Selling in Advance to Loss Averse Consumers
Web Appendix (Not for Publication)

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This Web Appendix consists of two sections providing omitted material for Section 6.3. In Section A, we provide mathematical details for the subsection on risk aversion (page 2 ff.). Section B contains mathematical details for the subsection on anticipated regret (page 4 ff.).

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A Risk Aversion

Assume that consumers have a CARA utility function $u(x) = -e^{-rx}$ where $r > 0$ denotes the consumers' (common) degree of risk aversion and x stands for the difference between the consumer's valuation of the purchased product and the price paid.

First, under *monopoly*, a consumer who purchases his favorite product in advance faces a lottery with two outcomes. In particular, $x = s+\frac{\sigma}{2}-p+z$ with probability γ and $x = s-\frac{\sigma}{2}-p+z$ with probability $1-\gamma$. The certainty equivalent of the consumer's utility from an advance purchase following his signal is then given by (see the textbook of Laffont and Martimort, 2002, page 237):

$$CE_1(\sigma) = s + (\gamma - \frac{1}{2})\sigma - p + z - R(\sigma) \tag{50}$$

with

$$R(\sigma) = \frac{1}{r}\ln[\gamma e^{-r(1-\gamma)\sigma} + (1-\gamma)e^{r\gamma\sigma}] \tag{51}$$

denoting a risk discount which is increasing in the spread σ between the lottery's outcomes.

A monopolist will price his two products identically. Thus, the purchase of the consumer's preferred product on the spot gives the certainty equivalent,

$$CE_2(\sigma) = s + \frac{\sigma}{2} - p. \tag{52}$$

Equating $CE_1(\sigma)$ and $CE_2(\sigma)$ in (50) and (52) yields the indifferent consumer σ_W as a non-linear function of z

$$\sigma_W = \frac{\ln\left(\frac{e^{rz}-\gamma}{1-\gamma}\right)}{r}. \tag{53}$$

The same complication as with loss aversion arises, in that advance purchase utility $CE_1(\sigma)$ can be decreasing in choosiness σ. This happens when γ is sufficiently small. We can explicitly solve for the critical level of γ^c where the advance purchase utility of the 0-type is equal to that of the σ_W-type. The former is larger than the latter for $\gamma < \gamma^c$. In contrast to the setup with loss aversion, with risk aversion γ^c is a function of σ_W,

$$\gamma \geq \gamma^c(\sigma_W) \equiv \frac{e^{\frac{r\sigma_W}{2}}}{e^{\frac{r\sigma_W}{2}} + 1}, \tag{54}$$

or equivalently, for a given γ,

$$\sigma_W \leq \sigma^c_W(\gamma) \equiv -\frac{2\ln\left(\frac{1}{\gamma} - 1\right)}{r} \in [0, \infty), \tag{55}$$

which is always satisfied for γ sufficiently close to 1 as $\sigma^c_W(\gamma)$ is strictly increasing in γ for $\gamma \in [1/2, 1]$ and approaches ∞ in the limit. In this case, the monopolist extracts the 0-type's surplus by setting $p - z = s$. For $\sigma_W > \sigma^c_W(\gamma)$, the monopolist extracts the σ_W-type's surplus by setting $p - z = s + (\gamma - \frac{1}{2})\sigma^M_W - R(\sigma^M_W)$.

The monopolist's profit equals

$$\frac{\Pi^{RA}}{2} = (p^M - z^M)F(\sigma^M_W) + p^M(1 - F(\sigma^M_W)) = (p^M - z^M) + z^M(1 - F(\sigma^M_W)). \tag{56}$$

Substitution of $p^M - z^M$ and z^M by $(1 - \gamma)\sigma^M_W + R(\sigma^M_W)$ from inverting (53) leads to

$$\frac{\Pi^{RA}}{2} = \begin{cases} s + (\gamma - \frac{1}{2})\sigma^M_W - R(\sigma^M_W) + \left((1 - \gamma)\sigma^M_W + R(\sigma^M_W)\right)(1 - F(\sigma^M_W)) & \text{if} \quad \sigma^M_W > \sigma^c_W(\gamma) \\ s + \left((1 - \gamma)\sigma^M_W + R(\sigma^M_W)\right)(1 - F(\sigma^M_W)) & \text{if} \quad \sigma^M_W \leq \sigma^c_W(\gamma). \end{cases} \tag{57}$$

Numerical results for F uniform are illustrated in Figure 6 in Section 6.3.

Second, suppose products A and B are offered by two *competing* firms. Following the same argumentation as in the setup with loss aversion we can show that the threshold $\bar{\sigma}$ is as in (3). σ_{WA} and σ_{WB} are given by $CE_2(\sigma_{Wi}, i) = CE_{1,i}(\sigma_{Wi}, i)$ or, equivalently,

$$\sigma_{Wi} \equiv \frac{\ln\left(\frac{\gamma e^{r(p_i+z_i)} - \gamma e^{rp_i} + (1-\gamma)e^{r(p_j+z_i)}}{1-\gamma}\right)}{r} - p_i, \tag{58}$$

and $CE_2(\sigma_{Wj}, j) = CE_{1,j}(\sigma_{Wj}, j)$ or, equivalently,

$$\sigma_{Wj} \equiv \frac{\ln\left(-\frac{\gamma\left(e^{2r(\gamma p_i + p_j)} - e^{r(2\gamma p_j + 2p_i + z_j)}\right)}{1-\gamma} + e^{r(2\gamma p_j + p_i + p_j + z_j)}\right)}{r} - 2(\gamma p_i + p_j). \tag{59}$$

If firms choose pricing policies (p_A, z_A) and (p_B, z_B) then for $i, j \in \{A, B\}, i \neq j$, firm i's profit is given as in (14).

