Advance Selling in the Wake of Entry

Nadia Ceschi* Marc Möller†

Abstract

This article provides a tractable model of inter-temporal price-discrimination by heterogeneous firms, imperative for our understanding of advance purchase markets in the wake of entry. The pricing schedule of an industry leader, whose product is more likely to match consumers' preferences, is found to differ systematically from the pricing schedule of a newcomer. By diverting competition to a stage where consumers face uncertainty about their preferences, advance selling reduces prices while increasing the newcomer's market share and profitability relative to the industry leader. Policies that curtail the firms' ability to sell in advance, although potentially beneficial for welfare, may have the adverse effects of consolidating an industry leader's position and of reducing the consumers' surplus.

JEL *classification*: D43, D80, L13.
Keywords: Competition, Price Discrimination, Individual Demand Uncertainty, Advance Purchase Discounts.

*Department of Economics, University of Bern, Schanzeneckstrasse 1, 3001 Bern, Switzerland. E-mail: nadia.ceschi@vwi.unibe.ch. Declarations of interest: None.

†Corresponding author. Department of Economics, University of Bern, Schanzeneckstrasse 1, 3001 Bern, Switzerland. Tel.: +41-31-6314510. E-mail: marc.moeller@vwi.unibe.ch. Declarations of interest: None.

1 Introduction

In markets characterized by individual demand uncertainty, firm entry is frequently accompanied by a surge in advance selling. Examples include the aggressive use of advance purchase discounts by low cost airlines, non-refundable hotel reservations becoming proliferated with the emergence of online booking platforms, and introductory offers being employed in the marketing of new technologies or services. Systematic differences in the dynamic pricing policies of new-coming and established firms seem to persist even for considerable time spans past entry.

To obtain a clearer picture about the nature of dynamic pricing in advance purchase markets, Table 1 presents data on train-ticket fees from northern Italy. The common

	Turin-Milan		Padua-Bologna		Milan-Bologna	
ΔT	Newcomer	Leader	Newcomer	Leader	Newcomer	Leader
21	11.66 (61%)	19.68 (47%)	10.62 (43%)	16.45 (37%)	27.08 (36%)	31.45 (34%)
14	12.43 (59%)	20.24 (45%)	10.59 (43%)	17.01 (35%)	27.80 (34%)	32.34 (33%)
7	15.25 (49%)	21.11 (43%)	10.55 (43%)	17.64 (33%)	28.36 (33%)	32.69 (32%)
0	30.15 (NA)	36.59 (NA)	19.03 (NA)	26.38 (NA)	42.24 (NA)	47.89 (NA)

Table 1: Price (Euro) and advance purchase discount (%) in dependence of the time to departure, ΔT (days). Depicted are means taken over all Thursday connections from November 2019 to February 2020 obtained from www.thetrainline.com. Roughly 70% of the connections are offered by the industry leader. The newcomer participates in the market since 2012.

feature of the selected routes is that they are serviced both by an established, formerly monopolistic, industry leader and a more recent newcomer. As can be seen from the table, prices are lower for the newcomer, independently of the lag between time of purchase and time of travel. Moreover, advance purchase discounts are larger for the newcomer than for the industry leader.[1] While lower prices could possibly derive from a cost advantage of the

[1]This finding is in line with the empirical evidence provided by Asplund et al. (2008) showing that

newcomer, which might be due to an improved technology (e.g. electronic- rather than over-the-counter-booking), a rationale for the systematic differences in advance purchase discounts is missing.

The objective of this article is to provide a stylized model of oligopolistic competition in advance purchase markets capable of explaining these stylized facts and to use the model to predict the consequences of a ban on inter-temporal price-discrimination. Shedding light on the desirability of advance purchase pricing is especially relevant in light of several recent policy interventions that have curtailed firms' ability to sell in advance. For instance, Spain has accompanied the opening of its railroad market for private operators by a restriction of its advance booking horizon. A similar policy has been introduced in Israel, where airlines must refund all tickets purchased within 14 days, ruling out the effective use of advance purchase discounts during the last two weeks before the date of travel. We will argue that such policies may have adverse effects not only for consumers but especially for firms that have recently entered the market.

