

Advance Purchase Discounts versus Clearance Sales *

Marc Möller[†]

Makoto Watanabe[‡]

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[†]Corresponding author. Mail: Department of Economics, University Carlos III Madrid, Calle Madrid 126, 28903 Getafe Madrid, Spain. Email: mmoller@eco.uc3m.es, Tel.: +34-91624-5744, Fax: +34-91624-9329.

[‡]Mail: Department of Economics, University Carlos III Madrid, Calle Madrid 126, 28903 Getafe Madrid, Spain. Email: mwatanab@eco.uc3m.es, Tel.: +34-91624-9331, Fax: +34-91624-9329.

Abstract

This paper considers advance selling problems. It explains why some goods (e.g. airline tickets) are sold cheap to early buyers, while others (e.g. theater tickets) offer discounts to those who buy late. We derive the profit maximizing selling strategy for a monopolist when aggregate demand is certain but buyers face uncertainty about their individual demands. When aggregate demand exceeds capacity, both Advance Purchase Discounts as well as Clearance Sales might be optimal. We determine how the comparison of these price discrimination strategies depends on the rationing rule, capacity costs, and the availability of temporal capacity limits, price commitment, and resale.

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In this paper we consider products that can be purchased in advance, i.e. long before their actual date of consumption. Common examples are airline travel, theater tickets, the right to participate in conferences or sports events, and seasonal products like the newest skiing equipment. In these examples consumers face a trade-off between buying early and buying late. By delaying their purchase consumers may get a better picture about their personal fit with the product but increase their risk to become rationed. This trade-off influences the way in which prices change over time.

To see this consider the extreme case where consumers face individual demand uncertainty but rationing risks are negligible. For example, potential participants of a conference or sports event are typically uncertain about their ability to attend but the maximum number of participants might be unlikely to be reached. In this case it is clear that consumers prefer to buy late rather than early since buying late maximizes their available information. A profit maximizing monopolist can therefore charge an information premium to those consumers that buy late and prices will increase over time. Hence those products for which consumers face individual demand uncertainty but rationing risks are absent will offer Advance Purchase Discounts.

Now consider the opposite case where consumers are certain about their personal fit with the product but face a positive risk to become rationed. For example, consumers

typically know their valuation of a new pair of skis but skis may become unavailable at the end of the season. In this case consumers prefer to buy early rather than late since buying early minimizes their risk to become rationed. As a consequence, a profit-maximizing seller may charge a supply security premium to those consumers who buy early and prices will decrease over time. Hence those products for which individual demand uncertainty is absent but rationing risks exist will be sold by use of a Clearance Sale.

In most advance selling problems both supply security and information will play a role and it is therefore unclear whether prices will be increasing or decreasing over time. For instance, the purchase of airline and theater tickets are both examples where individual demand uncertainty and rationing risks interfere. However, there exists empirical evidence which shows that airline ticket prices typically increase over time while theater tickets are often sold at a discount on the day of the performance. For example, investigating data on flight tickets for 12 different US routes, Stavins (2001) shows that postponing a ticket purchase by one day raises the fare by around 0.1% of the average fare.¹ In contrast, Leslie (2004) conducts a structural econometric analysis of price discrimination in Broadway performances and finds that 197 out of the 199 performances offered discounts of up to 50% on the day of the performance and that these discounts were applied to 14% of the attendants. In order to explain these characteristic differences in pricing patterns it is important to gain a better understanding of markets where both individual demand uncertainty and rationing risks are present and to derive the exact conditions under which Clearance Sales dominate Advance Purchase Discounts or vice versa.

For this purpose we propose a simple two period model with a monopolistic seller. In the first period buyers are uncertain about their valuation of the monopolist's good whereas in the second period all individual demand uncertainty has been resolved. Buyers differ in their expected valuations. We start our analysis by considering the (benchmark) case where overall capacity is exogenous, the seller is able to commit

¹For the European market, Giaume and Guillou (2004) offer evidence for flights from Nice Airport showing that the average price increases by 12.7% within the last 22 days prior to departure. Similar results have been found by Piga and Bachis (2006), who gathered a large data set containing 650 thousand European flights.

to prices in advance, per-period capacity limits are not feasible, buyers are rationed randomly, and resale is impossible. We show that as long as buyers face a positive risk to become rationed, both Clearance Sales as well as Advance Purchase Discounts constitute the monopolist's optimal selling strategy for some parameter values.

Next we change our assumptions one at a time and determine the effect on the relative profitability of Clearance Sales and Advance Purchase Discounts in comparison with the benchmark case. We find the following results. (1) Allowing the monopolist to use per-period capacity limits makes Advance Purchase Discounts more profitable but has no effect on the profitability of Clearance Sales. (2) If the monopolist can choose his overall capacity ex ante he will implement an Advance Purchase Discount when the marginal costs of capacity are relatively high. For the monopolist to employ a Clearance Sale marginal costs of capacity have to be small but sufficiently increasing. (3) When the monopolist is unable to commit to prices in advance, Clearance Sales (Advance Purchase Discounts) become more (less) likely to be observed. (4) Allowing consumers to resell, increases the profitability of Clearance Sales in comparison to Advance Purchase Discounts. (5) Finally, when consumers are rationed efficiently rather than randomly, Clearance Sales can never be optimal. When the monopolist can choose his rationing technology then for some parameter values he will choose random rationing over any more efficient form of rationing and implement a Clearance Sale.

Two of the main ingredients of our model are individual demand uncertainty and buyers' heterogeneity. So far most of the literature on dynamic monopolistic pricing has considered these two aspects in separation. One branch of the literature assumes that buyers are heterogeneous but know their individual demands, see for example Conlisk, Gerstner and Sobel (1984), Harris and Raviv (1981), Lazear (1986), Nocke and Peitz (2007), and Van Cayseele (1991). When consumers know their demands they have a clear preference for buying early as it lowers their risk to become rationed. As a consequence, in these papers Advance Purchase Discounts can never be profit-maximizing. Our model shows that this result depends crucially on the assumption that buyers know their individual demands. In the presence of individual demand uncertainty Advance Purchase Discounts become optimal. In a static setting Dana

(1999, 2001) shows that a seller may find it optimal to offer several prices for the same good at the same time and restrict the quantity to be sold at each price. Since consumers try to buy at the lowest price first (and buy at higher prices when they get rationed) one might interpret such a strategy as an Advance Purchase Discount. In our model an Advance Purchase Discount is used as a price discrimination device and some consumers buy late at higher prices because they prefer to do so and not because they have been rationed earlier on.

A second branch of the literature allows for individual demand uncertainty but assumes that buyers are homogeneous *ex ante*, see for example Courty (2003b) and DeGraba (1995). These papers find that the monopolist maximizes his profits by selling exclusively either before or after individual demand uncertainty has been resolved. Our model shows that when buyers are heterogeneous, the monopolist will price discriminate by selling to buyers with high expectations before individual demand uncertainty has been resolved and to buyers with low expectations after.

The only papers that combine individual demand uncertainty with buyers' heterogeneity are Courty and Li (2000), Gale and Holmes (1993), and recent work by Nocke and Peitz (2008).² Courty and Li (2000) allow the monopolist to screen types by offering a menu of refund contracts *before* demand uncertainty has been resolved. In this paper we rule out the use of refund contracts. This restriction is motivated by our focus on the temporal nature of advance selling problems and the implied price dynamics. In Courty and Li (2000) all "sales" take place before buyers know their demands. Moreover, even if one interprets contracts with zero refund and price P_1 as sales in period 1 and contracts with full refund and price P_2 as sales in period 2, decreasing price paths are not feasible since it has to hold that $P_1 \leq P_2$. Hence in order to explain the difference between Advance Purchase Discounts and Clearance Sales one has to abstract from the possibility of refund contracts. Gale and Holmes (1993) show that a monopolistic airline should offer an Advance Purchase Discount in order

²Dana (1998) also features these two ingredients but considers price discrimination in a competitive rather than a monopolistic market. The competitive equilibrium consists of a common low advance purchase price and several different but higher spot prices. Since consumers are always able to purchase the good in the spot market, although not always at the lowest price, rationing risks are absent and Clearance Sales never occur.

to divert demand from a peak time flight to an off-peak time flight. While Gale and Holmes (1993) consider the sale of two products with differing degrees of rationing risk, our framework is simpler in that we restrict attention to a single product. Nocke and Peitz (2008) derive a necessary and sufficient condition for the optimality of Advance Purchase Discounts in a model with a continuum of types. They show that Advance Purchase Discounts are used as a price discrimination device when buyers are sufficiently heterogeneous. While Nocke and Peitz assume that the seller can produce any quantity at any point in time, in our model capacity is fixed or has to be chosen *ex ante*. The resulting rationing risks for the buyers as well as for the seller make Clearance Sales an alternative to Advance Purchase Discounts as a price discrimination device.

