decreasing in the threshold σ_W , or equivalently in the fraction of advance sales $F(\sigma_W)$. For this reason, the fraction of advance sales, $F(\sigma_W)$, is the focus of our analysis in this section. Our goal is to understand how the relationship between advance selling $F(\sigma_W)$ and information γ is affected by the existence of consumer loss aversion l; we set $l_t = l_m = l$ from here onwards for simplicity.¹⁵ Understanding this relationship is important because, as can be seen from (7), information affects market performance not only directly (by reducing the likelihood of mismatches) but also indirectly through its influence on the inter-temporal allocation of sales. In Sections 4.1 and 4.2 we determine the fraction of advance sales $F(\sigma_W)$ separately for the cases of monopoly and competition before comparing the two market structures in Section 4.3. In both cases, accounting for loss aversion leads to a systematic change in the way advance selling depends on information.

4.1 Monopoly

We start our analysis by considering a monopolistic market where products *A* and *B* are sold by the same firm. Due to the model's symmetry, the monopolist will choose the same pricing strategy (p^M, z^M) for his two products.¹⁶ Consumers with choosiness $\sigma \leq \sigma_W^M = \frac{z^M}{(1-\gamma)(1+l\gamma)}$ will purchase their favorit product in advance whereas consumers with choosiness $\sigma > \sigma_W^M$ will postpone their purchase until the consumption period. From (1) it follows that a consumer's utility from purchasing his favorite product in advance is linear in his type σ . For

$$\gamma - \frac{1}{2} > l\gamma(1 - \gamma) \Leftrightarrow \gamma > \gamma^{M}(l) \equiv \frac{l - 1 + \sqrt{1 + l^{2}}}{2l},$$
(8)

¹⁵Kőzegi and Rabin (2006) propose narrow bracketing, i.e. a separate reference comparison in a money and a taste dimension, respectively. It seems natural to assign the same scale to both dimensions. Theoretically, different scales can be considered by introducing different degrees of loss aversion in the two dimensions, as we do in Section 3.2 (see Karle and Peitz, 2014 for an in-depth analysis of the competitive effects of loss aversion in the two dimensions). Empirically, if any, there is some evidence that loss aversion in the money dimension might be less pronounced in certain contexts than that in the taste dimension (cf. DellaVigna, 2009, page 326).

¹⁶ One may suspect that setting asymmetric (regular) prices, $p_A \neq p_B$, to loss-averse consumers could be optimal in spite of the model's symmetry. Indeed, when consumers are loss averse, setting $p_A \neq p_B$ renders purchasing in advance more attractive, i.e. asymmetric prices may serve as a substitute for a discount. Yet, as we show in the proof of Proposition 1, this presumption is incorrect, i.e. the monopolist's profit maximizing pricing strategy is symmetric.

advance-purchase utility is increasing in σ and hence minimal at $\sigma = 0$. For $\gamma < \gamma^{M}(l)$, advance-purchase utility is decreasing and minimal at $\sigma = \sigma_{W}^{M}$. Given our assumption of a covered market, all consumers who purchase in advance must obtain non-zero utility. To extract the maximum rent, the monopolist must make the consumer with the lowest advance-purchase utility indifferent between buying and not buying.¹⁷ Hence it must hold that

$$p^{M} - z^{M} = \begin{cases} s - [l\gamma(1-\gamma) - \gamma + \frac{1}{2}]\sigma_{W}^{M} & \text{if } \gamma < \gamma^{M}(l) \\ s & \text{if } \gamma \ge \gamma^{M}(l) \end{cases}$$
(9)

and substitution into the monopolist's profits

$$\Pi^{M} = 2\{(p^{M} - z^{M})F(\sigma_{W}^{M}) + p^{M}[1 - F(\sigma_{W}^{M})]\}$$
(10)

reveals that the monopolist's problem can be reduced to the choice of σ_W^M that maximizes

$$\frac{\Pi^{M}}{2} = \begin{cases} s - [l\gamma(1-\gamma) - \gamma + \frac{1}{2}]\sigma_{W}^{M} + (1-\gamma)(l\gamma+1)\sigma_{W}^{M}(1-F(\sigma_{W}^{M})) & \text{if } \gamma < \gamma^{M}(l) \\ s + (1-\gamma)(l\gamma+1)\sigma_{W}^{M}(1-F(\sigma_{W}^{M})) & \text{if } \gamma \ge \gamma^{M}(l). \end{cases}$$
(11)

It is immediate that for $\gamma \ge \gamma^M(l)$ the solution to this maximization problem is independent of the consumers' degree of loss aversion. Define this solution as

$$\sigma_W^0 \equiv \arg \max_{\sigma_W \in [0,1]} \sigma_W (1 - F(\sigma_W)).$$
(12)

Our assumption that f has an increasing hazard rate guarantees that σ_W^0 is uniquely defined and $\sigma_W^0 \in (0, 1)$. For $\gamma < \gamma^M(l)$, the need to compensate type σ_W^M for his loss aversion through a price reduction of size $[l\gamma(1 - \gamma) - \gamma + \frac{1}{2}]\sigma_W^M$ lowers the monopolist's incentive to sell in advance. In the Appendix we prove the following:

Proposition 1 (Advance Selling - Monopoly). For every $l \in (0, 2)$ there exists $\gamma^{M}(l) \in (\frac{1}{2}, 1)$ such that the following holds. For $\gamma \geq \gamma^{M}(l)$ the fraction of consumers a monopolist serves in advance is independent of l and the same as without loss aversion, i.e. $F(\sigma_{W}^{M}) = F(\sigma_{W}^{0})$. For $\gamma < \gamma^{M}(l)$, loss aversion reduces advance selling, i.e. $F(\sigma_{W}^{M}) < F(\sigma_{W}^{0})$.

¹⁷Note that a consumers' expected utility from purchasing in period 2 is always increasing in his type σ . Hence, participation of types in $[0, \sigma_W^M]$ guarantees participation of types in $(\sigma_W^M, 1]$

In the absence of loss aversion, the monopoly allocation of sales is independent of the consumers' information. This changes when loss aversion is taken into account. As can be seen from the left hand panel of Figure 1, loss aversion reduces monopolistic advance selling when consumer's are relatively uninformed about their preferences. The figure depicts the

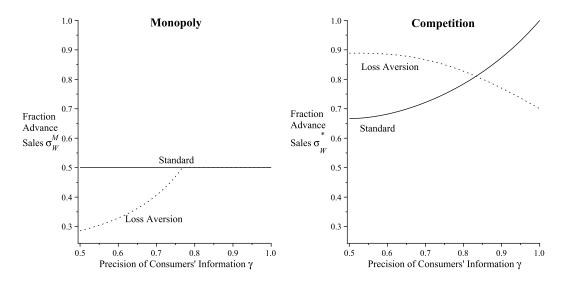


Figure 1: Fraction of advance sales. σ_W as a function of the consumers' quality of information γ when the distribution of consumer choosiness *F* is uniform. The left hand panel shows the monopoly benchmark, the right hand panel shows the case of competition. Solid curves depict standard preferences (l = 0), dashed curves depict loss aversion (l = 1.5).

case of a uniform distribution F of consumer types for which the fraction of advance sales $F(\sigma_W^M)$ is identical with the threshold type and is given by

$$\sigma_W^M = \begin{cases} \frac{1}{4(1-\gamma)(l\gamma+1)} & \text{if } \gamma < \gamma^M(l) \\ \frac{1}{2} & \text{if } \gamma \ge \gamma^M(l). \end{cases}$$
(13)

With a uniform distribution, or more generally for f non-increasing (see proof of Proposition 1), monopolistic advance selling becomes an increasing function of the quality of the consumers' information.

To understand this result, first note that a monopolist sets advance purchase prices to extract all surplus from the consumer with the lowest utility amongst the (relatively unchoosy) advance-customers. When consumers are loss averse and sufficiently uninformed, this consumer no longer coincides with the type of minimal choosiness but instead is given by the threshold type, i.e. the one indifferent with respect to the timing of purchase. Improving information reduces the compensation necessary to guarantee the participation of the threshold type. This allows the monopolist to induce advance purchases from consumers with higher degrees of choosiness, bringing his inter-temporal allocation of sales closer to the one that is optimal in the absence of loss aversion.

