Optimal Partnership in a Repeated Prisoner's Dilemma*

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Abstract

This paper studies informational partnerships in a repeated prisoner's dilemma with random matching. Assuming that players observe the play within but not across partnerships, we find the surprising result that the relation between the observability of actions and the sustainability of cooperation is non–monotonic. Increasing partnerships beyond a certain optimal size hardens cooperation although it improves observability.

Keywords: Prisoner's dilemma; Random matching; Observability; Cooperation.
JEL classification: C70; C73

1. Introduction

The sustainability of cooperation in a two–player repeated prisoner's dilemma is a well established fact. Fudenberg and Maskin (1986) were the first to prove a Folk Theorem for the case of perfectly observable actions. The extension of this result to the case of imperfectly observable actions has attracted considerable attention in recent years (e.g. Piccione (2002) and references therein). If the prisoner's dilemma is played in a population of uniformly randomly matched players further informational imperfections might exist. Kandori (1992) has considered two extreme cases of observability. For a fully informed population in which every player observes the identity and actions of all other players Kandori proves a Folk Theorem. Kandori also shows that cooperation can

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be sustained in a fully uninformed population in which players only observe the actions of their stage game opponents and the players' identities are unobserved. This paper extends Kandori's analysis to the intermediate case of partially informed populations in order to study the relationship between the sustainability of cooperation and the amount of available information in more detail. We assume that the population is partitioned into partnerships and that there is perfect observability within but not across partnerships. Surprisingly, it turns out that more information not necessarily implies that cooperation is easier to sustain. Increasing partnerships beyond a certain optimal size hardens cooperation although it improves observability.

So far the literature has found that additional information facilitates cooperation. Okuno-Fujiwara and Postlewaite (1995) for example assume that players possess observable labels like reputation or membership. They find that the introduction of this additional information improves cooperation. In a more applied setting Greif (1993) shows how the introduction of trade coalitions with specific information transmission mechanisms facilitated cooperation between 11th–century traders and their oversea agents. The present paper is the first to find a non–monotonistic relation between the amount of available information and the sustainability of cooperation.

Other papers have shown that cooperation in a population of players can be sustainable by departing from the assumption of uniform random matching. Harrington (1995) assumes that some players meet more frequently than others and Matsushima (1990) and Ghosh and Ray (1996) consider models in which opponents are chosen endogenously. Although these papers have explained the sustainability of cooperation they have failed to consider its dependence on the amount of available information.

The remainder is organized as follows. Section 2 presents the basic prisoner's dilemma framework and introduces partnerships as a particular form of informational structure. Section 3 shows how cooperation can be sustained in such a partially informed population. Section 4 discusses the dependence of the sustainability of cooperation on the partnership–size and Section 5 concludes.

2. The model

Consider an even number $N \geq 4$ of perfectly rational players. Time is discrete and periods are indexed by $t = 0, \ldots, \infty$. In each period players get uniformly randomly matched to play a 2-player prisoner's dilemma. After the prisoner's dilemmas have been played in period t and payoffs have been realized the pairs of players separate and random matching determines the new pairs for period $t+1$. Players discount future utility by a homogeneous discount factor $\delta \in (0,1)$. The one-period payoffs of the prisoner's dilemma are defined in Figure 1. They are normalized by setting the efficient payoff equal to 1 and the Nash payoff equal to zero. Payoffs are therefore completely characterized by two parameters; the gain from defecting, $G > 0$ and the loss from being defected, $L > 0$. We suppose that the population is partitioned into

	C	D
C	1,1	$-L$,$1+G$
D	$1+G$,$-L$	0,0

Figure 1: Payoff matrix of the normalized prisoner's dilemma

partnerships of n players, where n is a divisor of N such that $2 \leq n \leq \frac{N}{2}$. From the viewpoint of each player the population is therefore devided into two groups, the group of his partners and the group of non–partners which we also call *strangers*. We assume that in addition to the actions of their stage game opponents, players only observe the actions in matches amongst their partners. Players can distinguish partners from strangers but identify only partners. Note that the assumption that partnerships are of equal size allows us to express the observability of players' actions by a single parameter, n. It also implies that, as in Kandori (1992), players are homogeneous in the amount of information they possess. As common in this literature, I assume that players cannot communicate with each other.