Finally, our work is also related to the literature on information provision and marketing. Lewis and Sappington (1994) and Johnson and Myatt (2006) consider a seller's incentive to supply potential customers with information about their personal match with his product (e.g. a test drive). They identify general settings in which the seller will either supply perfect information and sell only to high valuation customers (niche production), or supply no information at all and sell to all customers (mass production). Translated into our setup this amounts to selling exclusively either before or after buyers have learned their demands. As we allow buyers to be heterogeneous *ex ante*, the monopolist may find it optimal to supply intermediate levels of (aggregate) information by selling to some buyers before they know their demands and to others after. This is similar to a recent result obtained by Bar-Isaac, Caruana, and Cuñat (2008) which shows that a firm's marketing policy can be used to discriminate between consumers with high and low expectations. However, while these authors assume that the firm can choose the cost at which consumers are able to obtain information explicitly, in our setup this cost is implicitly determined by the monopolist's choice of price schedule and capacity.

The plan of the paper is as follows. Section 1 describes our setup while Section 2 contains the analysis of the benchmark case. In Sections 3 to 7 we change the assumptions of the benchmark case and consider the effect on the relative profitability of increasing versus decreasing price schedules. Section 8 concludes.

1 The setup

We consider a monopolistic seller who faces a continuum of buyers with mass normalized to one. Buyers have unit demands and trade might take place in two periods; an advance purchase period 1 and the consumption period 2. We assume that demand exhibits the following two characteristics.

Individual demand uncertainty: Ex ante, buyers are uncertain about the consumption value they will derive from the good. A buyer's valuation might take two values; a high value normalized to 1 and a low value $u \in (0, 1)$. Valuations are distributed independently and buyers privately learn their own valuation between period 1 and period 2.

Heterogeneity: There are two types of buyers. A fraction $g \in (0, 1)$ has good type and a fraction $1 - g$ has bad type. Bad types face a greater risk to have a low consumption value than good types. In particular, we assume that a bad type's likelihood of having a low valuation is $r \in (0, 1)$ while for good types it is ar with $a \in (0, 1)$. We let $U_G \equiv 1 - ar + aru$ and $U_B \equiv 1 - r + ru$ denote the expected valuations of good and bad types respectively. Buyers' types are private knowledge.

Note that our assumptions on demand imply that a bad type's consumption value might turn out to be higher than a good type's expected valuation. We will see that this assumption is crucial for the optimality of increasing price schedules. Also note that since we are considering a continuum of buyers, there is no uncertainty about aggregate demand. Letting P_1 and P_2 denote the first and second period price respectively we make the following

Definition 1 *The monopolist implements a Clearance Sale (Advance Purchase Discount) if $P_1 > P_2$ ($P_1 < P_2$) and revenues are strictly positive in both periods.*

We assume that production costs are zero so that the monopolist's payoff, Π , is identical to his revenue minus capacity costs. Buyers are risk-neutral and have quasi-linear preferences. In particular, when a buyer purchases the good his payoff is equal to the difference between his (expected) consumption value and the price. Otherwise his payoff is zero. Following most of the literature mentioned in the Introduction we abstract from discounting by assuming that first and second period payoffs are valued

identically. The introduction of risk aversion and discounting is straight forward and would simply decrease a consumer's incentive to buy in advance.

2 Benchmark

In this section we derive the monopolist's optimal selling strategy in a benchmark setting. In this setting both Clearance Sales as well as Advance Purchase Discounts constitute the monopolist's optimal selling strategy for some parameter values. It is characterized by five assumptions which we will relax in the subsequent sections: (A1) the monopolist is unable to set per period capacity limits; (A2) the monopolist's capacity $k > 0$ is fixed exogenously; (A3) the monopolist can commit to a price schedule (P_1, P_2) in advance; (A4) consumers are unable to resell; and (A5) consumers are rationed randomly.

Since aggregate demand is never larger than 1, it is immediate that for $k > 1$ the monopolist's profit maximizing selling strategy is identical to the one for the case $k = 1$. We can therefore restrict our attention to the case where $k \in (0, 1)$ and obtain the monopolist's optimal selling strategy for the remaining cases by considering the limit as $k \rightarrow 1$.

A common element in the definitions of a Clearance Sale and an Advance Purchase Discount is the requirement that revenues have to be positive in both periods. Since the monopolist cannot set a first period capacity limit, the only way to achieve positive revenues in both periods is to price discriminate. The monopolist has to choose prices such that one type of consumer prefers to buy in advance while the other type prefers to postpone the purchase. The prices that make good types indifferent between buying early and buying late satisfy

$$U_G - P_1 = (1 - P_2)(1 - R)(1 - ar) \tag{1}$$

where R denotes the probability with which the consumer will be rationed in period 2. If the consumer buys early his utility is equal to his expected valuation minus the price. If he postpones his purchase, he will demand the good only if his valuation exceeds the second period price P_2 . Since $P_2 \geq u$, the consumer can obtain a rent

$1 - P_2$ only when his valuation turns out to be high and he fails to be rationed. The term $(1 - P_2)(1 - R)(1 - ar)$ can therefore be understood as the consumer's option value of postponing his purchase until his demand uncertainty has been resolved. If the risk to become rationed is positive then

$$U_G - U_B = (1 - u)(1 - a)r > (1 - P_2)(1 - R)(1 - a)r \quad (2)$$

implies that good types have a stronger propensity to buy in advance than bad types. Hence in order to implement a Clearance Sale or an Advance Purchase Discount the monopolist has to price discriminate by selling to good types before and to bad types after individual demand uncertainty has been resolved. An immediate consequence is that for $k \leq g$ neither a Clearance Sale nor an Advance Purchase Discount is implementable. Hence in the following we will concentrate on the case where $k > g$. Note that (1) can be rewritten as

$$P_1 = P_2 - (P_1 - u) \frac{ar}{1 - ar} + \frac{R}{1 - R} \frac{U_G}{1 - ar}. \quad (3)$$

From this equation we can see that good types are offered an information discount of size $(P_1 - u) \frac{ar}{1 - ar}$. By delaying their purchase, good types can avoid the mistake of buying the product when their valuation turns out to be smaller than the price and hence save the difference $P_1 - u$. The information discount is high when the consumers' likelihood of having a low valuation ar is large relative to the likelihood of having a high valuation $1 - ar$. In addition, good types are charged a supply security premium of size $\frac{R}{1 - R} \frac{U_G}{1 - ar}$. The premium increases in the rationing risk R and tends to zero when rationing risks become negligible. The consumers' trade-off between information and supply security will therefore be reflected in the monopolist's pricing decisions. When the information discount exceeds the supply security premium, prices will be increasing over time and price discrimination will take the form of an Advance Purchase Discount. If instead the supply security premium is larger than the information discount, prices will be decreasing and price discrimination takes the form of a Clearance Sale. In the following we derive the conditions under which the monopolist's profits are maximized by one of these two selling strategies.

Clearance Sales

Clearance Sales are employed for a broad variety of products ranging from bargain holidays to theater tickets. By definition a Clearance Sale requires that $P_2 < P_1$ and that revenues are positive in both periods which implies that $P_2 < 1$. Moreover, a Clearance Sale with $P_2 \in (u, 1)$ cannot be profit maximizing since P_2 can be increased without lowering neither second nor first period demand. Hence for a Clearance Sale to be optimal it has to hold that $P_2^C = u$. From (1) we get the optimal first period price:

$$P_1^C = U_G - (1 - u)(1 - ar)(1 - R) = U_G - (U_G - u)(1 - R). \quad (4)$$

In period 2 the monopolist's demand is given by the mass of consumers with bad type, $1 - g$, while his supply is $k - g$. Since rationing is random the probability to become rationed in period 2 is therefore $R = 1 - \frac{k-g}{1-g} = \frac{1-k}{1-g}$. Note that $P_1^C - P_2^C = (U_G - u)R$ which implies that in order to implement a Clearance Sale it has to hold that $R > 0$, i.e. there has to exist a positive risk to become rationed. Since $R \rightarrow 0$ for $k \rightarrow 1$ a Clearance Sale cannot be implemented when $k \geq 1$. When deciding whether to buy in advance or not, buyers trade off a lower risk to be rationed when buying early, with a price discount and better information when buying late. When rationing risks are absent, first period demand is zero for any decreasing price schedule.

