The Timing of Contracting with Externalities[*]

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Abstract

This paper endogenizes the timing of bilateral contracting between one principal and multiple agents in the presence of externalities. Contracting simultaneously with all agents is optimal for the principal if externalities become weaker the more an agent trades. If instead externalities become stronger, sequential negotiations might benefit the principal as they lower the agents' outside options. Under some linearity conditions, the principal's preferences with respect to different timings of contracting are opposed to their efficiency ranking.

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1 Introduction

When more than two parties are involved in bilateral negotiations externalities often exist. In his seminal paper Segal [23] states that "a shareholder tendering his shares to a superior corporate raider has a positive externality on other shareholders (Grossman and Hart [8]), [...] a buyer of a VCR has a positive network externality on owners of compatible VCRs (Katz and Shapiro [15]), [...] a private contributer to a public good has a positive externality on other consumers of the good (Bergstrom, Blume and Varian [2]), [...] a downstream firm purchasing an intermediate input from a manufacturer imposes a negative externality on competing firms (Hart and Tirole [10]; Katz and Shapiro [16])." Further examples include third party effects of bilateral WTO trade agreements (Bagwell and Staiger [1]) and externalities a plaintiff imposes on other plaintiffs through his acceptance of a settlement offer by a potentially insolvent defendant (Spier [27]).

In the presence of externalities a contract signed at an earlier date might change the parties' bargaining positions when contracting at later dates. The *timing* of contracting should therefore be considered as an important variable of multiparty contracting. The simple choice between simultaneous and sequential negotiations, for example, might influence the parties' payoffs. Applications as the ones mentioned above often exhibit significant differences in the timing of contracting. For example, while it is common practice for a corporate raider to make tender offers to many shareholders simultaneously,

supply contracts for intermediate goods are usually negotiated sequentially. In this paper we endogenize the timing of contracting between a single principal and many agents using the setup of Segal [23]. We derive conditions under which the principal prefers sequential to simultaneous negotiations or vice versa and determine the optimal order of sequential contracting. Moreover, we compare the welfare loss caused by different timings of contracting. This allows us to study whether the timing of contracting itself is chosen efficiently. For a practically relevant class of preferences we show that allowing the principal to choose the timing of contracting leads to a welfare loss.

What distinguishes intermediate good markets from corporate takeovers, leading to sequential negotiations for the former and simultaneous negotiations for the latter? This paper identifies a key property of externalities which determines the principal's preferences as well as the relative efficiency of the two timings. Note that the more a firm trades with the supplier of an intermediate good the larger is its share in the downstream market so that the externalities of other firms' purchases become stronger. In contrast, the more a shareholder trades with a corporate raider the smaller becomes his stake in the company so that the externalities of other shareholders' sales become weaker. It turns out that the relative efficiency and the principal's preferences over sequential versus simultaneous contracting depend on whether externalities become stronger or weaker the more an agent trades with the principal.

This paper is the first to endogenize the timing of contracting in a general model of contracting with externalities. Segal's [23] seminal paper is static and focuses on the efficiency of simultaneous contracting. Genicot and Ray [5], Gomes [6], and Segal and Whinston [25] consider dynamic models but restrict attention to binary trade sets. For the case of negative externalities these papers find that (some) sequentiality is profitable for the principal. Gomes [6] shows that contracting simultaneously with all agents is optimal for the principal if externalities are positive. Assuming a special form of payoffs, Bloch and Gomes [4] find similar results in a model of coalitional bargaining under unanimity. This paper provides a broader analysis by allowing for more general trade sets and payoffs. Moreover, with the exception of Segal and Whinston [25] the above papers assume discounting of future utilities, giving the principal an exogenous incentive to contract simultaneously. In this paper discounting is absent and the advantages or disadvantages of simultaneous negotiations originate directly from the existing externalities. Also related are the papers by Jéhiel and Moldovanu ([11], [12]) on the dynamics of the

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sale of an indivisible good in the presence of identity dependent externalities amongst buyers. They share our finding that externalities influence the timing of negotiations. However, whereas in our model the principal chooses the precise timing of contracting, in these papers principal and agents meet randomly and the principal merely decides between selling and waiting. In a more recent paper Jéhiel and Moldovanu [13] allow for the possibility of resale thereby making the identity of the principal itself a dynamic variable of the model. They find the strong result that the final allocation of the good is independent of the identity of the initial owner even though this allocation may not be efficient.

A key insight of our model is that the principal might improve his bargaining position by contracting with some agents and leaving temporarily aside others. This feature is also present in Gomes and Jéhiel's [7] model of dynamic contracting where it allows the extraction of rents from the *excluded* agents through future renegotiation. In contrast, in our model rents are extracted from the *included* agents via the principal's threat to implement certain future trades. Whereas in [7] the contract offering party is determined randomly in every period and renegotiation is possible, in our model the principal's identity is constant over time and renegotiation is ruled out. Interactions between dynamics and bargaining power are therefore a consequence of the implied commitment opportunities of the principal.

The influence of sequentiality on the efficiency of contracting with externalities has been investigated in some applications. In a model of vertical contracting, Marx and Shaffer [17] consider a situation in which simultaneous contracting is efficient but sequential contracting leads to an inefficient supply of intermediate goods. In a model with one borrower and many lenders, Bizer and DeMarzo [3] find that "sequential banking" leads to inefficiently high debt levels whereas "simultaneous banking" is efficient. This paper generalizes these results by considering the relative efficiency of the two timings in situations in which inefficiencies exist for *both*. Bagwell and Staiger [1] consider a model of sequential WTO negotiations between three countries and identify two sources of inefficiency which they call "forward manipulation" and "backward stealing". In this paper we show that it is the size of these two effects which determines the relative efficiency of simultaneous and sequential contracting. Whereas these applied papers take sequentiality as exogenously given, we are the first to consider the question of whether the timing itself is chosen efficiently.

The plan of the paper is as follows. Section 2 depicts the general model

of contracting with externalities. We start by comparing the equilibria of two alternative contracting games with *exogenous timing*; Simultaneous Contracting and Sequential Contracting. We focus on these two games as they are the most natural forms of contracting in the presence of laws and regulations about the timing of contracting. Moreover, they allow us to explain the crucial differences between simultaneity and sequentiality in contracting with externalities. In Section 3 we study a more general contracting game with *endogenous timing*. In every period of this game the principal can choose to contract with any subset of the agents he has not contracted with before. Our main results determine whether contracting simultaneously with all agents is an equilibrium of this game or not. We also endogenize the order of Sequential Contracting in an example with two agents. Section 4 compares the efficiency of the two contracting games introduced in Section 2. Our results allow us to determine conditions under which regulations concerning the timing of contracting improve its efficiency. For example, we identify a class of contracting problems for which Simultaneous Contracting is an equilibrium of the contracting game with endogenous timing but Sequential Contracting leads to a higher level of total surplus. In Section 5 we apply our general results to the theory of vertical contracting. Section 6 concludes. All proofs are contained in the Appendix.

2 Setup

2.1 The model

The paper extends the model of contracting with externalities introduced by Segal [23] by endogenizing the timing of contracting. A single principal offers bilateral contracts to N agents. A contract with agent $i \in \mathcal{N}$ specifies a monetary transfer, $t_i \in \Re$, from the agent to the principal and a "trade", $x_i \in \mathcal{X}_i \subset \Re_+$, taken from a compact set of nonnegative real numbers that contains 0. Contracting is thus restricted in the sense that one agent's contract cannot depend on other agents' contracts, in particular their trades with the principal. This assumption can be motivated by the fact that contingent contracts are seldom observed and difficult to enforce.[1] As in most

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[1]Implications of this assumption are discussed in Section 4. It is common, for example, in models of vertical contracting and corporate takeovers. To name some restrictions, it rules out the use of auctions in vertical contracting as well as conditional bids in corporate

applications, we rule out the possibility of renegotiation by assuming that the principal can contract only once with each agent. If renegotiation was permitted, the principal would have no opportunity to commit. As it is the objective of this paper to compare the commitment opportunities implied by different timings of contracting, ruling out renegotiation is crucial for our analysis.[2]

All bargaining power lies in the hands of the principal; each agent i merely has the choice between accepting a contract or rejecting it, thereby implementing his outside option $(x_i, t_i) = (0, 0)$. Externalities arise from the fact that each agent's utility depends on the full vector of trades $x \equiv (x_1, \ldots, x_N) \in \mathcal{X}_1 \times \ldots \times \mathcal{X}_N \equiv \mathcal{X}$. More specifically, agent i's utility, U_i, and the principal's utility, F, take the following quasi–linear form:

$$U_i = u_i(x) - t_i \quad \text{for all} \quad i \in \mathcal{N} \tag{1}$$
$$F = f(x) + \sum_{i \in \mathcal{N}} t_i. \tag{2}$$

I assume that the functions f and u_i are continuous on $\mathcal{X}$. To abbreviate notation let $u_i^R(x_{-i}) \equiv u_i(0, x_{-i})$ denote agent i's reservation utility and let $b_i(x) = f(x) + u_i(x)$ be the bilateral surplus of the principal and agent i. Total surplus is denoted by $W(x) \equiv f(x) + \sum_{i \in \mathcal{N}} u_i(x)$ and $\mathcal{X}^{eff} \subset \mathcal{X}$ is the set of "efficient trade profiles" maximizing $W(x)$. As W is continuous on the compact set $\mathcal{X}$, $\mathcal{X}^{eff}$ is non–empty. The following properties of the agents' payoffs have been defined in [24]. Note that throughout the paper *increasing* and *decreasing* will be used in the weak sense denoting non–decreasing and non–increasing respectively.