Investigating a firm's incentive to provide consumers with information about its product's characteristics, Lewis and Sappington (1994), Bar Isaac et al. (2010), and Gill and Sgroi (2012) show that a monopolistic seller may often find it optimal to provide consumers with either full information or none. The limiting cases of an uninformed ($\gamma \rightarrow \frac{1}{2}$) and an informed market ($\gamma \rightarrow 1$) therefore deserve some special attention:

Corollary 1. Without consumer loss aversion, a monopolist's inter-temporal allocation of sales is independent of the quality of consumers' information. When consumers are loss averse (l > 0), a monopolist practices more advance selling in an informed market than in an uninformed market, i.e. $\lim_{\gamma \to 1} F(\sigma_W^M) > \lim_{\gamma \to \frac{1}{2}} F(\sigma_W^M)$.

Proposition 1 and its corollary show that, when consumer loss aversion is accounted for, a monopolist's incentive to sell in advance depends on the quality of the consumer's information. For a monopolistic market, policies that improve the consumers' information during the advance purchase period can then have the adverse effect of increasing the fraction of consumers who purchase in advance. As we will argue next, a very different picture emerges in the presence of competition.

4.2 Competition

Consider the case where products *A* and *B* are offered by two competing firms. In any symmetric equilibrium firms must offer discounts, because already a small discount can secure the advance sale to a (relatively unchoosy) consumer who will become the rival firm's customer in the future with strictly positive probability $1 - \gamma$. If firms choose pricing policies (p_A, z_A) and (p_B, z_B) then for $i, j \in \{A, B\}, i \neq j$, firm *i*'s profit is given by

$$\Pi_{i} = (p_{i} - z_{i})[F(\sigma_{Wi}) \pm F(\bar{\sigma})] + p_{i}\{\gamma[1 - F(\sigma_{Wi})] + (1 - \gamma)[1 - F(\sigma_{Wj})]\}.$$
(14)

Here the thresholds $\bar{\sigma}$, σ_{WA} , and σ_{WB} are given by (3) and (5) respectively, and we have used the fact that for small deviations from a symmetric equilibrium pricing policy it must hold that $\bar{\sigma} < \min\{\sigma_{WA}, \sigma_{WB}\}$. Furthermore, $\pm F(\bar{\sigma})$ represents the following case distinction: $-F(\bar{\sigma})$ if $\bar{\sigma} > 0$ and $+F(|\bar{\sigma}|)$ if $\bar{\sigma} \le 0$. The firm's revenue has four parts. In period 1, firm *i* sells at discounted price $p_i - z_i$ to i-type consumers who are sufficiently unchoosy to purchase in advance. Depending on whether firm *i*'s first period price, $p_i - z_i$, is smaller or larger than firm *j*'s first period price, $p_j - z_j$, in period 1 firm *i* also gains (+) j-type or loses (-) i-type consumers who are unchoosy enough to prefer the less expensive product over their favorite product. In period 2, firm *i* sells at regular price p_i to those consumers who were too choosy to buy in advance. Firm *i* sells with probability γ to consumers whose favorite was *i* and with probability $1 - \gamma$ to consumers whose favorite was *j*.

Taking derivatives of (14) with respect to p_i and z_i and substituting $p_A = p_B = p^*$ and $z_A = z_B = z^*$ yields two first order conditions:

$$(p^* - z^*)\frac{f(0)}{2\gamma - 1} + [p^* - \gamma p^* + (1 - \gamma)p^*]\frac{f(\sigma^*_W)}{(1 + l\gamma)} = 1 + z^* f(\sigma^*_W)$$
(15)

$$(p^* - z^*)\frac{f(0)}{2\gamma - 1} + (p^* - \gamma p^*)\frac{f(\sigma_W^*)}{(1 - \gamma)(1 + l\gamma)} = F(\sigma_W^*) + z^*\frac{f(\sigma_W^*)}{(1 - \gamma)(1 + l\gamma)}.$$
 (16)

Conditions (15) and (16) equate firm i's marginal gain from attracting additional customers (LHS) with the marginal loss from charging lower prices (RHS), both for a decrease in the

price level p_i and for an increase in the discount z_i , respectively. Note from the left hand sides that the gain in the number of customers consists of two parts. The first part corresponds to *j*-type consumers with choosiness $\sigma \approx 0$ who become attracted by firm *i*'s reduction in first period price. As first period price is given by the difference between price-level and discount, the effect of a decrease in p_i is the same as the effect of an increase in z_i . The second part corresponds to firm *i*'s gain in sales that is due to the fact that consumers with choosiness $\sigma \approx \sigma_W^*$ change their timing of purchase. We refer to this part as the gain from inter-temporal business stealing. A decrease in p_i induces some *i*-types to buy early rather than late and some *j* types to buy late rather than early. In both cases the probability of a sale for firm *i* increases by $1 - \gamma$. An increase in z_i affects the timing of purchase only for *i*-types, because *j* types would purchase product *j* in advance. Note that the consumers' timing of purchase is more reactive to changes in discounts than to changes in price-levels because the latter affects prices in the advance *and* in the spot market.

Conditions (15) and (16) have to be satisfied by any (symmetric) equilibrium in which firms make positive sales in both periods, defined as a "price-discrimination equilibrium" in Möller and Watanabe (2016).¹⁸ Using the relation $z^* = (1 - \gamma)(1 + l\gamma)\sigma_W^*$ these two equations can be combined leading to the following (implicit) equation for the corresponding threshold

¹⁸Without loss aversion, existence of such an equilibrium has been shown by Möller and Watanabe (2016) for the cases where γ is sufficiently small or where *F* is uniform. By continuity, these results remain valid as long as *l* is not too large. In the proof of Proposition 4 we show that for *F* uniform, the profit function (14) is strictly concave for all *l* < 2. Although our derivation of (15) and (16) imposes symmetry, for *F* uniform the corresponding system of four linear first-order equations allows for a unique solution and this solution is symmetric, i.e. no asymmetric price-discrimination equilibrium exists. Finally, note that corner solutions, with one firm occupying the entire market, are ruled out by the following fact: for each equilibrium candidate in which the market-occupying firm makes positive profits there exists a profitable deviation by the other firm undercutting the price and exceeding the discount of the market-occupying firm by $\epsilon > 0$, respectively.

type σ_W^* :

$$0 = 1 + (1 - \gamma)(1 + l\gamma)\sigma_{W}^{*}\left[\frac{f(0)}{2\gamma - 1} + f(\sigma_{W}^{*})\right]$$

$$- \left[F(\sigma_{W}^{*}) + \sigma_{W}^{*}f(\sigma_{W}^{*}) + \sigma_{W}^{*}\frac{(1 - \gamma)(1 + l\gamma)}{2\gamma - 1}f(0)\right]\frac{(1 - \gamma)(4\gamma - 2)l\gamma f(\sigma_{W}^{*}) + (1 + l\gamma)f(0)}{(2\gamma - 1)f(\sigma_{W}^{*}) + (1 + l\gamma)f(0)}.$$
(17)

For a uniform distribution F of consumer types the solution σ_W^* to this equation can be obtained in closed form as

$$\sigma_W^* = \frac{\gamma(l+2)}{\gamma(3+4(1-\gamma)) - 1 + \gamma l[2 - (1-\gamma)\gamma(l+4)]}.$$
(18)

We depict it in the right hand panel of Figure 1, both for the case of standard preferences (l = 0) and loss aversion (l > 0). As can be seen from the figure, loss aversion increases advance selling when consumers are relatively uninformed but decreases advance selling when the consumers' information is relatively precise. The following proposition shows that this insight is not restricted to a uniform distribution of consumer-types but holds more generally:

Proposition 2 (Advance Selling - Competition). Under competition, advance selling depends on the consumers' loss aversion and their distribution of choosiness as follows:

- 1. Loss aversion increases advance selling in uninformed markets but reduces advance selling in informed markets, i.e. $\frac{\partial}{\partial l} \lim_{\gamma \to \frac{1}{2}} F(\sigma_W^*) > 0$ and $\frac{\partial}{\partial l} \lim_{\gamma \to 1} F(\sigma_W^*) \le 0$, with strict inequality for all $l > \frac{1}{f(0)} 1$.
- 2. If f_1 and f_2 are two densities of choosiness such that $\frac{f_2(\sigma)}{1-F_2(\sigma)} \leq \frac{f_1(\sigma)}{1-F_1(\sigma)}$ for all $\sigma \in [0, 1]$, then in both of the above limits, the equilibrium threshold type σ_W^* is (weakly) larger under f_2 than under f_1 .