By implementing a Clearance Sale the monopolist is able to sell his entire capacity and his profits are given by

$$\Pi^C = gP_1^C + (k - g)P_2^C. \quad (5)$$

Another way to achieve a sell-out is to set $P_2 = 1$ and to sell the product at price $P_1 = U_B$ to both types before individual demand uncertainty becomes resolved. For $k \rightarrow g$ the risk to become rationed, R , converges to one and a Clearance Sale sells the monopolist's entire capacity at a higher price $P_1^C \rightarrow U_G > U_B$. In contrast, for $k \rightarrow 1$, R converges to zero and a Clearance Sale sells everything at a lower price $P_1^C \rightarrow u < U_B$. It turns out that there exists a critical value of capacity

$$k^{C1} = \frac{g(U_G - u)}{gU_G + (1 - g)U_B - u} \in (g, 1) \quad (6)$$

such that the following lemma holds:

Lemma 1 *In the absence of rationing risks, i.e. when $k \geq 1$, a Clearance Sale cannot be implemented. For $k < 1$ a profit maximizing Clearance Sale sets $P_1^C = U_G - (U_G - u) \frac{k-g}{1-g} > u = P_2^C$, and leads the profits in (5). A Clearance Sale implements higher profits than selling exclusively in period 1 at $P_1 = U_B$ if and only if $k < k^{C1}$.*

Advance Purchase Discounts

Many products are sold by use of an Advance Purchase Discount. For example, airline tickets become more costly as the date of departure approaches. Similarly, academic conferences and sports events offer discounts for early registration. By definition an Advance Purchase Discount requires that $P_1 < P_2$. It is therefore immediate that in order to be profit maximizing the second period price has to be strictly larger than u . Moreover, a $P_2 \in (u, 1)$ cannot be optimal since by increasing P_2 the monopolist could lower the good types' option value, thereby raising the price good types are willing to pay in the first period, without changing the demand from bad types in the second period. Hence for an Advance Purchase Discount to be optimal it has to hold that $P_2^A = 1$. Given $P_2^A = 1$ the consumers' option value of postponing their purchase is zero and from (1) the optimal first period price is given by $P_1^A = U_G$. Whether the monopolist is able to sell his entire capacity depends on the comparison of second period supply $k - g$ and second period demand $(1 - g)(1 - r)$ and profits are given by

$$\Pi^A = gU_G + \min(k - g, (1 - g)(1 - r)). \quad (7)$$

Let us compare an Advance Purchase Discount with a strategy that sells exclusively after consumers have learned their demands by setting $P_1 > U_G$ and $P_2 = 1$. Such a strategy implements a higher price on average but restricts sales to those consumers who turn out to have high valuations. Profits are given by $\Pi = \min(k, g(1 - ar) + (1 - g)(1 - r))$. This strategy will dominate Advance Purchase Discounts when high valuations are sufficiently likely and capacity is sufficiently small. More precisely, the profits of an Advance Purchase discount turn out to be smaller than the profits from selling exclusively in period 2 at $P_2 = 1$, if and only if $r < \frac{1-g}{1-g(1-au)}$ and $k < k^{A2}$ where

$$k^{A2} = 1 - r[1 - g(1 - au)]. \quad (8)$$

On the other hand, when capacity is large and the fraction of good types is sufficiently low, an Advance Purchase Discount will fail to sell out and will be dominated by a strategy which sells the monopolist's entire capacity before consumers have learned their demands at $P_1 = U_B$. In particular, the profits from selling exclusively in period 1 at $P_1 = U_B$, $\Pi = kU_B$ are higher than Π^A if and only if $g < \frac{u}{1-a(1-u)}$ and $k > k^{A1}$ where

$$k^{A1} = \frac{gU_G + (1-g)(1-r)}{U_B}. \quad (9)$$

Our results are summarized in the following:

Lemma 2 *A profit maximizing Advance Purchase Discount sets $P_1^A = U_G < 1 = P_2^A$ and leads the profits in (7). It is dominated by selling exclusively in period 2 if and only if $r < \frac{1-g}{1-g(1-au)}$ and $k < k^{A2}$. It is dominated by selling exclusively in period 1 if and only if $g < \frac{u}{1-a(1-u)}$ and $k > k^{A1}$.*

Comparison

Using a Clearance Sale, the monopolist sells his entire capacity but pays information rents. He leaves the option value $(1-u)(1-ar)(1-R)$ to good types and a rent of $1-u$ to bad types who turn out to have a high consumption value. By implementing an Advance Purchase Discount the monopolist avoids to leave option values and information rents. If the monopolist is able to sell his entire capacity by use of an Advance Purchase Discount, i.e. if $k-g \leq (1-g)(1-r)$, he therefore strictly prefers an Advance Purchase Discount over a Clearance Sales. However, for $k-g > (1-g)(1-r)$ the elimination of information rents and option values comes at a cost since the monopolist fails to sell his entire capacity. Second period revenues for an Advance Purchase Discount and for a Clearance Sale are $(1-g)(1-r)$ and $(k-g)u$ respectively. If the former are larger than the latter the monopolist clearly prefers an Advance Purchase Discount. Otherwise the gain in first period revenue due to the elimination of option values, $g(1-u)(1-ar)$, has to be compared with the loss in second period revenues $(k-g)u - (1-g)(1-r)$. If the fraction of good types is sufficiently large or the likelihood of a high valuation is sufficiently high, the savings in option value from sales to

good types exceed the loss in revenues from bad types for all values of capacity. Only if g is sufficiently small and r is sufficiently large, Clearance Sales dominate Advance Purchase Discounts for high values of capacity. Defining

$$k^{AC} = g + \frac{1}{u}[(1-g)(1-r) + g(1-ar)(1-u)] \quad (10)$$

in the Appendix we prove the following:

Lemma 3 *Clearance Sales give higher profits to the monopolist than Advance Purchase Discounts if and only if $g < \frac{u}{1-a(1-u)}$, $r > \frac{1-u}{1-g[1-a(1-u)]}$ and $k > k^{AC}$.*

Optimality

From our analysis so far it follows that there exist four candidates for the monopolist's profit maximizing selling strategy. The monopolist can either sell exclusively before or after individual demand uncertainty has been resolved at $P_1 = U_B$ or $P_2 = 1$ respectively. Or he can price discriminate by implementing an Advance Purchase Discount or a Clearance Sale.

Selling exclusively in period 2 at $P = 1$ implements the highest price but sales can never exceed the number of consumers who turn out to have high valuations, $g(1-ar) + (1-g)(1-r)$. An Advance Purchase Discount increases potential sales to $g + (1-g)(1-r)$ but decreases the average price to some $P \in (U_G, 1)$. Finally, by use of a Clearance Sale or by selling exclusively in period 1 at $P_1 = U_B$ the monopolist sells his entire capacity at an even smaller average price $P \in [U_B, U_G)$. This ordering suggests that in order to fully characterize the monopolist's optimal selling strategy it is sufficient to consider the pairwise comparisons contained in Lemmas 1–3. For example, the comparison between selling exclusively in period 1 and selling exclusively in period 2 does not affect the characterisation of the monopolist's optimal strategy since for all parameter values Advance Purchase Discounts dominate at least one of these two selling strategies. Lemmas 1–3 therefore imply the following result:

Proposition 1 *For $g < \frac{u}{1-a(1-u)}$ an Advance Purchase Discount maximizes the monopolist's profits if and only if $\max(g, k^{A2}) < k < \min(k^{A1}, k^{AC})$ while a Clearance Sale is optimal if and only if $k^{AC} < k < k^{C1}$. For $g \geq \frac{u}{1-a(1-u)}$ an Advance Purchase*

Discount is profit maximizing if and only if $\max(g, k^{A2}) < k$ but a Clearance Sale can never be optimal. The monopolist's profit maximizing strategy is as in Figure 1.

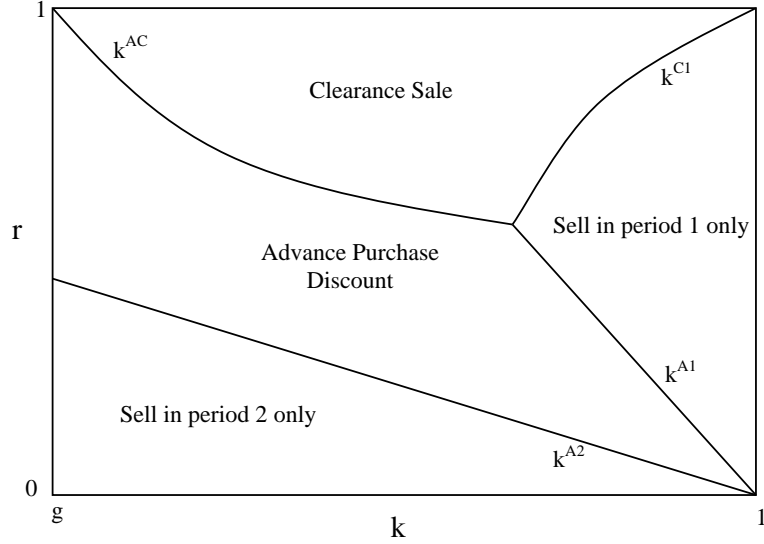


Fig. 1: *Profit maximizing selling strategy in the benchmark.* Note: The figure shows the case where $g < \frac{u}{1-a(1-u)}$. The thresholds k^{C1} , k^{A2} , k^{A1} , and k^{AC} are defined in (6), (8), (9), and (10).

Figure 1 provides a graphical representation of the monopolist's optimal selling policy. Both forms of price discrimination maximize the monopolist's profits for some parameters. While Clearance Sales are optimal when capacity is relatively high and low valuations are relatively likely, Advance Purchase Discounts are profit maximizing for lower values of capacity and when consumers are more likely to have high valuations.

In our model the total mass of consumers is constant and the monopolist's capacity is a parameter. In an equivalent formulation capacity could be taken as fixed and we might consider the monopolist's optimal selling strategy as a function of the market size. In this interpretation Clearance Sales would be employed when the market is relatively small while Advance Purchase Discounts would be optimal when the market is larger. Hence for markets whose size fluctuates seasonally our theory suggests observable changes in pricing patterns. For example our model may help to explain why

for package holidays Clearance Sales are more frequently observed during low season when aggregate demand is small relative to capacity.