Definition 1 Externalities are positive, negative, or absent at $x' \in \mathcal{X}$ if for all $i \in \mathcal{N}$, $u_i(x'_i, x_{-i})$ is increasing, decreasing, or constant in $x_{-i} \in \mathcal{X}_{-i}$ respectively. Externalities are positive, negative, or absent if they are positive, negative, or absent at all $x \in \mathcal{X}$.

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takeovers.

[2]Ruling out renegotiation is clearly a strong assumption. For example a shareholder who refuses to sell to a corporate raider at an earlier date might be reapproached at a later date. Ruling out renegotiation seems plausible when contracting is very costly and/or time consuming. A recent paper that allows for renegotiation in a model of coalitional bargaining with externalities is Gomes and Jéhiel [7]. Other papers consider intermediate cases in which agents can be reapproached after a rejection but not once they have accepted a contract offer (e.g. Genicot and Ray [5]) or in which coalitions can be extended but not break apart (e.g. Gomes [6]).

Definition 2 Externalities are (strictly) increasing [decreasing] if for each agent $i \in \mathcal{N}$, $u_i(x_i, x_{-i})$ has (strictly) increasing differences in (x_i, x_{-i}) $[(-x_i, x_{-i})]$ in the sense of Topkis [28], i.e., if for all $x_i, x_i' \in \mathcal{X}_i$ with $x_i' > x_i$, $u_i(x_i', x_{-i}) - u_i(x_i, x_{-i})$ is (strictly) increasing [decreasing] in $x_{-i} \in \mathcal{X}_{-i}$.

The N bilateral contracting problems are also linked by the principal's payoff as it might imply complementarities or substitutabilities in bilateral surplus.

Definition 3 Trades are (strict) complements [substitutes] if for each agent $i \in \mathcal{N}$, $b_i(x_i, x_{-i})$ has (strictly) increasing differences in (x_i, x_{-i}) $[(-x_i, x_{-i})]$.

Note that when externalities are (strictly) increasing [decreasing] then a sufficient but not necessary condition for trades to be (strict) complements [substitutes] is that f has increasing differences in (x_i, x_{-i}) $[(-x_i, x_{-i})]$ for all $i \in \mathcal{N}$.

Information is perfect. More specifically, agents observe the contracts offered to other agents and their decisions whether to accept or reject these contracts.

2.2 Equilibria

Before we endogenize the timing of contracting in Section 3, this section characterizes the equilibria of two extreme cases of contracting games with exogenously given timings; Simultaneous Contracting and Sequential Contracting.[3] Comparing these equilibria allows us to highlight the crucial differences between simultaneous and sequential negotiations. Simultaneous Contracting refers to the contracting game considered by Segal [23]. The principal first offers *each* agent a contract and agents then decide simultaneously and non-cooperatively whether to accept or reject their offers. Sequential Contracting consists of N periods. The order of contracting is fixed exogenously.[4] In each period $i \in \mathcal{N}$ the principal offers a contract to agent i and agent i subsequently accepts or rejects. Note that Simultaneous and Sequential Contracting are extreme in the sense that the former (latter) maximizes (minimizes) the principal's commitment power. When contracting simultaneously the principal can commit to the entire trade profile x whereas under Sequential Contracting he only commits to individual trades x_i. All other

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[3]Note that we use capitals to distinguish these games with exogenous timing from the particular outcomes of the game with endogenous choice of timing in Section 3.

[4]Section 3 endogenizes the order of Sequential Contracting for the case of two agents.

timings which combine simultaneous and sequential negotiations offer intermediate commitment opportunities. We now consider the subgame perfect equilibria of these two important timings.

Suppose that in an equilibrium the principal's strategy specifies a contract offer (x_i, t_i) which is rejected by agent i's strategy. If instead the principal offers the null contract $(0, 0)$ and agent i, being indifferent between accepting and rejecting, accepts this offer, the outcome of the game would be the same. For any subgame perfect strategy profile one can thus find another subgame perfect strategy profile which implements identical equilibrium trades and utilities and in which every contract offer is accepted. One can therefore, without loss of generality, restrict attention to equilibria in which the principal only offers contracts which are accepted.

2.2.1 Simultaneous Contracting

The following derivation is due to Segal [23]. Suppose that contracts offered by the principal specify the trade vector $x \in \mathcal{X}$ and all agents accept. All accept is a Nash equilibrium of the agents' simultaneous move game if and only if for every agent $i \in \mathcal{N}$ it holds that

$$u_i(x_i, x_{-i}) - t_i \geq u_i^R(x_{-i}). \tag{3}$$

The principal's optimal payment offers make these participation constraints binding and substituting them into the principal's utility gives

$$F^{sim}(x) = W(x) - \sum_{i \in \mathcal{N}} u_i^R(x_{-i}). \tag{4}$$

The principal's problem of finding the optimal contracts is reduced to the problem of choosing a trade vector in $\mathcal{X}^{sim} \equiv \arg\max_{x \in \mathcal{X}} F^{sim}(x)$ giving him the payoff $F^{sim} \equiv \max_{x \in \mathcal{X}} F^{sim}(x)$. Note that $\mathcal{X}^{sim} \neq \emptyset$ as $F^{sim}(x)$ is a continuous function on the compact set $\mathcal{X}$.

Segal [24] shows that the principal can implement the above equilibrium as the unique equilibrium of the agents' simultaneous move game if externalities are decreasing. In the case of increasing externalities the agents' simultaneous move game might have multiple equilibria as each agent is less eager to accept his offer if he expects other agents to reject. Here we assume that the principal can coordinate the agents on his preferred equilibrium

thereby obtaining the payoff F^{sim}. This assumption is made in most applications.[5] We will see that it is restrictive only in Propositions 1 and 4 for the case of increasing externalities.

2.2.2 Sequential Contracting

For every agent $i \in \mathcal{N}$, a strategy is a function $\alpha_i : \prod_{j=1}^{i-1}(\mathcal{X}_j \times \Re \times \{0,1\}) \times (\mathcal{X}_i \times \Re) \to \{0,1\}$, mapping contracting histories into actions $a_i \in \{0,1\}$, where rejection and acceptance are denoted by $a_i = 0$ and $a_i = 1$ respectively. A strategy of the principal is a vector $((\chi_1, \tau_1), \ldots, (\chi_N, \tau_N))$ of functions mapping contracting histories into trade and payment offers, that is $\chi_1 : \{0\} \to \mathcal{X}_1$, $\tau_1 : \mathcal{X}_1 \to \Re$ and $\chi_i : \prod_{j=1}^{i-1}(\mathcal{X}_j \times \Re \times \{0,1\}) \to \mathcal{X}_i$, $\tau_i : \prod_{j=1}^{i-1}(\mathcal{X}_j \times \Re \times \{0,1\}) \times \mathcal{X}_i \to \Re$ for $i = 2, \ldots, N$. Let a strategy profile of the principal and the N agents be denoted by σ. For every history of contracting up to period $i \in \mathcal{N}$, $h^i \equiv (x_1, t_1, a_1, \ldots, x_i, t_i, a_i)$ let $h_x^i \equiv (a_1 x_1, \ldots, a_i x_i)$ denote the corresponding profile of implemented trades. Finally let $x^\sigma(h_x^i)$ be the vector of trades which will be implemented by σ in periods $i+1, \ldots, N$ after the trades h_x^i have been realized in periods $1, \ldots, i$.