To understand the intuition for the first part of Proposition 2 consider first the limit of an informed market. When the consumers' uncertainty vanishes, discounts must converge to zero, because consumers are willing to pay only a tiny premium for an informed purchase. Hence $\lim_{\gamma \to 1} z^* = 0$ and from (15) the price level is readily determined as $\lim_{\gamma \to 1} p^* = \frac{1}{f(0)}$. Intuitively, in the limit of an informed market the price level is independent of consumer loss aversion. The effect of loss aversion on advance selling can thus be understood by considering the marginal effects of a discount as described by (16). Substitution of z^* from (6) and taking the limit $\gamma \to 1$ gives:

$$p^*f(0) + p^*\frac{f(\sigma_W^*)}{(1+l)} = F(\sigma_W^*) + \sigma_W^*f(\sigma_W^*).$$
(19)

From (19) it becomes clear that in the limit of an informed market, loss aversion's sole effect is a reduction in a firm's marginal gain from inter-temporal business stealing. Loss aversion reduces the number of consumers with choosiness $\sigma \approx \sigma_W^*$ who switch their timing of purchase in response to a larger discount. Hence, in an informed market, loss aversion reduces advance selling by making inter-temporal business stealing via discounts less profitable.

Now consider the limit of an uninformed market. For $\gamma \rightarrow \frac{1}{2}$ products appear homogenous in the advance purchase market and the first period price converges to marginal cost, i.e. $p^* - z^* \rightarrow 0$. Hence, in an uninformed market a firm's gain from a decrease in price or an increase in discount is entirely due to the corresponding changes in the consumers' timing of purchase. Moreover, for $\gamma \rightarrow \frac{1}{2}$, the gains from inter-temporal business stealing via price-level and the gains from inter-temporal business stealing via discount become identical, because price-level reductions are only half as effective but affect twice the number of consumers (*i* and *j* types instead of only *i* types). Hence in the limit of an uninformed market it must be the case that the costs of inter-temporal business stealing via price-levels (RHS of (15)) must be the same as the costs of inter-temporal business stealing via discounts (RHS of (16)). Using (6) it thus has to hold that

$$1 + \frac{1}{2}(1 + \frac{l}{2})\sigma_{W}^{*}f(\sigma_{W}^{*}) = F(\sigma_{W}^{*}) + \sigma_{W}^{*}f(\sigma_{W}^{*}).$$
(20)

The effect of an increase in loss aversion can be seen from equation (20). Loss aversion increases the cost of inter-temporal business stealing via price level relative to the cost of inter-temporal business stealing via discounts. In other words, in an uninformed market, loss aversion gives firms a stronger incentive to compete in discounts rather than in price levels, thereby increasing the prevalence of advance-selling.

While the first part of Proposition 2 is concerned with the influence of consumer loss aversion, the second part considers the effects of a change in the distribution of consumer types. When consumers become more choosy (F_2 dominates F_1 in terms of hazard rate dominance which implies first order stochastic dominance), firms respond by increasing their discounts, inducing consumers with higher levels of choosiness to purchase in advance. As there are less consumers with low degrees of choosiness under F_2 than under F_1 , the overall effect on the number of advance purchases, $F(\sigma_W^*)$, is indeterminate. Intuition would suggest that, as consumers become more choosy, competition is mitigated because the firms' products become more differentiated from the viewpoint of the average consumer. It turns out, however, that this conclusion requires additional assumptions on the shape of f.¹⁹

An immediate consequence of the first part of Proposition 2 is that loss aversion changes the way in which inter-temporal business stealing depends on the quality of consumers' information. In particular, Proposition 2 has the following implication:

Corollary 2. Without consumer loss aversion, competing firms practice more advance selling in an informed market than in an uninformed market, i.e. for l = 0 it holds that $\lim_{\gamma \to 1} F(\sigma_W^*) \ge \lim_{\gamma \to \frac{1}{2}} F(\sigma_W^*)$. When consumers are loss averse, this conclusion can be

¹⁹For instance, in the limit of an uninformed market, moving from f_1 to f_2 decreases the number of advance sales $F(\sigma_W^*)$ if $F_2^{-1}(x)f_2(F_2^{-1}(x)) \ge F_1^{-1}(x)f_1(F_1^{-1}(x))$ for all $x \in [0, 1]$ (see proof of Proposition 2). This requirement is more demanding than first order stochastic dominance which only guarantees that the quantile functions (inverse CDFs) satisfy $F_2^{-1}(x) \ge F_1^{-1}(x)$. Intuitively, the firms' incentive to sell to a certain mass \tilde{x} of consumers in advance is not only affected by the location of the corresponding threshold level of choosiness $\tilde{\sigma} = F^{-1}(\tilde{x})$ but also by the responsiveness $f(\tilde{\sigma})$ of the market's timing of purchase at that threshold.

reversed. In particular, if $f(0) > \frac{1}{3}$ then there exists $\underline{l} \in (0,2)$ such that for all $l \in (\underline{l},2)$, $\lim_{\gamma \to 1} F(\sigma_W^*) < \lim_{\gamma \to \frac{1}{2}} F(\sigma_W^*)$, i.e. improving consumers' information reduces advance selling.

Note that the weak inequality in Corollary 2 becomes strict under very mild conditions on the distribution of consumer types F. In particular, in the proof of Corollary 2 we show that, in the absence of loss aversion, there is *strictly* more advance selling in an informed market than in an uninformed market when the hazard rate of f is *strictly* increasing.

Proposition 2 and its corollary show that loss aversion introduces a systematic change in the way how firms' incentive to engage in advance selling depends on the consumers' information. With standard preferences, we obtain the conclusion that improving consumer information intensifies advance selling. Accounting for loss aversion shows that this must not be the case. Instead, improving consumer information can be double-beneficial, because it not only raises the accuracy of advance purchases but also because it *reduces* advance selling.

4.3 The effect of information on allocative efficiency

Equipped with our novel insights about the relation between information and advance selling, we now consider the effect of information on allocative efficiency. An increase in the consumers' precision of information, γ , has the direct and positive effect of reducing the likelihood with which an advance purchase results in a mismatch. In addition, γ affects allocative efficiency indirectly, through its influence on the intensity of advance selling, σ_W . The two effects can be seen nicely for the case of *F* uniform, for which allocative efficiency takes a particularly simple form:

$$V = 2s + 1 - 2(1 - \gamma)\sigma_W^2.$$
 (21)

In Figure 2 we depict allocative efficiency for F uniform, both under monopoly and competition, and with or without loss aversion. Focusing first on the case of a monopoly, we see

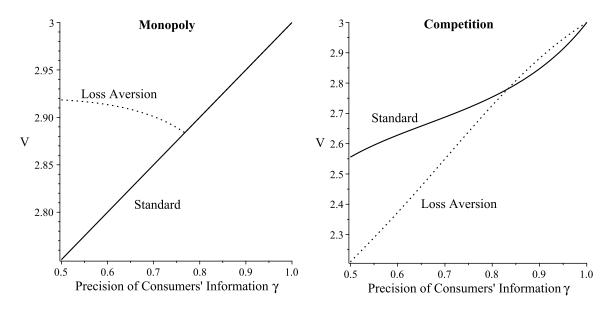


Figure 2: Allocative efficiency. V as a function of the quality of consumers' information γ for F uniform. The left hand panel shows monopoly, the right hand panel shows competition. Solid curves depict standard preferences (l = 0), dotted curves depict loss aversion (l = 1.5).

that without loss aversion (solid), allocative efficiency is monotonically increasing with information. This changes when consumers are loss averse (dotted). In particular, there exists a range of γ for which allocative efficiency is decreasing with information. In this range, the increase in the frequency of advance selling is strong enough to more than offset the reduced risk of a mismatch. Hence, for a monopoly, accounting for consumer loss aversion may change our assessment of information policies from positive to negative.

Quite the opposite happens in the case of competition, depicted in the right hand panel of Figure 2. Without loss aversion, allocative efficiency is increasing only moderately with information, because the increase in the number of advance purchases offsets, at least partly, the reduction in the likelihood of a mismatch. Note that while for the uniform distribution the overall effect is still positive, it is easy to find examples where information *reduces* allocative efficiency in the absence of consumer loss aversion.²⁰ When loss averse is accounted for, the effect of information on allocative efficiency is (more) positive, because advance selling increases less or even decreases with information. Hence, for the case of competition, accounting for loss aversion may change our assessment of information policies from negative to positive.

We can therefore conclude that accounting for consumer loss aversion and market structure constitute important elements in our assessment of information policy in advance purchase markets.