By considering the limit as $k \rightarrow 1$ we can derive the monopolist's optimal selling strategy for the remaining case $k \geq 1$. For $k \rightarrow 1$ Clearance Sales are no longer feasible and selling exclusively in period 2 ceases to be optimal. Hence when the monopolist's capacity exceeds the mass of consumers the monopolist will either price discriminate by offering an Advance Purchase Discount to consumers with high expectations or sell to all consumers before demand uncertainty becomes resolved. He will implement an Advance Purchase Discount if and only if the fraction of consumers with high expectations is sufficiently large, i.e. when $g \geq \frac{u}{1-a(1-u)}$.

Taking the limit $a \rightarrow 1$ we can relate our result to the literature which assumes that buyers are homogeneous ex ante. Since for homogeneous buyers price discrimination ceases to be feasible, the monopolist will either sell his entire capacity before individual demand uncertainty has been resolved or after. The resulting "buying frenzies" are the analog to the result of DeGraba (1995) and Courty (2003b) who show that selling both before and after buyers have learned their demands can never be optimal.

Finally we comment on our assumption that there are only two types of buyers. In a recent paper, Nocke and Peitz (2008) allow for a continuum of types. As in our model, Nocke and Peitz assume that each buyer's valuation may take either a high or a low value but they allow these values to differ across types. However, Nocke and Peitz abstract from the possibility of a capacity constraint. As a consequence Clearance Sales cannot be implemented and the only form of price discrimination is an Advance Purchase Discount. We expect that the main insights of our benchmark model survive in the presence of a continuum of types. In particular, Advance Purchase Discounts as well as Clearance Sales may both be optimal when the monopolist's capacity is restricted.

3 Temporal capacity limits

In this section we consider the possibility of temporal capacity limits. In particular, we relax assumption (A1) and assume instead that the monopolist can commit not

to sell more than a maximum of $k_1 \in [0, k]$ units in the first period. One example where temporal capacity limits are frequently employed is the sale of airline tickets. Airlines partition their capacity into fare classes and different fares are released into the electronic booking systems at different pre-determined points in time. Another example are sports events which sometimes reserve a certain number of slots for late entries.

Consider a Clearance Sale first. The introduction of a first period capacity limit decreases the consumers' risk to become rationed in the second period from $R = \frac{1-k}{1-g}$ to $R' = \frac{1-k}{1-k_1}$. This increases the option value for good types and therefore decreases the price the monopolist is able to charge in period 1. At the same time the fraction of capacity that is sold at the lower second period price increases. It follows that the introduction of a temporal capacity limit can only reduce the profitability of a Clearance Sale. In contrast, the profits of an Advance Purchase Discount might be increased with the help of a temporal capacity limit. To see this note that in an Advance Purchase Discount prices are independent of k_1 since a consumer's option value of postponing his purchase is zero. As $P_1^A < P_2^A$, the monopolist has an incentive to reduce his first period sales as long as this increases his second period revenues. This is possible as long as second period demand, $(g - k_1)(1 - ar) + (1 - g)(1 - r)$, exceeds second period supply, $k - k_1$. Optimally the monopolist will therefore choose k_1 to equate second period supply with second period demand. When second period demand exceeds supply in the absence of a capacity limit, i.e. when $(1 - g)(1 - r) > k - g$, the monopolist can achieve this by setting

$$k_1 = \frac{k - 1 + r[1 - g(1 - a)]}{ra} \in (0, g). \quad (11)$$

The availability of a temporal capacity limit raises the monopolist's profits from $\Pi^A = gU_G + k - g$ to $\Pi^{A'} = k_1U_G + k - k_1$. This affects the profit comparison with a strategy which sells exclusively in period 2 at $P_2 = 1$. In particular, an Advance Purchase Discount will dominate such a strategy whenever the latter fails to sell the monopolist's entire capacity, i.e. when $k > k^{A2'}$ where

$$k^{A2'} \equiv g(1 - ar) + (1 - g)(1 - r) \quad (12)$$

and $k^{A2'} < k^{A2}$. The monopolist increases his profits by selling some of the capacity that will remain unsold in period 2 with an Advance Purchase Discount in period 1. In contrast, for the above range of parameters, the comparison of an Advance Purchase Discount with a Clearance Sale remains unaffected by the availability of temporal capacity limits since the profits of the former, $\Pi^A = gU_G + k - g$, are larger than the profits of the latter, $\Pi^C = g[U_G - (U_G - u)\frac{1-k}{1-g}] + (k - g)u$, even in the absence of such limits.

Things are different when $(1 - g)(1 - r) \leq k - g$. In this case the monopolist has to sell more than g units in the first period in order to match his second period supply with second period demand. In order to do so the monopolist has to offer an even larger Advance Purchase Discount by offering $P_1 = U_B$ and limit his first period capacity by setting

$$k_1 = 1 - \frac{1 - k}{r[1 - g(1 - a)]}. \quad (13)$$

This selling strategy implements positive sales in both periods at increasing prices and therefore satisfies our definition of an Advance Purchase Discount. It differs from the Advance Purchase Discount we have considered so far in that it elicits demand from both types of consumers in period 1. For large values of capacity the monopolist will therefore implement an Advance Purchase Discount with $P_1^{A'} = U_B$ and $P_2^{A'} = 1$ leading the profits $\Pi^{A'} = k_1U_B + k - k_1$, where k_1 is as specified in (13). Since these profits are strictly larger than kU_B selling exclusively in period 1 cannot be optimal when temporary capacity limits are available. It simply makes no sense to sell everything in period 1 if a positive quantity can be sold at a higher price in period 2. Hence the last relevant comparison we have to discuss is the one between a Clearance Sale and an Advance Purchase Discount with $P_1^{A'} = U_B$. Both strategies sell the monopolist's entire capacity but do so at different prices. The Advance Purchase Discount price discriminates ex post by selling to consumers with high valuations only. In contrast, a Clearance Sale price discriminates ex ante by selling only to consumers with high expectations. When low valuations are relatively likely and the monopolist's capacity is small so that a large part of it can be sold to good types in advance, price discrimination ex ante is more profitable than price discrimination ex post. In

particular we find that Clearance Sales are more profitable than an Advance Purchase Discount with $P_1^{A'} = U_B$ if and only if $k < k^{C1'}$ where

$$k^{C1'} \equiv \frac{g^2 a(1 - ar) - (1 - g)^2(1 - r)}{ga[1 - r + gr(1 - a)]} \quad (14)$$

and $k^{C1'} < k^{C1}$. In the absence of temporal capacity limits an Advance Purchase Discount had the sole function of price discrimination between consumers with different expectations. The availability of such limits adds a second function. It allows the monopolist to satisfy the demands of consumers who turn out to have high valuations and to sell his “remaining capacity” in advance at a discount. We can summarize our findings as follows:

Proposition 2 *Temporal capacity limits increase the profitability of Advance Purchase Discount but will never be employed in a Clearance Sale. The monopolist’s profit maximizing strategy with and without temporal capacity limits is as depicted in Figure 2.*

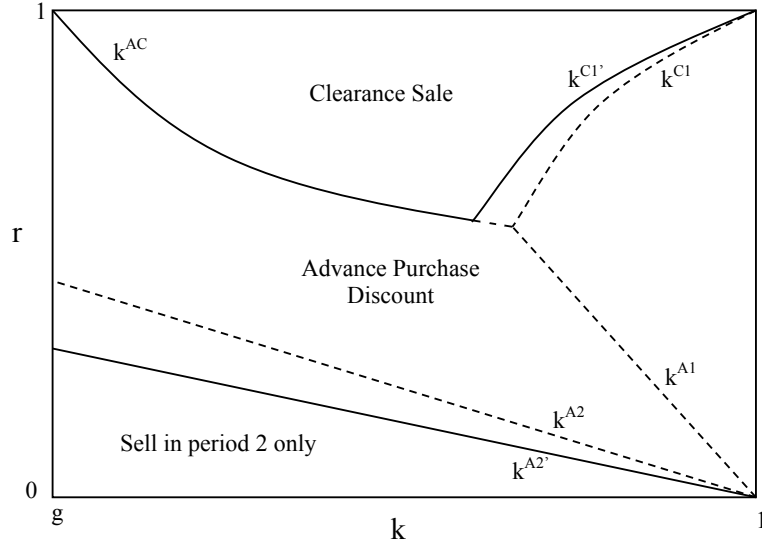


Fig. 2: Profit maximizing selling strategy with temporal capacity limits (bold lines) in comparison with the benchmark (dashed lines). Note: For explanatory notes see Figure 1. The thresholds $k^{A2'}$ and $k^{C1'}$ are defined in (12) and (14).

As can be seen in Figure 2, the presence of temporal capacity limits enlarges the set of parameters for which an Advance Purchase Discount is optimal, whereas for Clearance Sales the set becomes smaller. In this sense we can say that the availability of temporal capacity limits makes Advance Purchase Discounts (Clearance Sales) more (less) likely to be observed.