Suppose that σ^* is a subgame perfect equilibrium of Sequential Contracting and consider period i after some contracting history h^{i-1}. It is optimal for agent i to accept his offer, $\alpha_i^*(h^{i-1}, x_i, t_i) = 1$, if and only if

$$u_i(h_x^{i-1}, x_i, x^{\sigma^*}(h_x^{i-1}, x_i)) - t_i \geq u_i^R(h_x^{i-1}, x^{\sigma^*}(h_x^{i-1}, 0)). \tag{5}$$

The principal optimally chooses the transfer which makes (5) binding:

$$\tau_i^*(h^{i-1}, x_i) = u_i(h_x^{i-1}, x_i, x^{\sigma^*}(h_x^{i-1}, x_i)) - u_i^R(h_x^{i-1}, x^{\sigma^*}(h_x^{i-1}, 0)). \tag{6}$$

As in period i the principal has already received the payments $t_1, \ldots, t_{i-1}$ his objective is $f(h_x^{i-1}, x_i, x^{\sigma^*}(h_x^{i-1}, x_i)) + \sum_{j=i}^{N} t_j$. Substituting the optimal transfers, we find that the principal's optimal trade offer to agent i satisfies

$$\chi_i^*(h^{i-1}) \in \arg\max_{x_i \in \mathcal{X}_i} F_{\sigma^*}^i(h^{i-1}, x_i) \tag{7}$$

where

$$F_{\sigma^*}^i(h^{i-1}, x_i) = f(h_x^{i-1}, x_i, x^{\sigma^*}(h_x^{i-1}, x_i)) + \sum_{j=i}^{N} u_j(h_x^{i-1}, x_i, x^{\sigma^*}(h_x^{i-1}, x_i)) \tag{8}$$

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[5]Exceptions are Katz and Shapiro [15] in the setting of Network Externalities, Grossman and Hart [8] in Takeovers, and Segal and Whinston [25] in Exclusive Dealing. These papers assume that agents coordinate on *their* preferred equilibrium.

$$-\sum_{j=i}^{N} u_j^R(h_x^{i-1}, x_i, \ldots, x_{j-1}^{\sigma^*}(h_x^{i-1}, x_i), x^{\sigma^*}(h_x^{i-1}, x_i, \ldots, x_{j-1}^{\sigma^*}(h_x^{i-1}, x_i), 0)).$$

It follows from the one deviation property of subgame perfect equilibria that σ^* is an equilibrium if and only if conditions (5), (6) and (7) hold. An equilibrium of Sequential Contracting exists as action sets are compact and payoffs are continuous in histories (see Harris [9]).

2.2.3 Comparison

If for all $i \in \mathcal{N}$ and all $x_i, x_i' \in \mathcal{X}_i$, the first differences $u_i(x_i', x_{-i}) - u_i(x_i, x_{-i})$ and $f(x_i', x_{-i}) - f(x_i, x_{-i})$ are independent of $x_{-i} \in \mathcal{X}_{-i}$ then the timing of contracting plays no role. Under these conditions, the principal's optimal contract with one agent is independent of the other agents' contracts. In most applications, however, externalities are increasing or decreasing and trades are complements or substitutes. In general the N contracting problems are therefore interdependent and the timing of contracting is important.

In order to understand what distinguishes Sequential Contracting from Simultaneous Contracting it is useful to consider the principal's maximization program for the special case of two agents. For Simultaneous Contracting (4) implies:

$$\max_{x \in \mathcal{X}} \quad W(x_1, x_2) - u_1^R(x_2) - u_2^R(x_1). \tag{9}$$

Given any first period trade $x_1 \in \mathcal{X}_1$, in the second period of Sequential Contracting the principal solves

$$\max_{x_2 \in \mathcal{X}_2} f(x_1, x_2) + u_2(x_1, x_2) - u_2^R(x_1). \tag{10}$$

Letting $x_2^0 \in \arg\max_{x_2 \in \mathcal{X}_2} f(0, x_2) + u_2(0, x_2)$ denote an optimal trade offer to agent 2 in case of a rejection of agent 1, the maximization problem of Sequential Contracting becomes

$$\max_{x \in \mathcal{X}} \quad W(x_1, x_2) - u_1^R(x_2^0) - u_2^R(x_1) \tag{11}$$

$$\text{s. t. } x_2 \in \arg\max_{x_2' \in \mathcal{X}_2} f(x_1, x_2') + u_2(x_1, x_2').$$

Sequential Contracting and Simultaneous Contracting differ in the principal's ability to commit. Comparing (11) with (9) shows that the principal's inability to commit under Sequential Contracting has two consequences. First, the

principal's choice of $x \in \mathcal{X}$ is constrained by the fact that for any x_1 he has to select a trade x_2 that is optimal *after t_1 has been paid*. The principal fails to internalize the externality of his trade with the second agent on the first agent. Second, the objective function of Sequential Contracting contains the term $u_1^R(x_2^0)$ instead of $u_1^R(x_2)$. When contracting is sequential the principal can react to a rejection of agent 1 by implementing a different trade with agent 2 than in the case of an acceptance. In the next section we show that this flexibility might benefit the principal by lowering earlier agents' outside options.

3   Endogenous timing

In many applications, e.g. intermediate good markets, laws and regulations concerning the timing of contracting are absent and the timing is determined by the principal. It is therefore important to endogenize the timing of contracting. In this section we first consider a situation in which the principal's choice of timing is completely unrestricted. We derive conditions under which contracting will be simultaneous and show that for certain types of externalities, the principal profits from sequentiality. We then consider a situation in which the timing is restricted to be sequential but the order of contracting is determined by the principal. We determine the dependence of the principal's optimal order of Sequential Contracting on the agents' characteristics.

In order to endogenize the timing of contracting for the case of more than two agents, we have to specify whether or not the principal can commit to a particular timing. To see this, suppose that the principal offers a contract to some agent before contracting with the remaining ones. The trades with the remaining agents and thus the agent's decision whether to accept or reject his offer generally depend on the timing of contracting. In the absence of laws and regulations it is natural to assume that the principal cannot commit to a particular timing of contracting. As a consequence, when offering contracts to a subset of agents, the principal chooses the timing of contracting with the remaining agents which maximizes his utility.

In this section we therefore consider the following *contracting game with endogenous timing*. There are at most N contracting periods. In every contracting period the principal offers a contract to at least one of the agents who have not received a contract offer before and agents, who have been offered a contract, non–cooperatively and simultaneously decide whether to

accept or reject their offers. The game ends after every agent has accepted or rejected exactly one contract. We consider the subgame perfect equilibria of this game.

In Gomes [6] and Genicot and Ray [5] simultaneous contracting results from the principal's incentive to save time. Proposition 1 shows that simultaneous negotiations might be the result of strategic rather than temporal considerations. Even in the absence of discounting simultaneous negotiations emerge from the principal's incentive to minimize the agents' outside options.

Proposition 1 *Contracting simultaneously with all agents is an equilibrium of the contracting game with endogenous timing if either externalities are absent at $x = 0$ or if externalities are negative (positive) at $x = 0$ and increasing (decreasing), trades are strict complements (substitutes), and for all $i, j, k \in \mathcal{N}$ such that $i \neq j \neq k \neq i$ and all $x, x', y, y' \in \mathcal{X}$ such that $x_j = y_j > y'_j = x'_j$, $x_k = x'_k > y_k = y'_k$ and $x_l = y_l = y'_l = x'_l$ for all $l \in \mathcal{N} - \{j, k\}$, the second difference $[u_i(x) - u_i(y)] - [u_i(x') - u_i(y')]$ is increasing (decreasing) in $x_i \in \mathcal{X}_i$.*

Proposition 1 has a simple intuition.[6] Consider the possibility that the principal contracts with a subset $\mathcal{M} \subset \mathcal{N}$ of agents leaving temporarily aside agents in $\mathcal{N} - \mathcal{M}$. By induction we can assume that contracting simultaneously with all agents in $\mathcal{N} - \mathcal{M}$ is an equilibrium of the remainder of the game. It is easy to see that the principal cannot benefit from this sequential procedure as he could implement the same contracts as a Nash equilibrium of the simultaneous acceptance game amongst *all* agents in $\mathcal{N}$. This holds trivially when externalities at the agents' outside option are absent because no matter the timing, the principal has to guarantee each agent a payoff which is independent of the trade profile implemented. When externalities are increasing (decreasing) and trades are strict complements (substitutes) then under the assumption about the second differences, the principal's optimal trade with any agent in $\mathcal{N} - \mathcal{M}$ is increasing (decreasing) in the trades of agents in $\mathcal{M}$. It follows that under sequentiality the rejection of an agent in $\mathcal{M}$ decreases (increases) the trades of agents in $\mathcal{N} - \mathcal{M}$. When externalities at the agents' outside option are negative (positive) the agents' willingness to accept must therefore be stronger in the absence of sequentiality.