5 Pricing and consumer surplus

In this section we consider the effect of consumer loss aversion on prices. Our ultimate goal is to understand how the relation between information and consumer surplus is affected by the presence of consumer loss aversion. In line with the existing literature on this issue, we assume for the remainder that the distribution of consumer-types is uniform.

5.1 Monopoly

For F uniform, we can use (6), (9), and (13) to determine the monopolist's profit maximizing prices in closed form:

$$p^{M} - z^{M} = \begin{cases} s - \frac{l\gamma(1-\gamma) - \gamma + \frac{1}{2}}{4(1-\gamma)(1+l\gamma)} & \text{if } \gamma < \gamma^{M}(l) \\ s & \text{if } \gamma \ge \gamma^{M}(l) \end{cases}$$
(22)

$$p^{M} = \begin{cases} s + \frac{1}{4} - \frac{l\gamma(1-\gamma)-\gamma+\frac{1}{2}}{4(1-\gamma)(1+l\gamma)} & \text{if } \gamma < \gamma^{M}(l) \\ s + \frac{1}{2}(1-\gamma)(1+l\gamma) & \text{if } \gamma \ge \gamma^{M}(l) \end{cases}$$
(23)

where $\gamma^{M}(l)$ is as defined in (8).

²⁰Numerical analysis shows that this feature is rather generic for the family of truncated normal distributions. See the working paper version of this article available at https://ssrn.com/abstract=3126294.

Proposition 3 (Pricing - Monopoly). Suppose *F* is uniform. The monopolist's profit maximizing prices are given by (22) and (23). For $\gamma \ge \gamma^M(l)$, loss aversion increases regular price but has no effect on discounted price, i.e. $\frac{\partial p^M}{\partial l} > 0$ and $\frac{\partial p^M - z^M}{\partial l} = 0$. For $\gamma < \gamma^M(l)$, loss aversion has a negative effect on both prices i.e. $\frac{\partial p^M - z^M}{\partial l} < 0$ and $\frac{\partial p^M}{\partial l} < 0$.

To understand this result, recall that for small γ , the monopolist chooses the advance purchase price $p^M - z^M$ to make the consumer with type σ_W^M indifferent between participating and not participating in the market. Loss aversion lowers the consumers' expected utility from participating in the market. In order to compensate consumers for their increase in gain/loss disutility the monopolist reduces his prices $p^M - z^M$ and p^M (by equal amounts). For large enough γ , our analysis in the previous section has shown that the monopolist's profits are maximized when he sells to exactly a half of the consumers in advance. Loss aversion increases the discount that is necessary to achieve this and leads to an increase in the regular price p^M , given that advance purchase prices $p^M - z^M$ are set equal to *s*.

Accounting for loss aversion changes the way in which monopoly pricing depends on information, as summarized by the following corollary:

Corollary 3. Suppose F is uniform. Without consumer loss aversion, monopoly prices $p^M - z^M$ and p^M are constant and decreasing in the quality of consumers' information γ , respectively. When consumers are loss averse, there exists a range $(\frac{1}{2}, \gamma^M(l))$ in which monopoly prices are increasing in γ .

The effect of information on monopoly pricing is depicted in the left hand panel of Figure 4. Accounting for consumer loss aversion reveals that improving consumers' information may lead to an increase in monopoly prices. This insight is important because it implies that, in monopolistic advance purchase markets, information may have an adverse effect not only on (allocative) efficiency but also on consumer surplus. We will come back to this issue in Section 5.3.

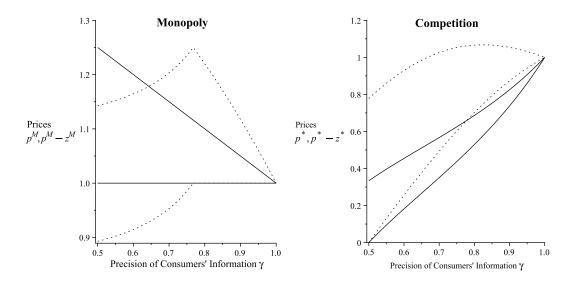


Figure 3: **Pricing.** Regular price p and discounted price p - z as a function of the quality of consumers' information γ for F uniform and s = 1. The left hand panel shows monopoly, the right hand panel shows the case of competition. Solid curves depict standard preferences (l = 0), dashed curves depict loss aversion (l = 1.5).

5.2 Competition

For *F* uniform, conditions (15) and (16) for a symmetric price-discrimination equilibrium (p^*, z^*) can be written as

$$0 = -1 + p^* \left[\frac{1}{2\gamma - 1} + \frac{2(1 - \gamma)}{1 + l\gamma} \right] - z^* \left[\frac{1}{2\gamma - 1} + 1 \right]$$
(24)

$$0 = \frac{p^* - z^*}{2\gamma - 1} - \frac{z^*}{(1 - \gamma)(1 + l\gamma)} - \frac{\gamma p^* - p^* + z^*}{(1 - \gamma)(1 + l\gamma)}.$$
(25)

Condition (24) shows the effect of a decrease in price-level on a firm's profit. Apart from the obvious loss from selling to a mass one of consumers at a lower price, a price cut has two effects; it increases the firm's total number of customers, and it raises the number of discounts the firm has to grant. From (24), the effect of loss aversion is straight forward. Loss aversion makes it harder to attract additional demand via a decrease in the price-level.

In addition, loss aversion also makes it less costly to attract additional demand via an

increase in discounts. This can be seen from condition (25) which shows how profit changes in response to an increase in discount. In equilibrium, the gain from attracting in advance consumers who favor the other firm's product must balance the losses from granting a higher discount to the firm's favoring consumers and from attracting more of those favoring consumers to purchase in advance rather than on the spot.

Loss aversion therefore induces a change to the nature of competition in advance purchase markets. It makes firms compete more strongly via discounts and less strongly via price-cuts. This intuition is formalized by Proposition 4 and follows from simple comparative statics applied to the solution of (15) and (16):

$$p^* = \frac{[3\gamma - 1 + l\gamma(1 - \gamma)](\gamma l + 1)}{-1 + 7\gamma - 4\gamma^2 + \gamma l[(\gamma - 1)\gamma(l + 4) + 2]}$$
(26)

$$z^* = \frac{(1-\gamma)\gamma(l+2)(\gamma l+1)}{-1+7\gamma-4\gamma^2+\gamma l[(\gamma-1)\gamma(l+4)+2]}.$$
(27)

Proposition 4 (Pricing - Competition). Let *F* be uniform. Loss aversion increases both prices and advance purchase discounts, i.e. p^* , $p^* - z^*$, and z^* are monotonically increasing in *l*.

Proposition 4 describes the effect of loss aversion on competing firms' choice of pricing policy. For oligopolistic markets without an advance purchase option it has been found that whether loss aversion has a pro-competitive or anti-competitive effect depends on the relative strength of loss aversion in the money and the taste dimensions. In particular, Karle and Peitz (2014) have shown that loss aversion acts anti-competitively in the taste-dimension but pro-competitively in the money-dimension. In light of this finding, it is important to note that Proposition 4 holds independently of whether loss aversion acts only in taste or only in money or receives equal weight in both dimensions.²¹ In an advance purchase setting, prices

²¹For details see the working paper version of this article, available at https://ssrn.com/abstract=3126294.

are increasing in loss aversion, independently of its dimension, because firms substitute competition in price levels by competition in advance-purchase discounts.

Finally, the following corollary shows that loss aversion also changes the way in which equilibrium prices depend on information:

Corollary 4. Suppose F is uniform. Without consumer loss aversion, equilibrium prices are increasing in the quality of consumers' information γ . For l > 1 there exists a range (γ^* , 1) where spot prices p^* are decreasing in γ .

The effect of information on equilibrium prices can be seen in the right hand panel of Figure 4. The reason why advance purchase prices $p^* - z^*$ are increasing in γ is that when consumers are relatively well informed, they differentiate strongly between products (already in the advance purchase market). Product differentiation mitigates competition, leading to higher prices. As shown by Proposition 4, loss aversion acts pro-competitively on discounts, i.e. it widens the gap between advance purchase prices $p^* - z^*$ and spot prices p^* . As this gap is largest for $\gamma \to \frac{1}{2}$ but vanishes for $\gamma \to 1$, loss aversion can make spot prices p^* decrease with information when consumers are relatively well informed.