4 Capacity choice

Our benchmark model assumes that the monopolist's capacity is fixed exogenously. Indeed, in many advance selling problems capacity is restricted, at least in the short run. For instance, due to logistic reasons, organizers of conferences or sports events are often unable to increase their capacity beyond certain limits. Similarly, the capacity of airline travel between two specific destinations might be restricted by the number of landing slots available. However, in the absence of such restrictions a seller can be expected to choose his capacity optimally. In this section we therefore relax assumption (A2) and assume instead that the monopolist chooses his capacity ex ante at a cost $C(k)$ with $C' \geq 0$ and $C'' \geq 0$.

In order to focus on the comparison of increasing versus decreasing price schedules we restrict attention to the set of parameters for which Clearance Sales are optimal for some values of capacity. In particular, we consider the case where $g < \frac{u}{1-a(1-u)}$ and $r > r^C$ where $r^C \in (0, 1)$ is the unique solution to $k^{AC} = k^{C1}$ (see Figure 1). The monopolist will employ a Clearance Sale when $k \in (k^{AC}, k^{C1})$. In this range, marginal revenue is therefore given by

$$\frac{d\Pi^C}{dk} = u - \frac{g}{1-g}(U_G - u). \quad (15)$$

Every additional unit of capacity increases second period revenues by u but decreases first period revenues by $\frac{g}{1-g}(U_G - u)$ as it increases the consumers' option value of waiting. For the monopolist to choose a $k \in (k^{AC}, k^{C1})$ and to implement a Clearance Sale it therefore has to hold that $C'(k^{AC}) < u - \frac{g}{1-g}(U_G - u)$. For $k > k^{C1}$ the monopolist will optimally sell his entire capacity to both types of consumers in period 1 and his marginal revenue is given by U_B . If $C'(k^{C1}) < U_B$ the monopolist will

therefore increase his capacity beyond k^{C1} and sell exclusively in period 1. Hence for a Clearance Sales to be employed the marginal costs of capacity have to be sufficiently small and sufficiently increasing. When $C'(k^{AC}) < u - \frac{g}{1-g}(U_G - u)$ and $C'(k^{C1}) > U_B$ the monopolist will choose the capacity $k_C \in (k^{AC}, k^{C1})$ which solves

$$C'(k_C) = u - \frac{g}{1-g}(U_G - u) \quad (16)$$

and implement a Clearance Sale. In contrast, Advance Purchase Discounts will be implemented when the marginal costs of capacity are relatively high. To see this, note that an Advance Purchase Discount leads a marginal revenue of 1 as long as the monopolist is able to sell his entire capacity, i.e. as long as $k < k_A \equiv g + (1-g)(1-r)$. Since marginal revenue is at most U_B for all $k > k_A$ the monopolist will choose $k = k_A$ and implement an Advance Purchase Discount when $1 > C'(k_A) > U_B$. The following proposition summarizes these results:

Proposition 3 *Suppose that $g < \frac{u}{1-a(1-u)}$ and $r > r^C$. If marginal costs of capacity are relatively high, i.e. $1 > C'(k_A) > U_B$, the monopolist will choose capacity $k_A < k^{AC}$ and implement an Advance Purchase Discount. If marginal costs are small but sufficiently increasing, i.e. $C'(k^{AC}) < u - \frac{g}{1-g}(U_G - u)$ and $C'(k^{C1}) > U_B$, then he will choose a $k_C \in (k^{AC}, k^{C1})$ and implement a Clearance Sale. If $C'(k^{C1}) < U_B$ the monopolist will choose a $k \in (k^{C1}, 1]$ and sell exclusively in period 1.*

In Section 2 we have seen that Clearance Sales cannot be implemented when rationing risks are absent, i.e. when $k \geq 1$. Nocke and Peitz (2007) have shown that the monopolist might therefore have an incentive to restrict his capacity ex ante even when capacity costs are absent. In contrast, Proposition 3 implies that in the absence of capacity costs, Clearance Sales will never be used and the monopolist will choose $k = 1$, implementing an Advance Purchase Discount if $g \geq \frac{u}{1-a(1-u)}$ and selling exclusively in period 1 otherwise. This difference stems from the fact that in Nocke and Peitz (2007) individual demands are known but aggregate demand is uncertain. Hence consumers prefer to buy early in order to minimize their risk to become rationed and it cannot be optimal to offer a price discount in period 1. In our setup, individual demands are uncertain but aggregate demand is constant so that rationing risks are absent for

$k \geq 1$. Hence consumers prefer to buy late and it cannot be optimal to offer a price discount in period 2. This comparison suggests that Advance Purchase Discounts are likely to be used for products whose value is uncertain to buyers at the time of purchase (e.g. airline tickets) while Clearance Sales can be expected to be employed when buyers know their individual valuations but aggregate demand is uncertain ex ante (e.g. for seasonal products).

5 Price commitment

In our benchmark model we have followed Courty (2003b), Harris and Raviv (1981), and Van Cayseele (1991) by assuming that the monopolist is able to commit to prices in advance. Indeed, there are many examples where price commitment is used in practice. For instance, the organizers of conferences and sports events often commit to prices by announcing participation fees as a function of the registration date. While in this example price commitment is explicit in other examples it is implied by the repeated nature of transactions. For example, although airlines do not commit to prices in advance, consumers do expect prices to increase as the departure date approaches.

In this section we change assumption (A3) and assume instead that the monopolist is unable to commit to prices in advance. In the absence of such commitment the monopolist faces a time consistency problem similar to the one studied in the durable goods literature (see Bulow (1982), Coase (1972), Gul, Sonnenschein and Wilson (1986), and Stokey (1981)). After selling to good types in the first period, the monopolist might have an incentive to adjust his second period price to second period demand. Under any price discrimination policy, the monopolist's lack of commitment therefore requires second period prices to maximize second period revenue.

This has consequences for the feasibility of an Advance Purchase Discount. In the benchmark model we have seen that an Advance Purchase Discount fails to sell the monopolist's remaining capacity in period 2 when $k - g \geq (1 - g)(1 - r)$. In this case an Advance Purchase Discount is feasible in the absence of commitment if and only if $P_2 = 1$ maximizes the monopolist's second period revenue, i.e. when

$(1 - g)(1 - r) \geq (k - g)u \Leftrightarrow k \leq k^{nc}$ where

$$k^{nc} \equiv g + \frac{1}{u}(1 - g)(1 - r). \quad (17)$$

In turn, a Clearance Sale does not require commitment when the opposite holds, i.e. when $k \geq k^{nc}$. When the monopolist's capacity is large, second period revenues are maximized by charging a low price and price discrimination has to take the form of a Clearance Sale. In contrast, for low values of capacity the monopolist will choose a high price in the second period and price discrimination takes to the form of an Advance Purchase Discount.

Since $k^{nc} < k^{AC}$, this implies that in the parameter range where Clearance Sales are optimal in the benchmark case, they do not require commitment power. On the other hand, for some subset of the parameter space for which Advance Purchase Discounts are optimal in the presence of price commitment, Advance Purchase Discounts cease to be feasible. Hence we can state the following result:

Proposition 4 *In the absence of price commitment Clearance Sales (Advance Purchase Discounts) are implementable if and only if $k \geq (\leq)k^{nc}$. The lack of price commitment makes Clearance Sales (Advance Purchase Discounts) more (less) likely to be observed. The monopolist's profit maximizing strategy is as depicted in Figure 3.*

To understand the intuition for this result consider the case where $k \leq k^{nc}$. By definition of k^{nc} , an Advance Purchase Discount leads the same second period revenues as a Clearance Sale. However, an Advance Purchase Discount implements higher first period revenues by eliminating option values. Hence for $k = k^{nc}$ an Advance Purchase Discount is more profitable than a Clearance Sale and if possible the monopolist would implement an Advance Purchase Discount for even larger values of capacity, i.e. for all $k \in (k^{nc}, k^{AC})$. Price commitment enables the monopolist to do so and therefore increases the set of parameters for which an Advance Purchase Discount will be observed.

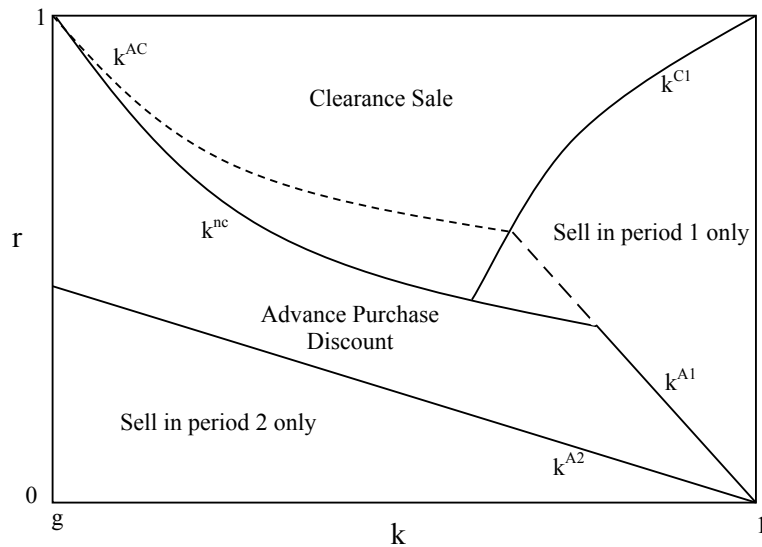


Fig. 3: Profit maximizing selling strategy without price commitment (bold lines) in comparison with the benchmark (dashed lines). Note: For explanatory notes see Figure 1. The threshold k^{nc} is defined in (17).