3. Sustaining cooperation

Consider the following strategy σ^* which combines undirected punishments of partners with a contagious punishment of strangers. Suppose that in period $t = 0$ players begin by cooperating. In period $t > 0$ a player's action depends on the identity of his opponent. If his opponent is a partner he cooperates if there has been no defection within his partnership so far, otherwise, he defects. If his opponent is a stranger he cooperates if no stranger has defected against him so far, otherwise, he defects. Note that σ^* separates behaviour with partners from behaviour with strangers. More specifically, the actions it prescribes against partners are independent of the history of play with strangers and vice versa. This property is intuitively appealing. We will say that a strategy is *separable* if it satisfies this property.

Our first result extends the findings of Kandori (1992) to the case where observability is neither perfect nor completely imperfect. It shows that σ^* sustains cooperation throughout the entire population and that no other separable strategy can do better.

Proposition 1 *There exists a $\delta^*(n) < 1$ such that for all $\delta \geq \delta^*(n)$, σ^* implements full cooperation as sequential equilibrium for sufficiently large L. For all $\delta < \delta^*(n)$, full cooperation cannot be sustained as a sequential equilibrium by any separable strategy.*

Proof. Consider player $i \in N$. By the Principle of Dynamic Programming (Abreu (1988)) one only has to check that one–shot deviations from σ^* are unprofitable after any history. If there was no defection amongst i's partners so far, a deviation against a partner is unprofitable if and only if

$$1+\frac{n-1}{N-1}\sum_{\tau=1}^{\infty}\delta^{\tau}\cdot 1\geq 1+G \quad\Leftrightarrow\quad \delta\geq\frac{G}{\frac{n-1}{N-1}+G}\equiv\delta_p. \tag{1}$$

If there was a defection in player i's partnership before, playing D is optimal for i as his action does not influence the future actions of any other player. Note that amongst all separable strategies, σ^* minimizes player i's payoff from deviating against a partner. It follows that for all $\delta<\delta_p$ cooperation amongst partners cannot be sustained. Now consider i's incentives to deviate against a stranger. If no stranger has defected against i so far, to be consistent with equilibrium, player i must believe that every other player still cooperates with strangers. Let p_τ and s_τ denote the (expected) proportion of i's partners and non–partners respectively who would defect against strangers τ periods after a deviation of i. It holds that $s_1=\frac{1}{N-n}$ and $p_1=\frac{1}{n}$ and for $\tau>1$ this contagious process develops in the following way: $s_\tau=s_{\tau-1}+(1-s_{\tau-1})(\frac{N-2n}{N-1}s_{\tau-1}+\frac{n}{N-1}p_{\tau-1})$ and $p_\tau=p_{\tau-1}+(1-p_{\tau-1})\frac{N-n}{N-1}s_{\tau-1}$. Player i has no incentive to deviate if and only if

$$1+\frac{N-n}{N-1}\sum_{\tau=1}^{\infty}\delta^{\tau}\cdot 1\geq 1+G+\frac{N-n}{N-1}\sum_{\tau=1}^{\infty}\delta^{\tau}(1+G)(1-s_\tau) \tag{2}$$

For δ close to one, the left hand side tends to infinity whereas the right hand side is bounded from above.[1] Hence, there exists a $\delta_s<1$ such that player i has no incentive to deviate if and only if $\delta\geq\delta_s$. If a stranger has defected against i before, then to be consistent with equilibrium player i has to believe that the probability of meeting a defecting stranger in the future is strictly positive. His incentive not to deviate from the punishment prescribed by σ^* then gives an upper bound on the discount factor. This is because by deviating an immediate loss (G for meeting a cooperating stranger, L for meeting a defecting stranger) is occurred which has to be compared to a future gain (depending on G, but not on L) from slowing down the contagion. For any given G one can therefore choose L large enough[2] such that for *every* consistent belief of player i the upper bound on δ is greater than 1 and hence not binding. Note that amongst all separable strategies σ^* transmits the information that a defection amongst strangers has occurred as fast as possible. The only way to speed up its transmission would be to answer a deviation of a stranger by a change in the behaviour with partners. Amongst all separable strategies, σ^* therefore minimizes player i's payoff from deviating against a stranger. It follows that for all $\delta<\delta_s$ cooperation amongst strangers cannot