5.3 The effect of information on consumer surplus

How does information affect consumer surplus in advance purchase markets? In order to answer this question consider (expected) aggregate consumer surplus:

$$CS = \int_0^{\sigma_W} [s + (\gamma - \frac{1}{2})\sigma - l\gamma(1 - \gamma) - (p - z)]d\sigma + \int_{\sigma_W}^1 [s + \frac{\sigma}{2} - p]d\sigma.$$
(28)

Consumer surplus consists of two parts; the surplus of consumers who purchase their favorite product in advance at price p - z, and the surplus of consumers who purchase their preferred product on the spot at price p. Note that, as common in the literature, the consumers' gain-loss utility is included in our calculation of consumer surplus.

Improving information has the obvious effect of enabling consumers to make better purchase decisions. When, in addition, information decreases prices the overall effect on consumer surplus is unambiguous. In this case, information increases consumer surplus. In contrast, when prices are increasing with information, the overall effect is unclear.

In Figure 5, we depict consumer surplus, both for the case of monopoly and the case of competition. In the absence of loss aversion, information affects consumer surplus in opposite directions, depending on market structure. For a monopoly, prices are (weakly) decreasing with information (Corollary 3) and hence, consumer surplus is increasing. Under competition, prices are increasing with information and the price increase turns out to be sufficiently strong to make consumer surplus a decreasing function of information. Account-

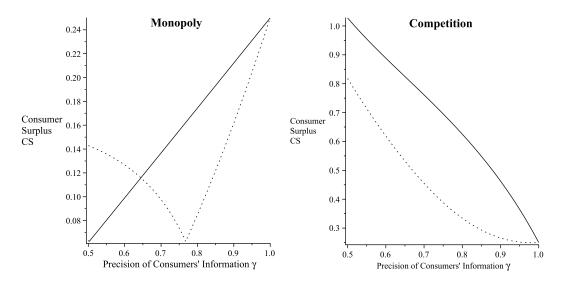


Figure 4: **Consumer surplus.** CS as a function of the quality of consumers' information γ for *F* uniform. The left hand panel shows monopoly, the right hand panel shows the case of competition. Solid curves depict standard preferences (l = 0), dashed curves depict loss aversion (l = 1.5).

ing for consumer loss aversion reveals that information can be detrimental for consumers not only in oligopolistic but also in monopolistic markets. While in an oligopolistic market, information reduces consumer surplus because it mitigates competition, under monopoly information is detrimental (for relatively small γ) because it decreases the gain/loss disutility of the "marginal" type (the one indifferent between buying and not buying), allowing the monopolist to extract a higher rent from all consumers.

6 Discussion

6.1 Firms' disclosure incentives

This article is concerned with regulatory policies aiming to improve consumers' information in advance purchase markets. The need for such policies is emphasized by a recent literature on firms' disclosure incentives. While information about *vertical* product characteristics (quality) is likely to be revealed due to unraveling (Milgrom, 1981), it has been argued that firms may lack incentives to disclose information regarding *horizontal* product features (Sun, 2011; Koessler and Renault, 2012; Celik, 2014; Janssen and Teteryatnikova, 2016). More specifically, in a pre-order setting, Chu and Zhang (2011) find that a monopolistic firm may supply some information or none, but will never reveal all information at the advance purchase stage.

In our setup, consumer information has been treated as an exogenous variable but we can derive some insights about firms' disclosure incentives by looking at profits as a function of γ . Figure 6 depicts profits for the case of a uniform distribution of consumer choosiness. For a monopolist, we find that in the absence of loss aversion, profits are monotonically decreasing with information. This is intuitive because improving information makes consumers more heterogeneous with respect to their willingness to pay, thereby increasing the difficulty of monopolistic screening. When consumers are loss averse, improving information has the additional effect of lowering the compensation necessary to make loss-averse consumers participate in the market. As argued before, for low γ , the consumers' participation constraint

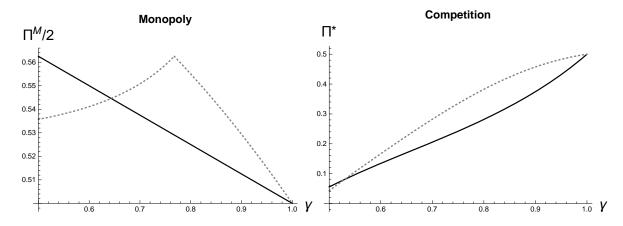


Figure 5: **Profit.** Profit as a function of the quality of consumers' information γ for *F* uniform and s = 1. The left hand panel shows monopoly, the right hand panel shows the case of competition. Solid curves depict standard preferences (l = 0), dashed curves depict loss aversion (l = 1.5).

becomes binding at $\sigma = \sigma_W^M$, making monopoly prices increase with information. Hence, under loss aversion profits first increase and then decrease with information. Similar to Chu and Zhang (2011), in our setting, a monopolist would reveal some information or none, depending on whether consumers are loss averse or not. Accounting for loss aversion reveals that a monopolist may have at least *some* incentive to inform consumers, so that improving information through regulation may not only be less effective but also less needed than otherwise expected.

Under competition, a very different picture emerges. Profits are increasing in γ because improving consumers' information makes products appear more heterogeneous at the advance purchase stage, thereby mitigating competition. Loss aversion affects only the intensity but not the direction in which profits depend on information. Under competition, profits and welfare are both maximized when consumers are perfectly informed. However, if information provision was costly, then firms would fail to internalize the positive externality of their information provision on their rival's profit and, in equilibrium, consumers might remain imperfectly informed. Accounting for loss aversion would then suggest that in such a situation, improving information through regulation will have an unambiguously positive effect on welfare. Note, however, that a comprehensive analysis of (competitive) information disclosure requires a different model, one with more than a single parameter measuring the consumers' quality of information. We leave this issue for future research.

6.2 Asymmetric firms

While our analysis of duopoly assumed firms to be identical, asymmetries can be easily accommodated into our framework. In this section we discuss the effects of introducing differences in the firms' unit cost of production, $c_j \ge 0$, $(c_A = c_B = 0$ in the main model) and in the firms' "prominence", i.e. the mass $k_j \in (0, 2)$, $k_A + k_B = 2$, of consumers who prefer firm j's product ($k_A = k_B = 1$ in the main model). The profit equation for firm *i* in (14) then becomes modified to

$$\Pi_{i} = (p_{i} - z_{i} - c_{i}) [k_{i}F(\sigma_{Wi}) \pm k_{j}F(\bar{\sigma})]$$

$$+ (p_{i} - c_{i}) \left\{ \gamma k_{i}[1 - F(\sigma_{Wi})] + (1 - \gamma)k_{j}[1 - F(\sigma_{Wj})] \right\}.$$
(29)

Taking derivatives with respect to prices and discounts yields four first order conditions which have to be satisfied by any price-discrimination equilibrium (Möller and Watanabe, 2016). For a uniform distribution of consumer choosiness, the resulting linear system of equations allows us to determine the unique candidate price schedules $(p_A^*, z_A^*), (p_B^*, z_B^*)$ for such an equilibrium in closed form. In the following we briefly discuss the properties of this candidate equilibrium in order to shed some light on the robustness of our results with respect to firm asymmetry.

Suppose that $c_A < c_B$ and $k_A = k_B$, so that firm A has a cost advantage over firm B but both firms share the same level of prominence. We can show that firm A will then set lower prices than firm B but offers the same discount $z_A = z_B$, i.e. discounts are independent of costs. This seems intuitive, because cost reductions increase a firm's profit margin equally in both periods. As this reasoning holds independently of the distribution of consumer types and consumers' degree of loss aversion, we therefore contemplate that, as long as $|c_A - c_B|$ is not too large for a price-discrimination equilibrium to exist, our results about advance selling are robust with respect to asymmetries in costs.

Now suppose that $k_A > k_B$ and $c_A = c_B$, so that firm A is more prominent than firm B but costs are identical. In this case, we find that firm A will set higher prices and offer a larger advance purchase discount than firm B. Intuitively, because more consumers favor firm A in advance, the firm has a stronger incentive to secure early purchases by way of a discount. Although differences in firm prominence affect the allocation of sales across firms, the allocation of sales across time remains largly unchanged. In particular, the dependence of the overall number of advance sales, $k_A \sigma_{WA}^* + k_B \sigma_{WB}^*$, on information and loss aversion is similar to the symmetric case $k_A = k_B$.

Although our results extend beyond the symmetric firm case, understanding asymmetries in advance purchase markets is important, especially in view of a potential analysis of firm entry. However, a detailed analysis of this issue is beyond the scope of this paper and is left for future research.