To build some intuition for the condition about the second differences consider the case of differentiable payoffs where it is equivalent to the require-

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[6]I thank one of the anonymous referees for pointing out this intuition.

ment that the cross derivatives $\frac{\partial^2 u_i(x)}{\partial x_j \partial x_k}$ are increasing (decreasing) in x_i. The influence of a rejection by agent $j \in \mathcal{M}$ on the trade with agent $k \in \mathcal{N} - \mathcal{M}$ depends directly on the sign of $\frac{\partial^2 b_k(x)}{\partial x_j \partial x_k} + \sum_{i \in \mathcal{N} - \mathcal{M} - \{k\}} \frac{\partial^2 u_i(x)}{\partial x_j \partial x_k} - \frac{\partial^2 u_i^R(x_{-i})}{\partial x_j \partial x_k}$ (and indirectly on its effect on trades with the other agents in $\mathcal{N} - \mathcal{M}$). The first term describes the effect on the marginal bilateral surplus between the principal and agent k. The sum represents the effect on the marginal surplus that the principal is able to extract from the remaining agents in $\mathcal{N} - \mathcal{M}$ via his trade with agent k. Our condition guarantees that both effects go in the same direction. That is, a rejection of agent j decreases (increases) the marginal bilateral surplus with agent k *and* the marginal surplus that can be extracted from the remaining agents in $\mathcal{N} - \mathcal{M}$. If the condition fails, the principal might have an incentive to increase (decrease) his trade with agent k in response to a rejection of agent j in order to extract more surplus from the remaining agents in $\mathcal{N} - \mathcal{M}$.

Note that for $N = 2$ the condition about the second differences is vacuous. For $N > 2$ it holds trivially if the agents' utility functions are quadratic polynomials. To give some examples for its limitations, in corporate takeovers with a superior raider, where the company's value V is an increasing function of the raider's aggregate purchase, it requires that $V'' \geq 0$ and $V''' \leq 0$. For public good provisions it holds as long as a consumer's marginal benefit from his own contribution is a decreasing and (weakly) concave function of the aggregate supply.

In the case of negative and increasing externalities Proposition 1 might fail to hold when under simultaneous contracting the principal is unable to coordinate the agents on his preferred equilibrium. To see this consider an example with two identical agents and binary trade and suppose the principal wishes to implement full trade. If contracting is simultaneous and the principal can coordinate the agents then he offers a price $P = -t_i > 0$ that makes each agent just willing to trade when he expects the other to do so. If he cannot coordinate the agents he has to offer one of them a price P' that makes him willing to trade when he expects the other not to. As externalities are increasing, $P' \geq P$. When contracting is sequential, the principal can guarantee trade with the last agent by offering P. The price that makes the first agent willing to trade depends on the principal's preferences. If he prefers no trade at all to trade with a single agent then he has to pay P' to guarantee trade with the first agent and Proposition 1 remains valid. Otherwise P is enough to guarantee trade with the first

agent and sequential contracting dominates simultaneous contracting when the principal is unable to coordinate. Sequentiality then serves as a substitute for the principal's lack of coordination. This observation has been made by Segal and Whinston [25] who show that an incumbent firm might improve its ability to exclude entry from a rival by offering exclusionary contracts to buyers sequentially rather than simultaneously.

Consider again the case where externalities are increasing (decreasing) and trades are strict complements (substitutes) but contrary to before suppose that externalities at the agents' outside option are positive (negative). Our next result shows that the principal profits from sequentiality if externalities at $x = 0$ are sufficiently strong.

Proposition 2 *Suppose that for all $i \in \mathcal{N}$, $\mathcal{X}_i = [0, \bar{x}_i]$ and f and u_i are twice continuously differentiable. Furthermore, suppose that for some $j \in \mathcal{N}$, $b_j(x)$ and $F^{sim}(x)$ are strictly concave in $x_j \in \mathcal{X}_j$ for any $x_{-j} \in \mathcal{X}_{-j}$. If externalities are increasing (decreasing) and positive (negative) at $x = 0$ with $\min_{x_{-i} \in \mathcal{X}_{-i}} |\frac{\partial u_i^R(x_{-i})}{\partial x_j}| > \delta_{ij}$ for all $i \in \mathcal{N} - j$ and trades are strict complements (substitutes) then contracting simultaneously with all agents cannot be an equilibrium of the contracting game with endogenous timing.*

The intuition for this result is as follows. Sequentiality offers flexibility to the principal as it allows him to make contract offers to later agents contingent on the acceptance of earlier agents. The principal can use this flexibility as a threat to punish a rejection by earlier agents. In particular he can threaten to decrease (increase) trades with later agents thereby decreasing earlier agents' reservation utilities when externalities at $x = 0$ are positive (negative). If externalities are increasing (decreasing) and trades are strict complements (substitutes) this threat is credible as the principal's optimal trade with later agents is increasing (decreasing) in earlier agents trades. The principal therefore gains from sequentiality due to its effect on the agents' outside option. However, for the principal to choose sequential offers, this gain has to more than compensate for his loss in commitment power which lowers earlier agents' willingness to accept. Sequentiality thus becomes optimal when the above threat is sufficiently powerful which is the case when externalities at the agents' outside option are sufficiently strong. The thresholds $\delta_{ij} > 0$ depend on the particular shape of the utility functions and are characterized in the Appendix. Note that the principal benefits from sequentiality through the extraction of additional rents from earlier agents. Sequentiality is detrimental for earlier agents.

Our assumption that the principal is able to coordinate agents is not necessary for Proposition 2. As F^{sim} represents an upper bound on the principal's payoff, the result holds for *all* equilibria of simultaneous contracting. Moreover, as our assumption about the principal's inability to commit to a particular timing restricts his choice, Proposition 2 also remains valid in the absence of this assumption. However, Proposition 1 might fail to hold. To see this consider the case of three agents. Suppose for example that when there are only agents 2 and 3 remaining, contracting simultaneously is optimal for the principal but leads to a higher aggregate trade than contracting sequentially. If externalities are negative a commitment to contract sequentially with agents 2 and 3 raises the rent the principal can extract from agent 1. The value of such commitment might be large enough to destroy the principal's incentive to contract simultaneously with all three agents.

Proposition 2 shows that there exist situations in which the principal's endogenous choice of timing involves sequential offers. Under the conditions of Proposition 2 it is not optimal for the principal to contract simultaneously with all agents. Empirical evidence for the sequentiality of negotiations stems from input sales in intermediate good markets as well as the adaptation of new technologies in the presence of network externalities.

For the remainder of this section we assume that contracting is restricted to be sequential but the principal is free to choose the order of contracting. For heterogeneous agents different orders of Sequential Contracting might lead to different outcomes. We derive the principal's optimal order of Sequential Contracting for the case of two agents under some restrictions on the form of payoffs. Let $F^{1,2}$ and $F^{2,1}$ denote the principal's maximized utility when contracting first with agent 1 or 2 respectively. We study the dependence of the principal's optimal order of contracting on the agents' characteristics. Imposing several symmetry requirements, we focus on differences in the externalities at the agents' outside option. Without loss of generality we assume that agent 1's reservation utility depends more strongly on agent 2's trade than vice versa.

Proposition 3 *Consider Sequential Contracting with $N = 2$ and $f(x_1, x_2) = f(x_2, x_1)$ for all x_1, $x_2 \in \mathcal{X}_1 = \mathcal{X}_2$. Furthermore, for $i, j \in \{1, 2\}$ such that $i \neq j$ suppose that $u_i(x_i, x_j) = v(x_i) + [e_i^R + e(x_i)]w(x_j)$ for all $x \in \mathcal{X}$ with w strictly increasing, $w(0) = e(0) = 0$, and $|e_1^R| > |e_2^R|$. If trades are strict complements (substitutes) it holds that $F^{1,2} \geq F^{2,1}$ if externalities at $x = 0$ are positive (negative) and $F^{2,1} \geq F^{1,2}$ if they are negative (positive).*

Proposition 3 demonstrates that the principal's optimal order of Sequential Contracting depends on the strength of externalities on non–traders. It assumes that an agent's utility can be additively separated into a function $v(.)$, only depending on his own trade and an externality part $w(.)$ depending on his opponent's trade. Externalities at agent i's outside option are represented by the parameter e_i^R. They are positive, negative, or absent if e_i^R is positive, negative, or zero respectively.

In order to understand the intuition of Proposition 3 suppose that externalities at the agents' outside option are positive (negative) and trades are strict complements (substitutes) so that the principal can credibly threaten to decrease (increase) the second agent's trade in case of a rejection by the first agent. In the absence of any other differences across agents the principal optimally contracts first with agent 1 who is most susceptible to such a punishment as his outside option depends more strongly on the other agent's trade. If externalities at the agents' outside option are negative (positive) and trades are strict complements (substitutes) no such credible threat is available to the principal. In this case the dependence of the last agent's trade on the first agent's decision even *raises* the first agent's outside option. In order to minimize this effect the principal contracts first with agent 2. In fact, the proof of Proposition 1 for the case of two agents shows that in this case it is optimal for the principal to contract with both agents simultaneously.