B Anticipated Regret

A consumer experiencing regret compares the price he ends up paying in period 2 for his preferred product with the price he would have paid if instead he had purchased his preferred product in period 1. This type of regret is inaction regret ($\eta_{ia} > 0$). An i-type consumer's expected utility (including anticipated inaction regret due to buying late) from the consumption plan of purchasing his preferred product in period 2 can be written as

$$U_2^r(\sigma, i) = s + \frac{\sigma}{2} - \gamma p_i - (1-\gamma)p_j + \eta_{ia}\gamma(-z_i) + \eta_{ia}(1-\gamma)(-z_j).$$

There is also action regret ($\eta_a > 0$). If the consumer happens to buy the wrong product then he regrets not having bought the other product. An i-type consumer's expected utility (including anticipated action regret due to buying the wrong product) from the consumption plan of buying his favorite product i in period 1 is

$$U_{1,i}^r(\sigma, i) = s + (\gamma - \frac{1}{2})\sigma - (p_i - z_i) + \eta_a(1-\gamma)\left(-\sigma - (p_i - z_i) + (p_j - z_j)\right).$$

An i-type consumer's expected utility (including anticipated action regret due to buying the wrong product) from the consumption plan of buying his non-favorite product $j \neq i$ in period 1 can be calculated analogously and is given by

$$U_{1,j}^r(\sigma, i) = s - (\gamma - \frac{1}{2})\sigma - (p_j - z_j) + \eta_a\gamma\left(-\sigma + (p_i - z_i) - (p_j - z_j)\right).$$

Following the same procedure as with loss aversion in Section 3, thresholds $\bar{\sigma}^r$, σ^r_{WA} and σ^r_{WB} can be derived with anticipated regret. We also find closed-form solutions for the model with anticipated regret. Solving for the fraction of advance sales σ^*_W in the case of *competition* for F uniform yields

$$\sigma^*_W = \frac{2\gamma\left((3-2\gamma)\eta_a - 2(1-\gamma)\eta_{ia} + 1\right) - \eta_a + \eta_{ia}}{(\gamma(7-4\gamma))(\eta_a+1) - 1}.$$

For $\eta_a = \eta_{ia}$, σ^*_W collapses to the case with standard preferences which is in accordance with the result of Nasiry and Popescu (2012)'s monopoly model. If we give a stronger weight to action regret than to inaction regret then results resemble those with loss aversion, i.e. advance selling decreases with information when action regret becomes sufficiently strong. As illustrated by Figure 7 (right panel), loss aversion is therefore comparable with action regret.

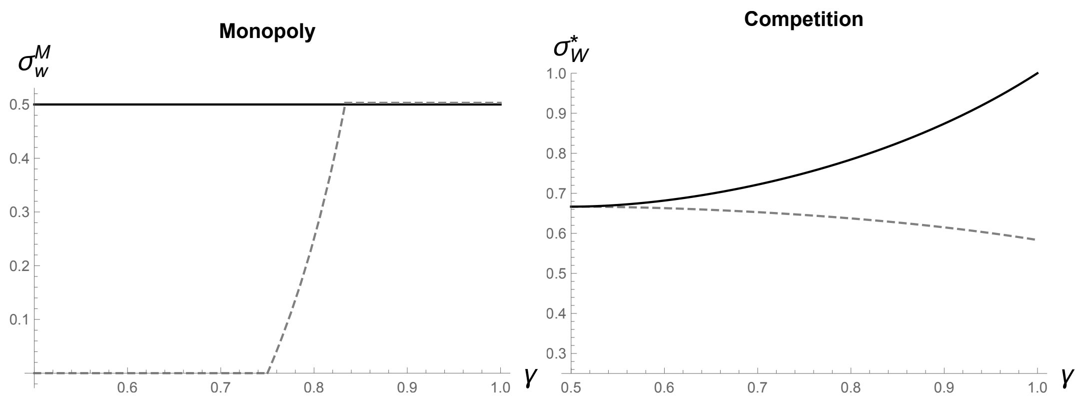

Figure 7: **Action regret.** σ_W as a function of the consumers' quality of information γ when the distribution of consumer choosiness F is uniform. The left hand panel shows the monopoly benchmark, the right hand panel shows the case of competition. Solid curves depict standard preferences ($\eta_a = \eta_{ia} = 0$), dashed curves depict *action regret* ($\eta_a = 5 > \eta_{ia} = 0$).

Solving for the fraction of advance sales σ^M_W in the case of *monopoly* for F uniform yields

$$\sigma^M_W = \begin{cases} \max\{\frac{\eta_{ia}(-2\gamma(\eta_a+1)+2\eta_a+1)-1}{4(\gamma-1)(\eta_a+1)}, 0\} & \text{if} \quad \gamma < \gamma^M(\eta_a) \\ \frac{1}{2} & \text{if} \quad \gamma \geq \gamma^M(\eta_a), \end{cases}$$

where $\gamma^M(\eta_a) \equiv 1 - \frac{1}{2(1+\eta_a)}$. This solution shows that also in our monopoly model, action regret resembles loss aversion. This is illustrated graphically in Figure 7 (left panel). In addition, it holds that without action regret, $\gamma^M(\eta_a = 0) = 1/2$ and therefore the condition $\gamma \geq \gamma^M(\eta_a)$ is satisfied for all η_{ia}. This implies that inaction regret alone has no impact on the fraction of advance sales.