6.3 Risk aversion and anticipated regret

The focus of this paper lies on expectation-based loss aversion which provides an empirically relevant and attractable candidate for modelling risk preferences in the context of advance purchases. Nevertheless, other classes of risk preferences such as risk aversion or anticipated regret could be considered. In this section we discuss the similarities and differences in results when considering risk aversion or anticipated regret instead of consumer loss aversion. All mathematical details are relegated to Web Appendix **??** and **??**.

Risk aversion. Assume that consumers have a CARA utility function $u(x) = -e^{-rx}$ where r > 0 denotes the consumers' (common) degree of risk aversion and x stands for the difference between the consumer's valuation of the purchased product and the price paid.

As there are no closed-form solutions, neither under monopoly nor competition, in Figure 6 we provide numerical results for F uniform. The left panel illustrates that under monopoly, advance selling is increasing in information when consumers are risk averse.²² This is reassuring because it is in line with our conclusion with loss aversion that under monopoly information can be detrimental (by intensifying advance selling).

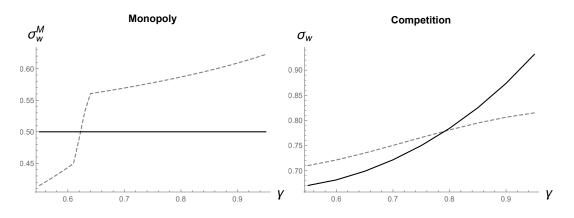


Figure 6: **Risk aversion.** σ_W as a function of the consumers' quality of information γ when the distribution of consumer choosiness *F* is uniform. The left hand panel shows the monopoly benchmark, the right hand panel shows the case of competition. Solid curves depict risk neutrality (*r* = 0), dashed curves depict CARA utility (*r* = 2).

The case of competition is depicted in the right panel of Figure 6. Similarly to consumer loss aversion, risk aversion increases (resp. decreases) advance sales relative to risk neutrality in uninformed (resp. informed) markets. We also find that with risk aversion, improving consumer information may decrease advance sales but, in contrast to consumer loss aversion

²²A degree of constant absolute risk aversion of above 0.5 could be classified as rather high, cf. Barseghyan et al. (2018). Nevertheless, we depict the figure with r = 2 because this allows to illustrate all three possible regions in one figure. For lower degrees of risk aversion, we predominantly obtain the third region.

(cf. Corollary 2), only for extremely high parameter values (i.e. for r around 10 or higher). In summary, consumer loss aversion yields results similar to classical risk aversion but requires less extreme assumptions in terms of parameter values (cf. Rabin, 2000's calibration theorem), while providing more tractability in terms of closed-form model predictions.

Anticipated regret. Expectation-based loss aversion compares realized utility with utility that could have been obtained under the same action but in different states of the world (Kőzegi and Rabin 2006, 2007). In contrast, regret compares realized utility with utility that could have been obtained by taking a different action in the same state of the world (Bell, 1985; Loomes and Sugden, 1986, 1987).

Hence the loss-averse consumer compares the price he ends up paying in period 2 for his preferred product with the price he was expecting to pay for his preferred product in period 2. In contrast, a consumer experiencing regret compares the price he ends up paying in period 2 for his preferred product with the price he would have paid if instead he had purchased his preferred product in period 1. This type of regret is inaction regret. There is also action regret. If the consumer happens to buy the wrong product then he regrets not having bought the other product.

If we introduce both types of regret with equal weights into our duopoly model then all results are as in the standard preference case (the mathematical details are relegated to Web Appendix ??). This is in accordance with Nasiry and Popescu (2012)'s monopoly model, thereby extending their preference independence result to oligopolistic environments. We further show that if we give a stronger weight to action regret than to inaction regret then results become as with loss aversion, i.e. advance selling increases with information under monopoly but decreases under competition when action regret becomes sufficiently strong. In our setting, loss aversion is therefore comparable with action regret.²³

²³Action regret resembles loss aversion also in our monopoly model. Inaction regret alone has no impact. For more details see Web Appendix **??**.

7 Conclusion

Contributing to the emerging literature on Behavioral Industrial Organization, this paper has introduced expectation-based loss aversion à la Kőzegi and Rabin (2006, 2007) into an advance purchase setting. Our starting point was the observation that in advance purchase markets this particular form of non-standard risk preferences can be expected to be especially relevant, because consumers are required to form expectations about prices and product match in order to decide on their timing of purchase. The paper's main results are concerned with the relation between information and market performance (fraction of uninformed purchases, allocative efficiency, prices, consumer surplus), with an emphasis on how this relation changes when consumer loss aversion is accounted for. The paper's analysis covers both the case of monopoly and the case of competition.

For a monopoly, we find that, in the absence of loss aversion, improving consumers' information has no effect on advance selling and thus a positive effect on allocative efficiency. Prices decrease and consumer surplus increases with information. Accounting for loss aversion reveals that improving information may increase advance selling and thus be detrimental for allocative efficiency. Moreover, prices may increase with information, leading to a reduction in consumer surplus.

For a duopoly, we find that, without consumer loss aversion, better information leads to more advance selling, making the overall effect on allocative efficiency indeterminate. In contrast, when consumers are (sufficiently) loss averse, information will reduce advance selling, and hence be unambiguously beneficial for allocative efficiency. Regarding prices and consumer surplus, loss aversion produces no (qualitative) change in their dependence on information.

In short, we can therefore conclude that accounting for consumer loss aversion affects our assessment of the desirability of information in advance purchase markets; negatively for a monopoly but positively in the presence of competition. More generally, our results demonstrate that accounting for consumers' risk preferences in markets where consumer decisions are made (at least partly) under uncertainty, may have a first-order impact on the prospective outcome of regulation.

Appendix: Proofs

Proof of Proposition 1. Consider the first order condition corresponding to the maximization program in (12):

$$\frac{1 - F(\sigma_W)}{f(\sigma_W)} - \sigma_W = 0.$$
(30)

Because f has an increasing hazard rate the LHS is strictly decreasing in σ_W . It is positive for $\sigma_W = 0$ and negative for $\sigma_W = 1$. By continuity there exists a unique solution, i.e. σ_W^0 is well-defined. For $\gamma \ge \gamma^M(l)$, σ_W^0 maximizes the monopolist's payoff, i.e. $\sigma_W^M = \sigma_W^0$ is independent of l. In the following, consider the remaining case where $\gamma < \gamma^M(l)$. The monopolist's problem implies the first order condition

$$\frac{1-F(\sigma_W)}{f(\sigma_W)} - \sigma_W - \frac{l\gamma(1-\gamma)-\gamma+\frac{1}{2}}{(1-\gamma)(l\gamma+1)f(\sigma_W)} = 0.$$
(31)

The LHS of (31) is strictly smaller than the LHS of (30) for all σ_W . This implies that for $\gamma < \gamma^M(l)$ the profit-maximizing allocation of sales must be such that $\sigma_W^M < \sigma_W^0$. If *f* is non-increasing then the LHS of (31) is strictly decreasing in σ_W . Because the last fraction in (31) can be written as $\frac{1}{f(\sigma_W)} - \frac{1}{2f(\sigma_W)(1-\gamma)(1+l\gamma)}$ and because $(1-\gamma)(1+l\gamma)$ is decreasing in γ but increasing in *l* it also holds that the LHS of (31) is increasing in γ but decreasing in *l*. Hence, for *f* non-increasing, the Implicit Function Theorem implies that (31) has a unique solution, σ_W^M , and that σ_W^M is increasing in γ but decreasing in *l*. When *F* is uniform, it follows from (31) that $\sigma_W^M = \frac{1}{4(1-\gamma)(l\gamma+1)}$.