4   Efficiency

This section considers the efficiency of the principal's choice of timing. For this purpose we first derive the welfare ranking of the two contracting games with exogenous timing introduced in Section 2.2 and then use our results from Section 3 to determine whether the principal would implement the most efficient of the two in the contracting game with endogenous timing. If this is the case then there is no need for regulation. Otherwise, restricting the timing of contracting would improve its efficiency.

In the presence of externalities contracting generally leads to inefficiencies. These inefficiencies can be attributed to the fact that contracts are restricted in the sense that transfer and trade of one agent cannot be contingent on other agents' trades. Allowing transfers alone to be contingent on other agents' trades eliminates the backward stealing effect described below, but does no suffice to restore efficiency. Only fully general contracts which

replicate multilateral contracts by making one agent's transfer *and* trade dependent on other agents' trades lead to efficiency. They allow the principal to implement a surplus maximizing trade profile and at the same time to minimize the agents' outside options. Note that this is an example of the celebrated Coase Theorem.

Different timings of contracting implement different levels of welfare. To see this, note from (4) that under Simultaneous Contracting the principal has an incentive to minimize the agents' outside options and that Simultaneous Contracting is efficient if externalities are absent at $x = 0$. The principal's objective in period $j \in \mathcal{N}$ of Sequential Contracting can be written as

$$F^j(x_j, \ldots, x_N) = W(x) - \sum_{i=j}^{N} u_i^R(x_{-i}) - \sum_{i=1}^{j-1} u_i(x). \tag{12}$$

As under Simultaneous Contracting the principal has an incentive to minimize the outside options of agents $j, \ldots, N$. Bagwell and Staiger [1] call this effect "forward manipulation". However, the absence of externalities at $x = 0$ is not a sufficient condition for the efficiency of Sequential Contracting. As represented by the second sum in (12), the principal fails to internalize the externalities of his trades with agents $j, \ldots, N$ on agents $1, \ldots, j-1$. This is the effect that Bagwell and Staiger call "backward stealing". It follows that Sequential Contracting is efficient if externalities are absent at $x = 0$ *and* at some $x^{eff} \in \mathcal{X}^{eff}$.

Although in (12) the principal fails to internalize externalities on agents $1, \ldots, j-1$ at positive trade levels, he has no incentive to minimize these agents' outside option. One can therefore expect that in comparison to Simultaneous Contracting, the efficiency of Sequential Contracting becomes more distorted by externalities on traders but less distorted by externalities on non–traders. The relative efficiency of Simultaneous and Sequential Contracting might therefore depend on the relative strength of these two types of externalities. If externalities are positive (negative) and increasing (decreasing), externalities at positive trade levels are stronger than externalities at zero trade. The above argument then implies that distortions to the efficient trade are stronger for Sequential than for Simultaneous Contracting. If, on the other hand, externalities are positive (negative) and decreasing (increasing), then externalities on non–traders are stronger than externalities on traders. In this case, distortions are stronger for Simultaneous Contracting. Proposition 4 states conditions under which the above intuition is correct. It

assumes that total surplus is a function of aggregate trade, $X = \sum_{i \in \mathcal{N}} x_i$, and that the strength of externalities on agent i only depends on his own trade x_i. The latter condition guarantees that we can compare the externalities at his outside option under Simultaneous Contracting, $(0, x_{-i}^{sim})$, with the externalities at his equilibrium trade under Sequential Contracting, $(x_i^{seq}, x_{-i}^{seq})$, as in the above argument. In applications this condition is often implied by the linearity of the externality generating function. For example in vertical contracting the condition holds if demand is linear and in corporate takeovers the linearity of the company's value function is sufficient.

Proposition 4 *Suppose that for all $i \in \mathcal{N}$, $\mathcal{X}_i = [0, \bar{x}_i]$ with $\bar{x}_i$ sufficiently large, f and u_i are differentiable, and for all $j \in \mathcal{N}$ such that $j \neq i$, $\frac{\partial u_i(x)}{\partial x_j}$ is independent of $x_{-i} \in \mathcal{X}_{-i}$. Furthermore, suppose that $W(x) = W(X)$ for all $x \in \mathcal{X}$ and that $W(X)$ is strictly concave in X. If externalities are increasing and positive (negative) or decreasing and negative (positive), then welfare under Simultaneous Contracting is at least (most) as high as under Sequential Contracting.*

Whereas in the case of increasing externalities Proposition 4 holds for all orders of Sequential Contracting, the case of decreasing externalities requires that the principal contracts last with an agent whose trade under Simultaneous Contracting is positive. Obviously, this is no restriction in the case of homogeneous agents where all orders of contracting are equivalent. For the case of increasing externalities Proposition 4 hinges on the principal's ability to coordinate agents. When externalities are positive (negative), Segal [24] shows that coordination lowers (raises) the efficiency of Simultaneous Contracting if the agents' payoff can be written in the form $u_i(x) = P(x) + g_i(x_{-i})$. It follows that under these conditions Proposition 4 might fail to hold if the principal cannot coordinate the agents.

We can now consider the efficiency of the principal's choice of timing. It is easy to see that the principal's and welfare preferences are aligned when externalities on non–traders are absent. In this case Sequential Contracting would generally be inefficient but according to Proposition 1 the principal chooses to contract simultaneously with all agents thereby implementing an efficient trade profile. We now apply Propositions 1, 2 and 4 to two practically relevant examples to show that this is not always the case. In many applications the principal's utility f consists of certain production or transaction costs so that $f(x) = -C(X)$ is a strictly concave function of aggregate trade. When

agents are buyers their utility often takes the form $u_i(x) = (x_i^E + x_i)u(X)$, where $x_i^E \geq 0$ denotes the buyer's endowment before trade. When agents are sellers it can often be written as $u_i(x) = (\bar{x}_i - x_i)u(X)$. For the special case where $u(X)$ is linear, the following corollary shows that in these examples the principal's and welfare preferences are opposed.[7]

Corollary 1 *Suppose that $f(x) = f(X)$ and f is twice continuously differentiable and strictly concave in X. For all $i \in \mathcal{N}$, let $\mathcal{X}_i = [0, \bar{x}_i]$ with $\bar{x}_i$ sufficiently large.*

1. *If for all $i \in \mathcal{N}$, $u_i(x) = (\bar{x}_i - x_i)u(X)$ and u is linearly increasing, then the principal chooses to contract simultaneously although Sequential Contracting is more efficient.*

2. *If for all $i \in \mathcal{N}$, $u_i(x) = (x_i^E + x_i)u(X)$ with $x_i^E \geq 0$, u is linearly decreasing, and for some $i \in \mathcal{N}$, x_i^E is sufficiently large, then the principal does not choose to contract simultaneously although it is more efficient than Sequential Contracting.*

The intuition for this result hinges on the interplay between commitment power and flexibility. As we have argued before, sequentiality decreases the principal's commitment power but increases his flexibility to react to rejections. Under the conditions of the first part of Corollary 1 it is efficient to restrict the principal's commitment opportunities as this weakens his incentive to minimize agents' outside options where externalities are strongest. However, although sequentiality increases total surplus, it decreases the principal's ability to extract rents from earlier agents. This is because the principal's loss in commitment power lowers these agents' willingness to accept but his gain in flexibility cannot raise their reluctance to reject due to the absence of a credible punishment. Under the conditions of the second part of Corollary 1 such a credible punishment exists. In particular, the principal can credibly threaten to increase his trades with later agents in response to a rejection of earlier agents. If this threat is sufficiently strong, which is the case when endowments x_i^E are sufficiently large, the principal prefers

[7]The principal's and welfare preferences are also opposed in Segal and Whinston's [25] model of exclusive dealing. However, this is due to their assumption that the principal cannot coordinate agents onto his preferred equilibrium when contracting simultaneously. As a consequence the principal profits from sequentiality even when it harms overall efficiency. Our result shows that the principal might profit from sequentiality even when it harms efficiency *and* the principal is able to coordinate agents.

the flexibility of sequential negotiations in spite of the reduction in overall surplus due to his loss of commitment power.

Corollary 1 shows that regulations concerning the timing of contracting might be welfare improving. For example, under the conditions of the first part, welfare would be higher in the presence of a law demanding the sequentiality of negotiations than in the absence of such a law.