6 Resale

While for some products (e.g. airline travel) resale amongst consumers is difficult or not feasible, for others (e.g. event tickets) resale markets are relatively well developed and abundant. In this section we relax assumption (A4) by allowing for resale amongst consumers. We consider a resale market in which consumers can exchange the good amongst each other after individual demand uncertainties have been resolved. The literature on monopoly with resale is relatively scarce and most papers assume that buyers know their valuations (see Zheng (2002) and Calzolari and Pavan (2006)).³

When resale is feasible, buyers who hold the good in period 2 and turn out to have a low valuation will try to resell to buyers with high valuations. We abstract from price competition in the resale market. Instead we assume that in the resale market buyers and sellers are matched randomly and following Zengh (2002) we suppose that

³An exception is Courty (2003b) but since buyers are assumed to be homogeneous ex ante, a comparison of the effects of resale on the profitability of different types of price discrimination is beyond the scope of the paper.

sellers have the entire bargaining power. Hence if a reseller is matched with a consumer with high valuation he will sell at price $P_R = 1$. Otherwise he will consume the good himself. Given the possibility of resale, the prices that make buyers with good types indifferent between buying early and buying late must satisfy

$$U_G + arQ(P_R - u) - P_1 = (1 - P_2)(1 - ar)(1 - R). \quad (18)$$

Q denotes the probability that the buyer is able to resell the good successfully. Q depends on the number of sellers and buyers in the resale market and hence on the monopolist's selling strategy. The possibility of resale has an important consequence. It provides early buyers with an insurance for the case where their valuation turns out to be low. In comparison with the benchmark case buyers therefore have a stronger incentive to buy early and the monopolist can charge an insurance premium to early buyers.⁴ On the other hand, when the monopolist is planning to sell to buyers with high valuations in the second period, the existence of a resale market has a detrimental effect on the demand faced by the monopolist since some buyers may purchase from the resale market rather than from the monopolist.

To see this consider an Advance Purchase Discount where $P_2^A = 1$ and $P_1^A = U_G + arQ^A(P_R - u)$ from (18). In period 2, the number of consumers willing to buy at $P_2 = 1$ is given by $(1 - g)(1 - r)$. On the other hand, gar consumers wish to resell and $k - g$ units are offered by the monopolist himself. Random rationing implies that each second period unit is sold with probability

$$Q^A = \min\left(\frac{(1 - g)(1 - r)}{gar + k - g}, 1\right). \quad (19)$$

Note that $P_1^A = 1$ when $Q^A = 1 \iff k < g(1 - ar) + (1 - g)(1 - r)$, i.e. Advance Purchase Discounts are not feasible when capacity is so small that the possibility of resale provides perfect insurance. When feasible, the profits of an Advance Purchase Discount are given by

$$\Pi^A = gP_1^A + (k - g)Q^A. \quad (20)$$

⁴Given that insurance is more valuable to bad types, the possibility of resale may induce bad types to have a stronger incentive to buy early than good types. In the Appendix we show that a strategy which induces bad types to buy early can never be profit maximizing.

Resale has the positive effect of increasing the first-period price by the insurance premium but also the negative effect of reducing second period sales. For high values of capacity, i.e. when $k \geq g + (1 - g)(1 - r)$, the loss in second period revenue exceeds the gain in first period revenue and the existence of a resale market has a negative effect on the profitability of an Advance Purchase Discount.

Under a Clearance Sale this negative effect is absent. Since $P_2^C = u < P_R$ all consumers prefer to buy from the monopolist rather than from the resale market. The consumers' resale probability under a Clearance Sale is given by

$$Q^C = \min \left(\frac{(1 - k)(1 - r)}{gar}, 1 \right). \quad (21)$$

and from (18) we have $P_1^C = U_G + arQ^C(P_R - u) - (1 - u)(1 - ar)(1 - R)$. Note that the existence of the resale market does not affect the consumers' option value of waiting since they obtain a surplus only if they are able to purchase from the monopolist which happens with probability $1 - R$ where $R = \frac{1 - k}{1 - g}$. Due to the insurance premium the monopolist's profits are greater than in the absence of resale and they are given by

$$\Pi^C = gP_1^C + (k - g)u. \quad (22)$$

Since resale markets increase the profits of Clearance Sales but their effect on the profitability of Advance Purchase Discounts may be negative, one may expect the former to become more profitable relative to the latter. Indeed, our next result confirms this intuition. Moreover, it is immediate that the availability of resale increases the profits of a strategy that sells exclusively in period 1 while the profits from selling exclusively in period 2 remain unaffected. In the Appendix we show that as a consequence, selling exclusively in advance becomes more profitable relative to any form of price discrimination. This is because by selling in advance the monopolist is able to charge a resale insurance premium to all consumers rather than only to good types. We have the following:

Proposition 5 *The possibility of resale amongst consumers makes Clearance Sales more profitable. Advance Purchase Discounts become more profitable for small values of capacity but less profitable for large values of capacity. The monopolist's profit*

maximizing strategy is as depicted in Figure 4. The relative profitability of Clearance Sales as compared to Advance Purchase Discounts increases.

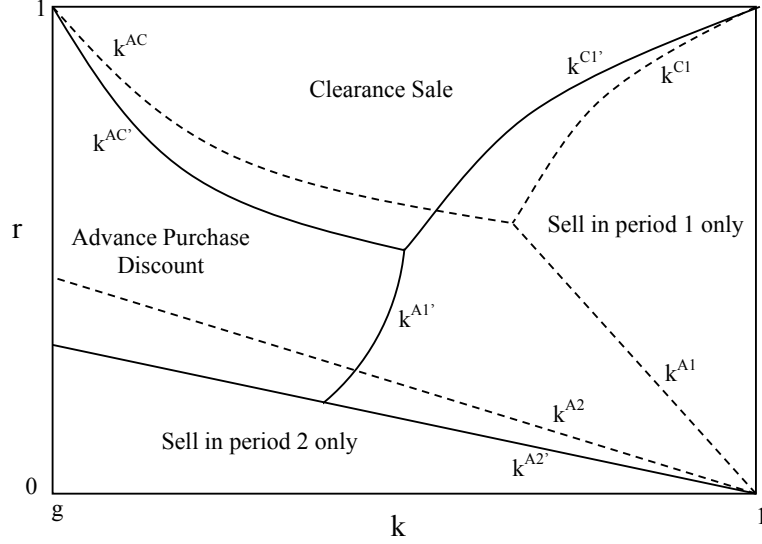


Fig. 4: Profit maximizing selling strategy with resale (bold lines) in comparison with the benchmark (dashed lines). Note: For explanatory notes see Figure 1. The thresholds $k^{C1'}$, $k^{A1'}$, $k^{AC'}$, and $k^{A2'}$ are defined in the Appendix.

As one can see in Figure 4 there exists a non-empty set of parameters for which Advance Purchase Discounts are optimal in the absence of resale but Clearance Sales are profit maximizing when resale is allowed. The opposite is not the case. Hence when resale is introduced one might observe a switch from increasing to decreasing price schedules but not the other way around. This result might help to understand why Clearance Sales are frequently observed for goods for which resale amongst consumers is possible (e.g. theater tickets) while Advance Purchase Discounts are employed for goods whose resale is costly or difficult to implement (e.g. airline tickets).

Finally let us comment on the assumption that in the resale market all bargaining power resides with the sellers. If instead buyers have some bargaining power then the resale price will be such that $P_R \in (u, 1)$. This will decrease the insurance premium the monopolist is able to charge to early buyers no matter whether price discrimination takes the form of a Clearance Sale or an Advance Purchase Discount. On the other

hand, the reduction in demand from late buyers under an Advance Purchase Discount becomes even stronger. For $P_R < 1$ consumers prefer to buy in the resale market rather than from the monopolist at $P_2^A = 1$. Hence a decrease in the sellers bargaining power would make Clearance Sales even more profitable relative to Advance Purchase Discounts.