Asymmetric pricing: As mentioned in footnote 16, we show next that the monopolist has no incentive to set asymmetric regular prices. For this purpose, suppose that $p_B = p_A + \Delta p$ with $\Delta p \ge 0$ and $z_A \ge z_B$. Then, the monopolist's profit is given by

$$\Pi^{M} = (p_{A} - z_{A})[F(\sigma_{WA}) + F(\bar{\sigma})] + p_{A}\{\gamma[1 - F(\sigma_{WA})] + (1 - \gamma)[1 - F(\sigma_{WB})]\}$$
(32)
+ $(p_{B} - z_{B})[F(\sigma_{WB}) - F(\bar{\sigma})] + p_{B}\{\gamma[1 - F(\sigma_{WB})] + (1 - \gamma)[1 - F(\sigma_{WA})]\}.$

For $\gamma > \gamma^{M}(l)$, it holds that $p_{i} - z_{i} = s$ for all i = 1, 2 by (9). Thus, the monopolist maximizes (32) over $(z_{A}, z_{B}, p_{A}, p_{A} + \Delta p)$ s.t. $p_{i} = s + z_{i}$ for all i. By substituting all constraints into (32), (32) becomes a function of z_{A} and Δp only. By continuity, for $\Delta p \ge 0$ sufficiently low, the first-order condition with respect to z_{A} characterizes the optimal z_{A} ,

$$[(1 - F(\sigma_{WA})) + (1 - F(\sigma_{WB}))] - z_A \frac{f(\sigma_{WA}) + f(\sigma_{WB})}{(1 - \gamma)(1 + l\gamma)} - \Delta p \frac{(1 - \gamma)f(\sigma_{WA}) + \gamma f(\sigma_{WB})}{(1 - \gamma)(1 + l\gamma)} = 0.$$

At $\Delta p = 0$, this equation elapses to (30). In addition, at $\Delta p = 0$, the first derivative of the monopolist's profit with respect to Δp equals

$$-z_A f(\sigma_W) \left[1 - \frac{(1-\gamma)(1-l\gamma)}{(1-\gamma)(1+l\gamma)} \right].$$
(33)

Because $z_A > 0$, this derivative is negative for l > 0 and zero only for l = 0, i.e., it is optimal to set $\Delta p = 0$.

It remains to consider the case where $\gamma \leq \gamma^M(l)$. In this case it holds that $p_i - z_i = s - [l\gamma(1 - \gamma) - \gamma + 1/2]\sigma_{Wi}$ for all i = 1, 2 by (9). The monopolist maximizes (32) over $(z_A, z_B, p_A, p_A + \Delta p)$ s.t. $p_i = z_i + s - [l\gamma(1 - \gamma) - \gamma + 1/2]\sigma_{Wi}$ for all i. Using the fact that σ_{Wi} only depends on z_i and Δp , it follows from the constraints that $z_B = z_A + 2\Delta p(1 - \gamma)\gamma[(2\gamma - 1)l + 2]$. By substituting into (32), (32) becomes again a function of z_A and Δp only. For $\Delta p \ge 0$ sufficiently low, the first-order condition with respect to z_A characterizes the optimal z_A . At $\Delta p = 0$, the first derivative of the monopolist's profit with respect to Δp can be expressed as follows (using the first-order condition with respect to z_A),

$$-(1-\gamma)l\gamma \left[1-F\left(\frac{z_A}{(1-\gamma)(1+l\gamma)}\right)\right].$$
(34)

This derivative is negative for l > 0 and zero only for l = 0, i.e., it is optimal to set $\Delta p = 0$.

Proof of Corollary 1. The result is an immediate consequence of Proposition 1 and the fact that $\lim_{l\to 0} \gamma^M(l) = \frac{1}{2}$.

Proof of Proposition 2. For $\gamma \rightarrow \frac{1}{2}$ the first order condition (17) determining the equilibrium fraction of advance sales is given by

$$G_{\frac{1}{2}}(\sigma_{W}, l) \equiv \frac{1 - F(\sigma_{W})}{f(\sigma_{W})} - \sigma_{W} + \frac{1}{2}(1 + \frac{l}{2})\sigma_{W} = 0.$$
(35)

As f has an increasing hazard rate and l < 2, $G_{\frac{1}{2}}$ is strictly decreasing in σ_W , positive for $\sigma_W = 0$, and negative for $\sigma_W = 1$. Hence $\lim_{\gamma \to \frac{1}{2}} \sigma_W^*$ is well defined as the unique solution to (35) in (0, 1). As $G_{\frac{1}{2}}$ is strictly increasing in l, it follows that $\lim_{\gamma \to \frac{1}{2}} \sigma_W^*$ is strictly increasing in l. Similarly, $G_{\frac{1}{2}}$ is larger for $f = f_2$ than for $f = f_1$ for any σ_W by hazard rate dominance, which implies that $\lim_{\gamma \to \frac{1}{2}} \sigma_W^*$ is (weakly) larger for $f = f_2$ than for $f = f_1$. Defining $x = F(\sigma_W)$, (35) can be written as

$$1 - x - (\frac{1}{2} - \frac{l}{4})F^{-1}(x)f(F^{-1}(x)) = 0$$
(36)

and a sufficient condition for the solution to this equation $x^* = F(\sigma_W^*)$ to be smaller for $f = f_2$ than for $f = f_1$ is that $F_2^{-1}(x)f_2(F_2^{-1}(x)) \ge F_1^{-1}(x)f_1(F_1^{-1}(x))$ for all $x \in [0, 1]$.

For $\gamma \rightarrow 1$, (17) becomes

$$G_1(\sigma_W, l) \equiv \frac{1 - F(\sigma_W)}{f(\sigma_W)} - \sigma_W + \frac{1}{(1 + l)f(0)} = 0.$$
(37)

As f has an increasing hazard rate, G_1 is strictly decreasing in σ_W . It is positive for $\sigma_W = 0$. It is negative for $\sigma_W = 1$ if and only if $\frac{1}{(1+l)f(0)} < 1 \Leftrightarrow \frac{1}{f(0)} - 1 < l$. If this condition holds, then $\lim_{\gamma \to 1} \sigma_W^*$ is given by the unique solution to (37) in (0, 1). Otherwise $\lim_{\gamma \to 1} \sigma_W^* = 1$. As G_1 is strictly decreasing in l, it follows that $\lim_{\gamma \to 1} \sigma_W^*$ is strictly decreasing in l as long as $\lim_{\gamma \to 1} \sigma_W^* < 1$. To see that $\lim_{\gamma \to 1} \sigma_W^*$ is (weakly) larger for $f = f_2$ than for $f = f_1$ note that from hazard rate dominance evaluated at $\sigma = 0$ it follows that $f_2(0) \le f_1(0)$ so that G_1 is larger for $f = f_2$ than for $f = f_1$ for any σ_W .

Proof of Corollary 2. Consider first the case without loss aversion, i.e. let l = 0. The functions $G_{\frac{1}{2}}(\sigma_W, 0)$ and $G_1(\sigma_W, 0)$ in (35) and (37), defining (implicitly) the equilibrium fraction of advance sales in the limits $\gamma \to \frac{1}{2}$ and $\gamma \to 1$, are both decreasing in σ_W but do so at different rates. They intersect once at $\sigma_W = \tilde{\sigma} = \frac{2}{f(0)}$ and G_1 crosses $G_{\frac{1}{2}}$ from above, i.e. $G_{\frac{1}{2}}(\sigma_W, 0) < G_1(\sigma_W, 0)$ if and only if $\sigma_W < \tilde{\sigma}$. If $\frac{2}{f(0)} > 1$ then $\sigma_{\frac{1}{2}} < \sigma_1$. Consider the remaining case where $\frac{2}{f(0)} \leq 1$. In this case it holds that $\sigma_{\frac{1}{2}} < \sigma_1$ if and only if $G_{\frac{1}{2}}$ and G_1 intersect below the zero axis, that is, if and only if

$$\frac{1-F(\tilde{\sigma})}{f(\tilde{\sigma})} - \frac{1}{2}\tilde{\sigma} < 0.$$
(38)

Note that

$$\frac{1 - F(\tilde{\sigma})}{f(\tilde{\sigma})} - \frac{1}{2}\tilde{\sigma} = \frac{1 - F(\frac{2}{f(0)})}{f(\frac{2}{f(0)})} - \frac{1}{f(0)} \le \frac{1 - F(0)}{f(0)} - \frac{1}{f(0)} = 0$$
(39)

where the inequality follows because f has an increasing hazard rate. Hence we have shown that $\sigma_{\frac{1}{2}} \leq \sigma_1$ and that this inequality becomes strict when f(0) < 2 or when the hazard rate of f is *strictly* increasing.