In the example of vertical contracting $f(x) = -C(X)$ is the upstream supplier's cost of production and, $u(X) = P(X) - c$ is the difference between the price and the downstream firms' constant marginal cost. In corporate takeovers $f(x) = XV(X) - C(X)$ and $u(X) = V(X)$ where $V(X)$ denotes the company's value when X shares are sold to the corporate raider and $C(X)$ are his transaction costs. Regarding the provision of a public good, $f(x) = -C(X)$ is the provider's cost of production and $u(X) = B(X)$ is the buyers' benefit. The assumptions of Corollary 1 hold in these examples when $P(.)$, $V(.)$ and $B(.)$ are linear. The following section discusses the implications of our results for the example of vertical contracting. Applications to takeovers, public goods, and network externalities can be found in an earlier version of this paper contained in [21].

5   Application to vertical contracting

Suppose the principal is a monopolistic supplier of an intermediate good and the agents are firms which buy the good as an input and compete in a common downstream market. A contract specifies the quantity of the input, x_i, to be sold to firm i and a payment, t_i, from the firm to the seller. Let $x_i^E \geq 0$ denote firm i's endowment of input before trade. Suppose that the supplier's cost of production is $C(X)$ so that his payoff is $F(x) = -C(X) + \sum_{i \in \mathcal{N}} t_i$. Let the demand in the downstream market be characterized by a strictly decreasing inverse demand function $P(Y)$, where Y denotes the aggregate output of the final good. Each downstream firm i has a one–to–one–technology of transforming the intermediate good into final output, y_i at a total cost, $c_i(y_i)$. Firm i's payoff is $U_i(x) = (x_i^E + x_i)P(X^E + X) - c_i(x_i^E + x_i) - t_i$.[8] Externalities are negative. Total surplus is $W(x) = (X^E + X)P(X^E + X) - C(X) - \sum_{i=1}^N c_i(x_i^E + x_i)$.[9] Similar models have been considered by Hart and Tirole

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[8] These profits result for example when downstream firms are capacity constrained by $x_i^E + x_i$ and compete in prices.

[9] Note that our efficiency considerations neglect consumers of the final product.

[10], Kamien, Oren and Tauman [14], Katz and Shapiro [16], Marx and Shaffer [17], McAfee and Schwartz [19], and O'Brian and Shaffer [22].

Consider first the case in which $x_i^E = 0$ for every downstream firm i so that externalities on non–traders are absent. For example, firms might be potential entrants which do not participate in the downstream market if they reject the upstream firm's supply offer. We have seen that in this case Simultaneous Contracting would be efficient whereas Sequential Contracting might lead to inefficiency. The potential inefficiency of sequential vertical contracting has been noted by Marx and Shaffer [17]. However, Marx and Shaffer fail to endogenize the timing of contracting. Proposition 1 shows that the supplier has no incentive to contract sequentially, if externalities at the firms' outside option are absent.

For the remainder of this section suppose that downstream firms have positive endowments, $x_i^E > 0$, identical and constant marginal costs, demand is linear and the supplier's cost function is strictly convex. In this case externalities are present at the firms' outside option and strictly decreasing and trades are strict substitutes. Contracting will generally be inefficient, whether it is done sequentially or simultaneously. Proposition 4 implies that the total profits of the vertical structure are higher under Simultaneous Contracting than under Sequential Contracting. Proposition 4 therefore extends the result of Marx and Shaffer [17] to cases where Simultaneous Contracting might be inefficient. Furthermore, under the above conditions Proposition 2 implies that contracting with all firms simultaneously cannot be optimal for the supplier if their endowments are large enough. Letting the supplier choose the timing of contracting then indeed leads to sequentiality which might be detrimental for efficiency.

Finally, consider the case of a duopoly in the downstream market and suppose that one of the two downstream firms is larger than the other in the sense that it has a larger endowment x_i^E. Under the above conditions the downstream firms' profits take the parameterized form of Proposition 3 with $e_i^R = x_i^E$. Proposition 3 implies that the supplier's profit is maximized by selling first to the larger downstream firm. In this example it is the downstream firms' market share that determines the optimal order of contracting for the upstream supplier.

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6　Conclusion

This paper has endogenized the timing of contracting in a general model of contracting with externalities. We have shown that under some mild assumptions the principal chooses to contract simultaneously with all agents if externalities become weaker the more an agent trades with the principal. If on the other hand externalities become stronger then the principal profits from sequential negotiations if externalities on non–traders are sufficiently strong. Unfortunately welfare comparisons are less clear cut. Here our results require that welfare is a function of aggregate trade and that agents' payoffs are linear in other agents' trades. These assumptions are quite restrictive, but they allow us to derive implications for some practically relevant cases, e.g. vertical contracting or corporate takeovers. Although generally efficient, in these examples the principal's choice of timing might lead to inefficiencies. Giving the principal the right to choose the timing might therefore be detrimental for the efficiency of multiparty contracting.

　　To conclude, this paper has highlighted the importance of the timing of bilateral negotiations for the theory of multiparty contracting with externalities. The wide variety of applications of the general model of multiparty contracting suggest that the results of this paper have implications beyond the examples we have discussed.

Appendix

Proof of Proposition 1

The claim follows by induction over the number of agents N. For $N = 2$ and $i \in \mathcal{N}$ let (x_i^*, t_i^*) be equilibrium contracts when the principal contracts sequentially. If he offers the same contracts simultaneously only agent 1's participation constraint changes. He accepts if $u_1(x_1^*, x_2^*) - t_1^* \geq u_1(0, x_2^*)$ whereas sequentially, acceptance required $u_1(x_1^*, x_2^*) - t_1^* \geq u_1(0, x_2^0)$. Subgame perfection implies that $x_2^* \in \mathcal{X}_2^*(x_1^*)$ and $x_2^0 \in \mathcal{X}_2^*(0)$ where $\mathcal{X}_2^*(x_1) \equiv \arg\max_{x_2 \in \mathcal{X}_2} b_2(x_1, x_2)$. As trades are strict complements (substitutes), b_2 satisfies strictly increasing (decreasing) differences in (x_1, x_2). From the Monotone Selection Theorem (see [20]) it follows that $x_2^0 \leq x_2^*$ $(x_2^0 \geq x_2^*)$. As externalities are negative (positive) at $x = 0$, $u_1(0, x_2^0) \geq u_1(0, x_2^*)$. Hence the principal can implement (x_i^*, t_i^*) as a Nash equilibrium of the agents' simultaneous acceptance game. Note that this holds trivially when externalities are absent at $x = 0$. Hence contracting simultaneously with both agents has to be

an equilibrium of the contracting game with endogenous timing and two agents.

For $N > 2$ suppose that for all $N' < N$ contracting simultaneously with all agents is an equilibrium of the contracting game with endogenous timing with N' agents. If the principal contracts first simultaneously with a subset $\mathcal{M} \subset \mathcal{N}$ of agents then by the above assumption contracting simultaneously with the remaining $N - M$ agents is an equilibrium of the remainder of the game. The principal's objective when contracting simultaneously with these agents is $\tilde{F}(x) \equiv f(x) + \sum_{i \in \mathcal{N}-\mathcal{M}} u_i(x) - u_i^R(x_{-i})$. Our assumption about the second cross differences of u_i implies that $\tilde{F}(x)$ satisfies the conditions of the Monotone Selection Theorem. To see this note that for all $x, x', y, y' \in \mathcal{X}$ such that $x_j = y_j > y'_j = x'_j$ and $x_k = x'_k > y_k = y'_k$ for some $j, k \in \mathcal{N} - \mathcal{M}$ such that $j \neq k$ and $x_i = x'_i = y_i = y'_i$ for all $i \in \mathcal{N} - \{j, k\}$, it holds that $\tilde{F}(x) - \tilde{F}(x') - (\tilde{F}(y) - \tilde{F}(y')) = b_k(x) - b_k(x') - (b_k(y) - b_k(y')) + u_j(x) - u_j(x') - (u_j(y) - u_j(y')) + \sum_{i \in \mathcal{N}-\mathcal{M}-\{j,k\}} u_i(x) - u_i(x') - (u_i(y) - u_i(y')) - (u_i^R(x_{-i}) - u_i^R(x'_{-i}) - (u_i^R(y_{-i}) - u_i^R(y'_{-i})))$. The term under the sum is positive (negative) due to our assumption about the second differences. As trades are complements (substitutes) and externalities are increasing (decreasing) it therefore follows that $\tilde{F}(x)$ is supermodular (submodular) in $(x_i)_{i \in \mathcal{N}-\mathcal{M}}$ for each $(x_i)_{i \in \mathcal{M}}$. Furthermore, for all $x, x', y, y' \in \mathcal{X}$ such that $x_j = y_j > y'_j = x'_j$ and $x_k = x'_k > y_k = y'_k$ for some $j \in \mathcal{M}$ and $k \in \mathcal{N} - \mathcal{M}$ and $x_i = x'_i = y_i = y'_i$ for all $i \in \mathcal{N} - \{j, k\}$, it holds that $\tilde{F}(x) - \tilde{F}(x') - (\tilde{F}(y) - \tilde{F}(y')) = b_k(x) - b_k(x') - (b_k(y) - b_k(y')) + \sum_{i \in \mathcal{N}-\mathcal{M}-\{k\}} u_i(x) - u_i(x') - (u_i(y) - u_i(y')) - (u_i^R(x_{-i}) - u_i^R(x'_{-i}) - (u_i^R(y_{-i}) - u_i^R(y'_{-i})))$. Our assumption about the second differences guarantees that the term under the sum is positive (negative). As trades are strict complements (substitutes), $\tilde{F}(x)$ therefore satisfies strictly increasing (decreasing) differences in $((x_i)_{i \in \mathcal{M}}, (x_i)_{i \in \mathcal{N}-\mathcal{M}})$. As for the case $N = 2$ it thus follows that for every M such that $1 \leq M < N$ the principal cannot gain from separating M agents from the remaining one(s). ∎