7 Rationing

In line with most of the literature on monopolistic pricing we have so far assumed that buyers are rationed randomly. Random rationing is prevalent in many markets and has often been justified by its fairness. Moreover, Gilbert and Klemperer (2000) show that a monopolist may even have an incentive to commit to random rather than efficient rationing in order to make low valuation buyers undertake seller specific investments. Nevertheless, in recent years the development of internet auctions and electronic market places has provided sellers with an easy way to implement more efficient forms of rationing. It is therefore important to understand the influence of the rationing technology on the monopolist's optimal selling strategy. In this section we change assumption (A5) and suppose instead that the monopolist can use an efficient rationing technology which allows him to serve the most eager buyers first. In particular we assume that in period 1 good types are served prior to bad types and in period 2 consumers with high valuations are served prior to consumers with low valuations.⁵

The switch from random to efficient rationing has consequences only for the profitability of Clearance Sales. Since Advance Purchase Discounts serve only good types in period 1 and sell only to buyers with high valuations in period 2, profits are independent of the rationing rule. Similarly, the profitability of any of the strategies that sells exclusively in one of the two periods remains the same. In contrast, in a Clearance Sale buyers with high valuations are served with a larger probability in period 2 than buyers with low valuations when rationing is efficient. The resulting increase in the

⁵A rationing rule under which consumers are served in the opposite order can be motivated by the fact that consumers with low valuations might have smaller costs of time and are therefore more willing to queue. However, such a rule is more appropriate in models where the rationing rule influences the price paid rather than the likelihood to be served. See Sherman and Visscher (1982) for details.

consumers' option value of waiting forces the monopolist to charge a lower first period price and decreases the profitability of a Clearance Sale.

More precisely, under a Clearance Sale efficient rationing implies that the second period capacity $k - g$ is allocated to $(1 - g)(1 - r)$ high valuation buyers prior to the remaining low valuation buyers. If $k - g \geq (1 - g)(1 - r)$, a consumer who chooses not to buy in advance and whose valuation turns out to be high will be served in period 2 at $P_2 = u$ with certainty. Hence in this case rationing risks are absent and Clearance Sales are not implementable. If $k - g < (1 - g)(1 - r)$, the monopolist can induce good types to buy in advance at price $P_1^C = U_G - (1 - ar)(1 - u)(1 - R') > u$ where

$$R' \equiv 1 - \frac{k - g}{(1 - g)(1 - r)} \quad (23)$$

is the rationing probability for a consumer with high valuation under the efficient rationing rule. Note that R' is strictly smaller than the corresponding value under random rationing, $R = \frac{1-k}{1-g}$. As a consequence P_1^C is strictly smaller than in the benchmark case. Note however that in the parameter range in which Clearance Sales are implementable, i.e. for $k - g < (1 - g)(1 - r)$, the monopolist is able to sell his entire capacity even with the help of an Advance Purchase Discount. Since Advance Purchase Discounts implement higher prices, Clearance Sales can therefore never be optimal.

Proposition 6 *When buyers are rationed efficiently rather than randomly, Clearance Sales can never be profit maximizing.*

Since all other selling strategies remain unaffected by the change in the rationing rule, after eliminating Clearance Sales the monopolist's profit maximizing selling strategy is as depicted in Figure 1. Note that the profitability of a Clearance Sale depends negatively on the consumers' probability to become rationed. This implies that any rationing technology which increases the likelihood with which consumers with high valuations are served will decrease the profits of a Clearance Sale. Hence a direct implication of Proposition 6 is that in the parameter range in which Clearance Sales are profit maximizing under a random rationing rule, the monopolist would have an incentive to choose random rationing over any technology that leads to a more efficient

allocation of the monopolist's product. Hence our model provides a justification for the use of random rationing based on the presence of individual demand uncertainty rather than seller specific investments as in Gilbert and Klemperer (2000).

8 Conclusion

In this paper we have considered a monopolist's profit maximizing selling strategy when buyers are heterogeneous and uncertain about their individual demands. Our analysis fills an important gap in the literature on dynamic monopolistic pricing. With a few exceptions, this literature has concentrated exclusively on the effects of either individual demand uncertainty or buyers' heterogeneity. As a consequence authors have either found that it is optimal to sell exclusively before or after demand uncertainty has been resolved or that prices should be decreasing. We have shown that in a model which combines the two effects, none of these predictions survives. In particular, in our model, selling exclusively before or after buyers have learned their demands might be profit maximizing and price discrimination may take the form of a Clearance Sale or an Advance Purchase Discount.

From an applied viewpoint our model is particularly suitable to study the trade-off between increasing and decreasing price schedules. The model's simplicity has allowed us to consider the relative profitability of these two forms of price discrimination in dependence of important market characteristics. We have shown that Clearance Sales (Advance Purchase Discounts) are more (less) likely to be observed in markets where (1) temporal capacity limits are difficult to implement, (2) capacity costs are low but sufficiently convex, (3) prices can be committed to in advance, (4) resale is feasible, and (5) rationing is random rather than efficient.

A further issue concerning advance sales problems is the presence of third parties, so called brokers or middlemen. Brokers stock an inventory in advance and stand ready for buyers to sell close to the consumption date. The existence of brokers will therefore influence the consumers' trade-off between buying early versus buying late and hence the monopolist's optimal selling strategy. A closer look at this issue is left for future research.

Appendix

In this Appendix we include the more technical proofs. All remaining proofs can be found in the main text.

Proof of Lemma 3

$\Pi^C > \Pi^A$ if and only if $gP_1^C + (k - g)u > gU_G + (1 - r)(1 - g) \Leftrightarrow$

$$g(1 - ar)(1 - u)\frac{k - g}{1 - g} < (k - g)u - (1 - r)(1 - g) \quad (24)$$

Both sides are linearly increasing in k and for $k \rightarrow g$ the LHS is strictly larger than the RHS. Hence there exists a $k^{AC} \in (g, 1)$ such that $\Pi^C > \Pi^A \Leftrightarrow k > k^{AC}$ if and only if

$$g(1 - ar)(1 - u) < (1 - g)u - (1 - r)(1 - g) \quad (25)$$

This inequality holds if and only if $g < \frac{u}{1 - a(1 - u)}$ and $r > \frac{1 - u}{1 - g[1 - a(1 - u)]}$. k^{AC} is as defined in (10). ■

Proof of Proposition 5

In the following we derive the monopolist's profit maximizing selling strategy in the presence of resale. Note that the monopolist can obtain the profit $\Pi^E = kP_1^E$ by inducing both types to buy early at a price

$$P_1^E = U_B + rQ^E(P_R - u) \quad (26)$$

where

$$Q^E = \min\left(1, 1 - \frac{k - g(1 - ar) - (1 - g)(1 - r)}{k[gar + (1 - g)r]}\right) \quad (27)$$

is the resale probability when the monopolist sells his entire capacity in period 1.

Step 1: Clearance Sales which induce bad types to buy in period 1 and good types to buy in period 2 cannot be optimal.

For such a strategy to be implementable it has to hold that $k > 1 - g$. The probability of resale is given by $Q = \min\left(1, \frac{(1 - k)(1 - ar)}{(1 - g)r}\right)$. The price that makes bad types indifferent between buying early and buying late is $P_1 = U_B + rQ(1 - u) - (1 - r)(1 - u)(1 - R)$ where $R = \frac{1 - k}{g}$. Good types do not buy at this price if and only if $P_1 > U_G + arQ(1 - u) - (1 - ar)(1 - u)(1 - R)$, which is equivalent to $R < Q$ or $r < \frac{g}{1 - g + ga}$. Profits are $\Pi = (1 - g)P_1 + [k - (1 - g)]u$. For

$k \leq g(1-r) + (1-g)(1-ar)$ we have $\Pi^E = k > \Pi$. Otherwise it holds that $\Pi^E = kP_1^E > \Pi$ if and only if

$$\frac{g(1-k)(1-a)[g-r(1-g+ga)]}{1-g+ga} + (1-r)[k-(1-g)] > 0. \quad (28)$$

Both terms are positive whenever the above Clearance Sale is feasible i.e. when $r < \frac{g}{1-g+ga}$ and $k > 1-g$. Hence for all parameter values the above Clearance Sale is dominated by selling exclusively in period 1 to both types.

Step 2: Selling exclusively in period 2 leads to higher profits than an Advance Purchase Discount if and only if $k \leq g(1-ar) + (1-g)(1-r) \equiv k^{A2'}$. Moreover $k^{A2'} < k^{A2}$.

Selling exclusively in period 2 at $P_2 = 1$ leads the profits $\Pi = \min(k, k^{A2'})$ and cannot be improved upon when $k \leq k^{A2'}$. For $k > k^{A2'}$, Advance Purchase Discounts are implementable and lead the profits $\Pi^A = g[U_G + arQ^A(1-u)] + (k-g)Q^A > k^{A2'}$. It is straightforward to show that $k^{A2'} < k^{A2}$.

Step3: Selling exclusively in period 1 leads to higher profits than a Clearance Sale if and only if $k \geq k^{C1'} \equiv \frac{ga\{1-r[1-g(1-a)]\}}{(1-g)(1-r)[1-g(1-a)]+ga\{1-r[1-g(1-a)]\}}$. Moreover $k^{C1'} < k^{C1}$.