In the remainder, we determine a lower bound \underline{l} on the consumers' degree of loss aversion such that $\sigma_1 < \sigma_{\frac{1}{2}}$ for all $l > \underline{l}$. For this purpose, note that $\sigma_1 < \sigma_{\frac{1}{2}}$ if and only if $G_{\frac{1}{2}}$ and G_1 intersect at $\tilde{\sigma} < 1$ and this intersection lies above the zero-axis. More precisely, $\sigma_1 < \sigma_{\frac{1}{2}}$ if and only if

$$\tilde{\sigma}(l) = \frac{2}{(1+l)(1+\frac{l}{2})f(0)} < 1 \Leftrightarrow l > -\frac{3}{2} + \sqrt{\frac{1}{4} + \frac{4}{f(0)}} \equiv \tilde{l}$$
(40)

and

$$K(l) \equiv G_{\frac{1}{2}}(\tilde{\sigma}(l)) = \frac{1 - F(\tilde{\sigma}(l))}{f(\tilde{\sigma}(l))} - (\frac{1}{2} - \frac{l}{4})\tilde{\sigma}(l) > 0.$$
(41)

Note that $\tilde{l} < 2$ if and only if $f(0) > \frac{1}{3}$. If $K(\tilde{l}) > 0$ then, because K(l) is strictly increasing, $\sigma_1 < \sigma_{\frac{1}{2}}$ for all $l \in (\tilde{l}, 2)$. If $K(\tilde{l}) < 0$ then the fact that K(2) > 0 implies that there exists a $\tilde{l} \in (\tilde{l}, 2)$ such that K(l) > 0 if and only if $l > \tilde{l}$. Defining $\underline{l} \equiv \max{\{\tilde{l}, \tilde{l}\}} < 2$ we have therefore shown that $\sigma_1 < \sigma_{\frac{1}{2}}$ if and only if $l > \underline{l}$.

For the example of a uniform F we have $\tilde{l} = \frac{\sqrt{17}-3}{2}$ and with $\tilde{\sigma} = \frac{2}{(1+l)(1+\frac{l}{2})}$ we get $K(l) = 1 - \frac{3-\frac{l}{2}}{(1+l)(1+\frac{l}{2})}$. From K(l) = 0 we find $\tilde{l} = \sqrt{8} - 2$. Hence, for F uniform $\sigma_1 < \sigma_{\frac{1}{2}}$ if and only if $l > \underline{l} = \max\{\frac{\sqrt{17}-3}{2}, \sqrt{8}-2\} = \sqrt{8} - 2 \approx 0.83$.

Proof of Proposition 3. The monopolist's profit maximizing prices in (22) and (23) have been derived in the main text and in the proof of Proposition 1. The comparative statics follow from the fact that

$$\frac{\partial}{\partial l} \left[\frac{l\gamma(1-\gamma) - \gamma + \frac{1}{2}}{4(1-\gamma)(1+l\gamma)} \right] = \frac{\gamma}{8(1-\gamma)(1+l\gamma)^2} > 0.$$

$$\tag{42}$$

Proof of Corollary 3. For l = 0 we have $\frac{\partial}{\partial \gamma}[p^M - z^M] = \frac{\partial}{\partial \gamma}[s] = 0$ and $\frac{\partial}{\partial \gamma}[p^M] = \frac{\partial}{\partial \gamma}[s + \frac{1}{2}(1 - \gamma)] = -\frac{1}{2}$. If l > 0 then $\gamma^M(l) \in (\frac{1}{2}, 1)$ and for all $\gamma \in (\frac{1}{2}, \gamma^M(l))$ it holds that

$$\frac{\partial}{\partial \gamma} [p^M - z^M] = \frac{\partial}{\partial \gamma} [p^M] = -\frac{\partial}{\partial \gamma} \left[\frac{l\gamma(1-\gamma) - \gamma + \frac{1}{2}}{4(1-\gamma)(1+l\gamma)} \right] = \frac{l(2\gamma-1) + 1}{8(1-\gamma)^2(1+l\gamma)^2} > 0.$$
(43)

Proof of Proposition 4. First note that (24) and (25) are both increasing in p^* but decreasing in z^* . Moreover, (24) is decreasing in l, whereas (25) is increasing in l. It thus follows that the solutions p^* and z^* must be both increasing in l. It thus remains to prove the comparative statics for $p^* - z^*$, because, as both p^* and z^* are increasing, the overall effect of loss aversion on the first period price is unclear. From (27) and (26)

$$p^* - z^* = \frac{(1+l\gamma)\gamma(1+2\gamma) - 1}{\gamma \left[7 + 2l - (1-\gamma)\gamma(l+4)l - 4\gamma\right] - 1}$$
(44)

and

$$\frac{\partial(p^* - z^*)}{\partial l} = \frac{\gamma(1 - \gamma)(2\gamma - 1)(1 + \gamma)[(1 + l\gamma)^2 + 4(2\gamma - 1)]}{\{\gamma \left[7 + 2l - (1 - \gamma)\gamma(l + 4)l - 4\gamma\right] - 1\}^2} > 0$$
(45)

for all $\gamma \in (\frac{1}{2}, 1)$.

The remainder of this proof is concerned with the existence of a price-discrimination equilibrium, i.e. an equilibrium in which firms make positive sales in both periods. First, we show that the profit function in (14) is strictly concave for all l < 2. For *F* uniform and $p_A - z_A \le p_B - z_B$, the profit function in (14) equals

$$\Pi_A = (p_A - z_A)[\sigma_{WA} + \bar{\sigma}] + p_A \{\gamma [1 - \sigma_{WA}] + (1 - \gamma)[1 - \sigma_{WB}]\}.$$

This profit function is valid for the following set of prices and discounts,

$$P_a \equiv \left\{ (p_A, z_A, p_B, z_B) \in \mathbb{R}^4_+ | \bar{\sigma} \le \sigma_{WA} \le 1, \bar{\sigma} \le \sigma_{WB} \le 1 \right\}.$$

The Hessian matrix is given by

$$H = \begin{pmatrix} \frac{\partial^2 \Pi_A}{\partial p_A^2} & \frac{\partial^2 \Pi_A}{\partial z_A \partial p_A} \\ \frac{\partial^2 \Pi_A}{\partial z_A \partial p_A} & \frac{\partial^2 \Pi_A}{\partial z_A^2} \end{pmatrix} = \begin{pmatrix} -\frac{2(1-\gamma)}{\gamma l+1} - \frac{1}{2\gamma - 1} & \frac{\gamma(2\gamma l+l+4)}{2(2\gamma - 1)(\gamma l+1)} \\ \frac{\gamma(2\gamma l+l+4)}{2(2\gamma - 1)(\gamma l+1)} & -\frac{1}{(1-\gamma)(\gamma l+1)} - \frac{1}{2\gamma - 1} \end{pmatrix}.$$
 (46)

The *i*-th leading principal minor of the Hessian, denoted by H(i), is given by $H(1) = \frac{\partial^2 \Pi_A}{\partial p_A^2} = -\frac{2(1-\gamma)}{\gamma l+1} - \frac{1}{2\gamma - 1} < 0$ and $H(2) = \frac{\partial^2 \Pi_A}{\partial p_A^2} \frac{\partial^2 \Pi_A}{\partial z_A^2} - (\frac{\partial^2 \Pi_A}{\partial z_A \partial p_A})^2 = \frac{\gamma (l(-(1-\gamma)\gamma((2\gamma+3)l+16)+4)+4)}{4(1-\gamma)(2\gamma-1)(\gamma l+1)^2} > 0$ for all γ

if l < 2. Thus, *H* is negative definite and so the profit function Π_A is strictly concave in P_a for all l < 2. In particular, this rules out the profitability of any "simultaneous" deviation in price-level *and* discount within P_a . A similar argument applies to the opposite case where $p_A - z_A > p_B - z_B$.

Finally, the non-profitability of deviations for which the conditions in P_a fail to hold has been shown by Möller and Watanabe (2016) for l = 0. Profits' continuity in the loss aversion parameter l then implies that (15) and (16) are not only necessary but also sufficient conditions for an equilibrium as long as loss aversion is not too strong.

Proof of Corollary 4. For l = 0 we have

$$\frac{\partial p^* - z^*}{\partial \gamma} = \frac{6(1 - 2\gamma + 3\gamma^2)}{(1 - 7\gamma + 4\gamma^2)^2} > 0$$
(47)

$$\frac{\partial p^*}{\partial \gamma} = \frac{4(1 - 2\gamma + 3\gamma^2)}{(1 - 7\gamma + 4\gamma^2)^2} > 0$$
(48)

for all $\gamma \in (\frac{1}{2}, 1)$. To see that for l > 1, p^* is decreasing in γ for large γ note that

$$\lim_{\gamma \to 1} \frac{\partial p^*}{\partial \gamma} = \frac{2 - l - l^2}{1 + l} \tag{49}$$

is negative if and only if l > 1.

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