Proof of Proposition 2

This proof assumes that all equilibrium trades are interior so that we can use first order conditions. We focus on the case of positive and increasing externalities. The proof for the case of negative and decreasing externalities is analogous. Let x^{sim} be an equilibrium trade when the principal contracts simultaneously with all agents and let F^{sim} denote the principal's corresponding utility. For each $i \in \mathcal{N}$ let $x^i = (x_1^{sim}, \ldots, x_{i-1}^{sim}, 0, x_{i+1}^{sim}, \ldots, x_N^{sim})$. Consider the following alternative timing of contracting. In a first period the principal contracts simultaneously with agents in $\mathcal{N} - j$. In a second period he then contracts with agent j. Let x^* be an equilibrium trade of this particular timing and denote by F^* the principal's corresponding utility. For any $x_{-j} \in \mathcal{X}_{-j}$, denote by $x_j^*(x_{-j}) = \arg \max_{x_j \in \mathcal{X}_j} b_j(x)$ the subgame

perfect equilibrium trade with agent j. $x_j^*(.)$ is a continuous function as b_j is strictly concave on the convex set $[0, \bar{x}_j]$. As trades are strict complements the Monotone Selection Theorem implies that $x_j^*(x_{-j}^{sim}) \geq x_j^*(x_{-j}^i)$. Furthermore $x_j^{sim} \geq x_j^*(x_{-j}^{sim})$ as $x_j^{sim} \in \arg\max_{x_j \in \mathcal{X}_j} b_j(x_{-j}^{sim}, x_j) + \sum_{i \neq j}[u_i(x_{-j}^{sim}, x_j) - u_i(x_{-j}^i, x_j)]$ and the term under the sum is increasing in x_j due to increasing externalities. We first derive a lower bound on $x_j^*(x_{-j}^{sim}) - x_j^*(x_{-j}^i)$. Starting with the first order condition for $x_j^*(x_{-j}^{sim})$ one gets

$$
\begin{aligned}
0 &= \frac{\partial b_j(x_{-j}^{sim}, x_j^*(x_{-j}^{sim}))}{\partial x_j} \\
&= \int_{x_j^*(x_{-j}^i)}^{x_j^*(x_{-j}^{sim})} \frac{\partial^2 b_j(x_{-j}^{sim}, t)}{\partial x_j^2} dt + \int_0^{x_i^{sim}} \frac{\partial^2 b_j(x_{-j-i}^{sim}, t, x_j^*(x_{-j}^i))}{\partial x_i \partial x_j} dt \\
&\geq [x_j^*(x_{-j}^{sim}) - x_j^*(x_{-j}^i)] \min_{x \in \mathcal{X}} \frac{\partial^2 b_j(x)}{\partial x_j^2} + x_i^{sim} \min_{x \in \mathcal{X}} \frac{\partial^2 b_j(x)}{\partial x_i \partial x_j}.
\end{aligned}
\tag{13}
$$

The last equality follows from the first order condition for $x_j^*(x_{-j}^i)$. Thus

$$
x_j^*(x_{-j}^{sim}) - x_j^*(x_{-j}^i) \geq -x_i^{sim} \frac{\min_{x \in \mathcal{X}} \frac{\partial^2 b_j(x)}{\partial x_i \partial x_j}}{\min_{x \in \mathcal{X}} \frac{\partial^2 b_j(x)}{\partial x_j^2}} > 0.
\tag{14}
$$

Next we derive an upper bound on $x_j^{sim} - x_j^*(x_{-j}^{sim})$. Starting with the first order condition for $x_j^*(x_{-j}^{sim})$ one gets

$$
\begin{aligned}
0 &= \frac{\partial W(x_{-j}^{sim}, x_j^*(x_{-j}^{sim}))}{\partial x_j} - \sum_{i \neq j} \frac{\partial u_i(x_{-j}^{sim}, x_j^*(x_{-j}^{sim}))}{\partial x_j} \\
&= \frac{\partial W(x^{sim})}{\partial x_j} - \int_{x_j^*(x_{-j}^{sim})}^{x_j^{sim}} \frac{\partial^2 W(x_{-j}^{sim}, t)}{\partial x_j^2} dt - \sum_{i \neq j} \frac{\partial u_i^R(x_{-i}^{sim})}{\partial x_j} \\
&\quad + \sum_{i \neq j} \int_{x_j^*(x_{-j}^{sim})}^{x_j^{sim}} \frac{\partial^2 u_i(x_{-j}^i, t)}{\partial x_j^2} dt - \sum_{i \neq j} \int_0^{x_i^{sim}} \frac{\partial^2 u_i(t, x_{-j-i}^{sim}, x_j^*(x_{-j}^{sim}))}{\partial x_i \partial x_j} dt \\
&= -\int_{x_j^*(x_{-j}^{sim})}^{x_j^{sim}} \frac{\partial^2 F^{sim}(x_{-j}^{sim}, t)}{\partial x_j^2} dt - \sum_{i \neq j} \int_0^{x_i^{sim}} \frac{\partial^2 u_i(t, x_{-j-i}^{sim}, x_j^*(x_{-j}^{sim}))}{\partial x_i \partial x_j} dt \\
&\geq -[x_j^{sim} - x_j^*(x_{-j}^{sim})] \max_{x \in \mathcal{X}} \frac{\partial^2 F^{sim}(x)}{\partial x_j^2} - \sum_{i \neq j} x_i^{sim} \max_{x \in \mathcal{X}} \frac{\partial^2 u_i(x)}{\partial x_i \partial x_j}.
\end{aligned}
\tag{15}
$$

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The last equality follows from the first order condition for x_j^{sim}. Thus

$$[x_j^{sim} - x_j^*(x_{-j}^{sim})] \leq -\frac{\sum_{i \neq j} x_i^{sim} \max_{x \in \mathcal{X}} \frac{\partial^2 u_i(x)}{\partial x_i \partial x_j}}{\max_{x \in \mathcal{X}} \frac{\partial^2 F^{sim}(x)}{\partial x_j^2}}. \qquad (16)$$

Finally we derive a sufficient condition for $F^* > F^{sim}$. From the optimality of x_{-j}^* it follows that

$$F^* \geq W(x_{-j}^{sim}, x_j^*(x_{-j}^{sim})) - \sum_{i \neq j} u_i^R(x_{-i-j}^i, x_j^*(x_{-j}^i)) - u_j^R(x_{-j}^{sim}). \qquad (17)$$

The optimality of $x_j^*(x_{-j}^{sim})$ implies

$$W(x_{-j}^{sim}, x_j^*(x_{-j}^{sim})) - \sum_{i \neq j} u_i(x_{-j}^{sim}, x_j^*(x_{-j}^{sim})) \geq W(x^{sim}) - \sum_{i \neq j} u_i(x^{sim}). \qquad (18)$$

It follows that $F^* > F^{sim}$ if

$$\sum_{i \neq j} [u_i(x^{sim}) - u_i(x_{-j}^{sim}, x_j^*(x_{-j}^{sim}))] < \sum_{i \neq j} [u_i^R(x_{-i}^{sim}) - u_i^R(x_{-i-j}^{sim}, x_j^*(x_{-j}^i))]. \qquad (19)$$

Using differentiability we find that $F^* > F^{sim}$ if for all $i \neq j$:

$$\int_{x_j^*(x_{-j}^{sim})}^{x_j^{sim}} \frac{\partial u_i(x_{-j}^{sim}, t)}{\partial x_j} dt - \int_{x_j^*(x_{-j}^i)}^{x_j^{sim}} \frac{\partial u_i^R(x_{-i}^{sim}, t)}{\partial x_j} dt < 0 \qquad (20)$$

which is equivalent to

$$\int_{x_j^*(x_{-j}^{sim})}^{x_j^{sim}} \frac{\partial u_i(x_{-j}^{sim}, t)}{\partial x_j} - \frac{\partial u_i(x_{-j}^i, t)}{\partial x_j} dt < \int_{x_j^*(x_{-j}^i)}^{x_j^*(x_{-j}^{sim})} \frac{\partial u_i^R(x_{-i-j}^i, t)}{\partial x_j} dt. \qquad (21)$$