For good types to buy early and bad types to buy late it has to hold that $Q^C = \frac{(1-k)(1-r)}{gar} < R = \frac{1-k}{1-g} \Leftrightarrow r > \frac{1-g}{1-g(1-a)}$. This implies that $Q^E < 1$ and $P_1^E < 1$. A straightforward comparison shows that $\Pi^C = gP_1^C + (k-g)u > \Pi^E = kP_1^E$ if and only if $k < k^{C1'}$. $k^{C1'}$ is strictly increasing in $r \in (0, 1)$ and satisfies $k^{C1'}\left(\frac{1-g}{1-g+ga}\right) = g$ and $k^{C1'}(1) = 1$. It is easy to check that $k^{C1'}(r) < k^{C1}(r) \equiv \frac{g(U_G-u)}{gU_g+(1-g)U_B-u}$ for all $r \in (0, 1)$.

Step 4: Suppose that $k > k^{A2'}$. Selling exclusively in period 1 leads to higher profits than Advance Purchase Discounts if and only if $k > k^{A1'} \equiv \frac{g[u-g(1-a)+gar(1-a)(1-u)]}{u-g(1-a)}$ and $g < \frac{u}{1-a}$. Moreover $k^{A1'} < k^{A1}$.

Since $k > k^{A2'}$, Advance Purchase Discounts can be implemented, i.e. $P_1^A = U_G + arQ^A(1-u) < 1$. Moreover $P_1^E = U_B + rQ^E(1-u) < 1$. Some straightforward algebra yields $\Pi^E = kP_1^E > \Pi^A = gP_1^A + (k-g)Q^A$ if and only if

$$k[u-g(1-a)] > g[u-g(1-a)+gar(1-a)(1-u)].$$

For $g \geq \frac{u}{1-a}$ there exists no $k \in (k^{A2'}, 1)$ that satisfies this inequality so that $\Pi^E < \Pi^A$ for all $k > k^{A2'}$. For $g < \frac{u}{1-a}$ we find $\Pi^E > \Pi^A$ if and only if $k > k^{A1'}$. It is easy to show that $k^{A1'} < k^{A1}$.

Step 5: Suppose that $k > k^{A2'}$. Clearance Sales lead to higher profits for the monopolist than Advance Purchase Discounts if and only if $g < \frac{u}{1-a(1-u)}$, $r > \frac{1-u-u(1-g)}{[1-g(1-a)](1-u)}$, and $k > k^{AC'} \equiv \frac{\{1-r[1-g(1-a)]\}[u-g+gar(1-u)]}{u(1-g)-(1-u)\{1-r[1-g(1-a)]\}}$. Moreover $k^{AC'} < k^{AC}$.

Some algebra shows that $\Pi^C > \Pi^A$ if and only if

$$k(u(1-g) - (1-u)\{1-r[1-g(1-a)]\}) > \{1-r[1-g(1-a)]\}[u-g+gar(1-u)].$$

Suppose that $g \geq \frac{u}{1-a(1-u)}$. Then both sides of the above inequality are negative for all $r \in (0, 1)$. Hence $\Pi^C > \Pi^A \Leftrightarrow k < k^{AC'}$. Since $g < \frac{u}{1-a(1-u)}$ implies that $k^{AC'}(r) < g$ for all r it cannot hold that $\Pi^C > \Pi^A$. Now consider $g < \frac{u}{1-a(1-u)}$. In this case the left hand side of the above inequality is positive if and only if $r > \frac{1-u-u(1-g)}{[1-g(1-a)](1-u)}$ and $\Pi^C > \Pi^A \Leftrightarrow k > k^{AC'}$ where $k^{AC'} > g$. Moreover we have

$$k^{AC'} - k^{AC} = \frac{(1-u)\{1-r[1-g(1-a)]\}[u - (1-r\{1-g[1-a(1-u)]\})]}{u(u(1-g) - (1-u)\{1-r[1-g(1-a)]\})} + garu > 0$$

since the first term is positive for $g < \frac{u}{1-a(1-u)}$.

This completes the characterisation of the monopolist's profit maximizing strategy as depicted in Figure 4 and proves the claim of Proposition 5. ■

References

- [1] Bar-Isaac, H., Caruana, G., Cuñat, V. (2008) 'Information Gathering and Marketing', NYU Working Paper No. 2451/26026.
- [2] Bulow, J. I. (1982) 'Durable Goods Monopolists', *Journal of Political Economy*, vol. 90(2), pp. 314-332.
- [3] Calzolari, G. and Pavan, A. (2006) 'Monopoly with Resale', *RAND Journal of Economics*, vol. 37(2), pp. 362-375.
- [4] Coase, R. H. (1972) 'Durability and Monopoly', *Journal of Law and Economics*, vol. 15, pp. 143-149.
- [5] Conlisk, J., Gerstner, E., Sobel, J. (1984) 'Cyclic Pricing by a Durable Goods Monopolist', *Quarterly Journal of Economics*, vol. 99(3), pp. 489-505.
- [6] Courty, P. and Li, H. (2000) 'Sequential Screening', *Review of Economic Studies*, vol. 67(4), pp. 697-717.

- [7] Courty, P. (2003a) ‘Some Economics of Ticket Resale’, *Journal of Economic Perspectives*, vol. 17(2), pp. 85-97.
- [8] Courty, P. (2003b) ‘Ticket Pricing under Demand Uncertainty’, *Journal of Law and Economics*, vol. 46, pp. 627-652.
- [9] Dana Jr., J. D. (1998) ‘Advance Purchase Discounts and Price Discrimination in Competitive Markets’, *Journal of Political Economy*, vol. 106, pp. 395-422.
- [10] Dana Jr., J. D. (1999) ‘Equilibrium Price Dispersion under Demand Uncertainty: The Roles of Costly Capacity and Market Structure’, *RAND Journal of Economics*, vol. 30(4), pp. 632-660.
- [11] Dana Jr., J. D. (2001) ‘Monopoly Price Dispersion under Demand Uncertainty’, *International Economic Review*, vol. 42, pp. 649-670.
- [12] DeGraba, P. (1995) ‘Buying Frequencies and Seller-Induced Excess Demand’, *RAND Journal of Economics*, vol. 26(2), pp. 331-342.
- [13] Gale, I. L. and Holmes, T. J. (1993) ‘Advance Purchase Discounts and Monopoly Allocation of Capacity’, *American Economic Review*, vol. 83(1), pp. 135-146.
- [14] Giaume, S., and Guillou, S. (2004) ‘Price Discrimination and Concentration in European Airline Markets’, *Journal of Air Transport Management*, vol. 10(5), pp. 305-310.
- [15] Gilbert, R. J. and Klemperer, P. (2000) ‘An Equilibrium Theory of Rationing’, *RAND Journal of Economics*, vol. 31 (1), pp. 1-21.
- [16] Gul, F., Sonnenschein, H., Wilson, R. (1986) ‘Foundations of Dynamic Monopoly and the Coase Conjecture’, *Journal of Economic Theory*, vol. 39(1), pp. 155-190.
- [17] Harris, M. and Raviv, A. (1981) ‘A Theory of Monopoly Pricing Schemes with Demand Uncertainty’, *American Economic Review*, vol. 71(3), pp. 347-365.
- [18] Johnson, J. P. and Myatt, D. P. (2006) ‘On the Simple Economics of Advertising, Marketing, and Product Design’, *American Economic Review*, vol. 96(3), pp. 756-784.
- [19] Lazear, E. (1986) ‘Retail Pricing and Clearance Sales’, *American Economic Review*, vol. 76(1), pp. 14-32.
- [20] Leslie, P. (2004) ‘Price Discrimination in Broadway Theater’, *RAND Journal of Economics*, vol. 35(3), pp. 520-541.
- [21] Lewis, T. R. and Sappington, D. E. M. (1994) ‘Supplying Information to Facilitate Price Discrimination’, *International Economic Review*, vol. 35(2), pp. 309-327.

- [22] Nocke, V. and Peitz, M. (2007) ‘A Theory of Clearance Sales’, *Economic Journal*, vol. 117, pp. 964-990.
- [23] Nocke, V. and Peitz, M. (2008) ‘Advance–Purchase Discounts as a Price Discrimination Device’, CEPR Discussion Paper No. 6664.
- [24] Piga, C. and Bachis, E. (2006) ‘Pricing Strategies by European Low Cost Airlines: Or, When is it the Best Time to Book Online?’, Loughborough University Department of Economics Working Paper WP 2006-14.
- [25] Sherman, R. and Visscher, M. (1982) ‘Nonprice Rationing and Monopoly Price Structures when Demand is Stochastic’, *Bell Journal of Economics*, vol. 13, pp. 254-262.
- [26] Stavins, J. (2001) ‘Price Discrimination in the Airline Market: The Effect of Market Concentration’, *Review of Economics and Statistics*, vol. 83(1), pp. 200-202.
- [27] Stokey, N. L. (1981) ‘Rational Expectations and Durable Goods Pricing’, *Bell Journal of Economics*, vol. 12(1), pp. 112-128.
- [28] Van Cayseele, P. (1991) ‘Consumer Rationing and the Possibility of Intertemporal Price Discrimination’, *European Economic Review*, vol. 35, pp. 1473-1484.
- [29] Zheng, C. (2002) ‘Optimal Auctions with Resale’, *Econometrica*, vol. 70, pp. 2197-2224.