[1]As $\lim_{\tau\to\infty}\frac{1-s_\tau}{1-s_{\tau-1}}=1-\frac{N-n}{N-1}<1$ the Ratio Test criterion implies that $\sum_{\tau=1}^{\infty}(1-s_\tau)$ is finite.

[2]For a completely uninformed population Ellison (1994) shows that cooperation can be sustained as sequential equilibrium for sufficiently patient players *for every value of L*. His equilibrium strategies make the contagious punishment less severe by spreading it out over time. Our aim is to find the *smallest* discount factor for which cooperation can be sustained *for some values of L*.

be sustained by a separable strategy. We have thus shown that σ^* is a sequential equilibrium for all $\delta \geq \delta^*(n) \equiv \max(\delta_p, \delta_s)$ if L is sufficiently large and that full cooperation cannot be sustained by a separable strategy for $\delta < \delta^*(n)$. ■

Note that answering a defection of a stranger by defecting forever against all strangers has two effects; the direct punishment of the deviator if he is met again, and the transmission of the information that a defection amongst strangers has occurred throughout the population. It is the dependence of the speed of this information transmission on the partnership–size which drives our next result.

4. Optimal partnership–size

Cooperation is easiest to sustain in a fully informed population. In a fully uninformed population cooperation is harder to enforce as the observational restrictions imply less effective punishments. In other words, in a fully uninformed population players have to be more patient in order to make full cooperation sustainable in equilibrium. One might therefore think that for partially informed populations there is a monotone relationship between the amount of available information and the sustainability of cooperation. Surprisingly, our next result shows that this is not the case. We show that for generic parameter values the partnership–size n^* which minimizes $\delta^*(n)$ is strictly smaller than $\frac{N}{2}$. The threshold $\delta^*(n)$ is increasing between n^* and $\frac{N}{2}$. This means that cooperation becomes harder to enforce although information improves. The size n^* is optimal in the sense that it requires players to be least patient for full cooperation to become an equilibrium.

Proposition 2 *For generic parameter values $\delta^*(n)$ is minimized at n^* with $2 < n^* < \frac{N}{2}$. The partnership–size n^* optimally promotes cooperation in a partially informed population.*

Proof. First note that δ_p is strictly decreasing in n. For larger n the probability of meeting a partner, $\frac{n-1}{N-1}$, is higher and therefore a deviation against a partner implies a higher future loss making it less tempting. Second, note that δ_s is strictly increasing in n. This is because the probability to meet a stranger, $\frac{N-n}{N-1}$, is strictly decreasing in n which has a direct and an indirect effect on the expected future payoff of a deviating player. It decreases the fraction of future payoffs the player derives from encounters with strangers but it also decreases the speed with which the news of a deviation is transmitted through the population which makes the punishment less effective. For $n = 2$ a player's incentive to deviate is stronger against his partner than against a stranger so that $\delta_p(2) > \delta_s(2)$. Both deviations cost the cooperation of one player immediately but the deviation against a stranger also leads to a (delayed) breakdown of cooperation with $N-3$ additional players. For $n = \frac{N}{2}$ the number of players who will punish a deviator is nearly identical for deviations against partners and strangers.