A sufficient condition for (21) to hold is $[x_j^{sim} - x_j^*(x_{-j}^{sim})] x_i^{sim} \max_{x \in \mathcal{X}} \frac{\partial^2 u_i(x)}{\partial x_i \partial x_j} < [x_j^*(x_{-j}^{sim}) - x_j^*(x_{-j}^i)] \min_{x_{-i} \in \mathcal{X}_{-i}} \frac{\partial u_i^R(x_{-i})}{\partial x_j}$. Inserting the bounds derived above and using the fact that trades are bounded we finally find that $F^* > F^{sim}$ if for all $i \neq j$, $\min_{x_{-i} \in \mathcal{X}_{-i}} \frac{\partial u_i^R(x_{-i})}{\partial x_j} > \delta_{ij}$ where

$$\delta_{ij} = \frac{\min_{x \in \mathcal{X}} \frac{\partial^2 b_j(x)}{\partial x_j^2}}{\max_{x \in \mathcal{X}} \frac{\partial^2 F^{sim}(x)}{\partial x_j^2}} \frac{\max_{x \in \mathcal{X}} \frac{\partial^2 u_i(x)}{\partial x_i \partial x_j}}{\min_{x \in \mathcal{X}} \frac{\partial^2 b_j(x)}{\partial x_i \partial x_j}} \sum_{k \neq j} \bar{x}_k \max_{x \in \mathcal{X}} \frac{\partial^2 u_k(x)}{\partial x_k \partial x_j} > 0. \qquad (22)$$

Thus contracting simultaneously with all agents cannot be an equilibrium of the contracting game with endogenous timing if externalities at the agents' outside options are sufficiently positive. ∎

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Proof of Proposition 3

For $i, j \in \{1, 2\}$ such that $i \neq j$, the set of optimal second period trades when contracting in order (i, j) is:

$$\mathcal{X}_j^{(i,j)}(x_i) = \arg\max_{x_j \in \mathcal{X}_j} f(x_i, x_j) + v(x_j) + w(x_i)e(x_j). \tag{23}$$

The symmetry of the principal's preferences implies $\mathcal{X}_j^{(i,j)}(.) = \mathcal{X}_i^{(j,i)}(.)$. Let $l(.)$ denote any selection from $\mathcal{X}_j^{(i,j)}(.)$. If the principal offers a first period trade $x \in \mathcal{X}_i$ when contracting is in order (i, j) his payoff is

$$\begin{aligned} F^{i,j}(x) = \;& f(x, l(x)) + v(x) + v(l(x)) + w(x)(e_j^R + e(l(x))) \\ & + w(l(x))(e_i^R + e(x)) - 2v(0) - w(x)e_j^R - w(l(0))e_i^R. \end{aligned} \tag{24}$$

By the symmetry of f we find that for any first period trade $x \in \mathcal{X}_1 = \mathcal{X}_2$:

$$F^{1,2}(x) - F^{2,1}(x) = (e_1^R - e_2^R)[w(l(x)) - w(l(0))]. \tag{25}$$

The claim follows from the fact that $l(x) \geq l(0)$ ($l(x) \leq l(0)$) if trades are strict complements (substitutes). ∎

Proof of Proposition 4

As $W(x) = W(X)$ for all $x \in \mathcal{X}$ one can rewrite the maximization of Simultaneous Contracting using the "aggregation method" of Milgrom and Shannon [20]:

$$\max_{X \in [0, \bar{X}]} \left(W(X) - \min_{x \in \mathcal{X}} \left\{ \sum_{i \in \mathcal{N}} u_i^R(x_{-i}) \text{ s. t. } \sum_{i \in \mathcal{N}} x_i = X \right\} \right). \tag{26}$$

As $\frac{\partial u_i}{\partial x_j}$ is independent of $x_{-i} \in \mathcal{X}_{-i}$ we can define $u_{ij}(x_i) \equiv \frac{\partial u_i(x_i)}{\partial x_j}$. The sum of the agents' reservation utilities is minimized by trading only with agents in $\mathcal{N}^{sim} = \arg\min_{j \in \mathcal{N}} \sum_{i \neq j} u_{ij}(0)$. To see this let $j \in \mathcal{N}^{sim}$ and $k \notin \mathcal{N}^{sim}$. If $x_k = \epsilon > 0$ then the principal can lower k's and raise j's trade by ϵ. This leaves X and thus $W(X)$ unchanged but reduces the second term in (26) by $(\sum_{i \neq k} u_{ik}(0) - \sum_{i \neq j} u_{ij}(0))\epsilon > 0$. If the agents' maximum feasible trade levels are sufficiently large then their equilibrium trade x^{sim} therefore satisfies $x_i^{sim} = 0$ for all $i \notin \mathcal{N}^{sim}$ and X^{sim} solves

$$\frac{\partial W(X^{sim})}{\partial X} = \min_{j \in N} \sum_{i \neq j} u_{ij}(0). \tag{27}$$

Let x^{seq} be an equilibrium trade of Sequential Contracting. Consider first the case of increasing externalities. I will show that $X^{seq} \leq X^{sim}$. Let agent n be the last agent whose equilibrium trade is positive. If for all $m > n$, $x_m^{seq} = 0$ is a strict corner solution (first order condition slack) to the principal's problem in period m then the following first order condition holds in period n:

$$\frac{\partial W(X^{seq})}{\partial X} = \frac{\partial W(\sum_{i=1}^{n-1} x_i^{seq} + x_n^{seq})}{\partial x_n} \geq \sum_{i=1}^{n-1} u_{in}(x_i^{seq}) + \sum_{i=n+1}^{N} u_{in}(0). \quad (28)$$

Otherwise (28) holds with equality for the last period $n' > n$ in which the first order condition is binding at $x_{n'}^{seq} = 0$. As externalities are increasing, $\frac{\partial W(X^{seq})}{\partial X} \geq \frac{\partial W(X^{sim})}{\partial X}$ and the strict concavity of W implies $X^{seq} \leq X^{sim}$. If externalities are positive both timings implement inefficiently low aggregate trades but Simultaneous Contracting is more efficient. If externalities are negative it is easy to see that $X^{eff} \leq X^{seq}$ so that both timings implement inefficiently high aggregate trade but Sequential Contracting is more efficient.

Next we consider decreasing externalities and show that $X^{seq} \geq X^{sim}$ if the order of Sequential Contracting is such that $N \in \mathcal{N}^{sim}$. If $\sum_{i=1}^{N-1} x_i^{seq} < X^{sim}$ then

$$\frac{\partial b_N(x_{-N}^{seq}, x_N)}{\partial x_N} = \frac{\partial W(x_{-N}^{seq}, x_N)}{\partial x_N} - \sum_{i=1}^{N-1} u_{iN}(x_i^{seq}) > \sum_{i=1}^{N-1} u_{iN}(0) - u_{iN}(x_i^{seq}) \quad (29)$$

for all $x_N < X^{sim} - \sum_{i=1}^{N-1} x_i^{seq}$. The inequality uses the strict concavity of W and the fact that $N \in \mathcal{N}^{sim}$. As externalities are decreasing it follows that $x_N^{seq} \geq X^{sim} - \sum_{i=1}^{N-1} x_i^{seq}$. If externalities are negative then both timings lead to inefficiently high aggregate trades but Simultaneous Contracting is more efficient. For positive externalities one easily finds $X^{seq} \leq X^{eff}$ so that both timings implement inefficiently low aggregate trade but Sequential Contracting is more efficient. ∎

Proof of Corollary 1

Consider $u_i(x) = (\tilde{x}_i \pm x_i)u(X)$. For all $i, j, k \in \mathcal{N}$ such that $i \neq j$ and $i \neq k$ and all $x \in \mathcal{X}$, $\frac{\partial u_i(x)}{\partial x_j} = (\tilde{x}_i \pm x_i)u'$, $\frac{\partial^2 u_i(x)}{\partial x_j \partial x_i} = \pm u'$ and $\frac{\partial^2 u_i(x)}{\partial x_j \partial x_k} = 0$. Furthermore, for all $i \in \mathcal{N}$ and all $x \in \mathcal{X}$, $\frac{\partial^2 b_i(x)}{\partial x_i^2} = \frac{\partial^2 F^{sim}(x)}{\partial x_i^2} = \frac{\partial^2 W(X)}{\partial X^2} = f'' \pm 2u'$. The claim then follows directly from Propositions 1, 2 and 4. ∎

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