However, partners punish a deviator immediately whereas the punishment by strangers needs time to develop. It follows that $\delta_p(\frac{N}{2}) < \delta_s(\frac{N}{2})$. For generic parameter-values the situation therefore looks as depicted in Figure 2. As $\delta^*(n) = \max\{\delta_p, \delta_s\}$, full

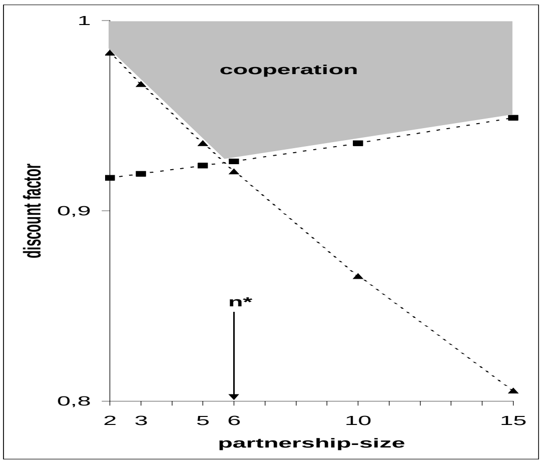

Figure 2: Dependence of $\delta^*(n) = \max\{\delta_p, \delta_s\}$ on the partnership–size n for a population of $N = 30$ players and $G = 2$. δ_p (triangles) is decreasing in n whereas δ_s (squares) is increasing. The optimal partnership–size is $n^* = 6$.

cooperation is sustainable as sequential equilibrium in the shaded area above the two thresholds δ_p and δ_s. The optimal partnership–size n^* is one of the two divisors of N which are closest to the "intersection" of the two thresholds. The negative correlation between the partnership–size and the speed of information transmission guarantees that $n^* < \frac{N}{2}$. Without this effect we would have $n^* = \frac{N}{2}$. ■

Note that n^* is second best in the sense that it is easiest to sustain cooperation in a fully informed population ($n = N$). It is optimal however, if observational imperfections exist ($n < N$). In the presence of observational restrictions there exists an advantage of having small partnerships. Smaller partnerships allow for faster information transmission. The news of a defection amongst strangers is transmitted sufficiently fast to make the contagious punishment a powerful threat.

5. Conclusion

This paper has focused on the influence of informational structure on the sustainability of cooperation in a prisoner's dilemma played in a population of randomly matched players. By restricting attention to an intuitive class of strategies we found the surprising result that too much information can be detrimental for the sustainability of cooperation.

This paper provides an explanation for the coexistence of cooperative and non–cooperative behaviour based on observational imperfections. It sheds some light on the question why cooperative behaviour seems to be contingent on the affiliation to certain groups. We have focussed on partnerships as a simple form of informational structure. Networks and other more complicated informational structures could be considered in future research along these lines.

References

Abreu, D., (1988). On the Theory of Infinitely Repeated Games with Discounting. *Econometrica* **56**, 383-396.

Ellison, G., (1994). Cooperation in the Prisoner's Dilemma with Anonymous Random Matching. *Review of Economic Studies* **61**, 567-588.

Fudenberg, D., Maskin, E., (1986). The Folk Theorem in Repeated Games with Discounting or with Incomplete Information. *Econometrica* **54**, 533-556.

Ghosh, P., Ray, D., (1996). Cooperation in Community Interaction Without Information Flow. *Review of Economic Studies* **63**, 491-519.

Greif, A., (1993). Contract Enforceability and Economic Institutions in Early Trade: The Maghribi Traders' Coalition. *American Economic Review* **83**, 525-548.

Harrington, J. E. Jr., (1995). Cooperation in a One-Shot Prisoner's Dilemma. *Games and Economic Behaviour* **8**, 364-377.

Kandori, M., (1992). Social Norms and Community Enforcement. *Review of Economic Studies* **59**, 63-80.

Matsushima, H., (1990). Long-term Partnership in a Repeated Prisoner's Dilemma with Random Matching. *Economics Letters* **34**, 245-248.

Piccione, M., (2002). The Repeated Prisoner's Dilemma and Imperfect Private Monitoring. *Journal of Economic Theory* **102**(1), 70-83.

Okuno-Fujiwara, M., Postlewaite, A., (1995). Social Norms and Random Matching Games. *Games and Economic Behaviour* **9**, 79-109.