Our starting point is the observation that new-coming and established firms typically differ in their *prominence* amongst consumers. Net of prices, a larger share of consumers prefer the established firm's product over the newcomer's. For example, given its greater variety of connections, a larger number of travelers may prefer a departure time offered by the leading railroad. However, while consumers may be aware of such a preference at the time of consumption (the day of travel), at the advance purchase stage, their preferences may be subject to uncertainty. In particular, a consumer may judge the industry leader's product as more likely to become his most preferred option, but he cannot discard the possibility that he may turn out to prefer the newcomer's product. In the presence of individual demand uncertainty, differences in prominence thus loom larger at the consumption stage and new-coming firms may have an incentive to divert competition to the advance purchase stage, with the help of a discount.

small newspapers are more prone to offer discounts to new readers than their larger competitors.

Based on the above intuition, in Section 2, we introduce our model of a duopolistic advance purchase market, featuring a prominent industry leader and a newcomer. Consumers have unitary demands and can make their purchase during two periods; an advance purchase period and a consumption period. In the advance purchase period, consumers are uncertain about the identity of their preferred product. All they know is that, on aggregate, a larger fraction of consumers will turn out to prefer the industry leader's product. The identity of their preferred product becomes revealed to consumers (privately) at the start of the consumption period. Consumers differ in their level of choosiness, which constitutes their private information. More choosy consumers weigh the difference in the products' characteristics more heavily, making choosiness a key determinant of a consumer's timing of purchase. We assume that firms can commit to a price schedule, consisting of an advance purchase price and a regular price. To permit a more nuanced welfare analysis our model allows for differences in the firms' constant marginal cost of production. It is important to note, however, that our main results are driven by the firms' difference in prominence, and remain valid when cost-differences are absent.

In accordance with the stylized facts outlined above, and in line with the aforementioned intuition, our theory explains why a new-coming firm will charge lower prices and offer larger discounts than a prominent industry leader. More importantly, via comparison with the uniform pricing benchmark, our model sheds light on the effects of inter-temporal price discrimination on market performance. Price discrimination is often under scrutiny when a market is dominated by a prominent firm. One of the worries is that price-discrimination may serve incumbents as an anti-competition instrument to prevent the rise of rival firms. We show that, contrary to this view, advance purchase pricing allows new-coming firms to *increase* their market share and their profitability relative to an industry leader. The reason is that advance selling allows newcomers to move competition to a stage where an industry leader's advantage in consumer prominence weighs less

heavily. Moreover, as products appear more homogeneous at the advance purchase stage, competition is intensified and prices are reduced, leading to an increase in consumer surplus. Hence, in markets subject to individual demand uncertainty, inter-temporal price discrimination turns out to be doubly beneficial: It benefits consumers by lowering prices and helps new-coming firms to close the gap to established industry leaders.

From a welfare perspective, advance selling has been shown to be detrimental for a market with homogeneous firms (Möller and Watanabe, 2016). In contrast to uniform pricing, advance selling induces sub-optimal purchases and the corresponding mismatch between consumer preferences and product characteristics leads to a loss in surplus. However, for a market with heterogeneous firms, as advance selling increases the market share of newcomers, advance selling could be welfare improving if newcomers have a cost-advantage over established firms. We conclude our analysis by showing that even in the presence of such welfare gains from a more efficient supply, the overall effect of inter-temporal price-discrimination on welfare is negative. This means that, from a policy perspective, the benefits of advance selling that accrue to new-coming firms and consumers must be confronted by the associated losses in terms of the market's overall efficiency.

Related literature

This article contributes to the growing literature on advance purchase markets and intertemporal price-discrimination. While the early literature has focused on perfectly competitive industries (e.g. Dana Jr., 1998) and monopoly (e.g. Gale and Holmes, 1993; Nocke and Peitz, 2007; Möller and Watanabe, 2010; Nocke et al., 2011), more recent contributions have turned their attention to the more relevant analysis of oligopolistic competition (e.g. Gale, 1993; Möller and Watanabe, 2016; Karle and Möller, 2020). While a common assumption of the existing work is that firms are homogeneous, we add to this literature the first tractable model of oligopolistic competition between *heterogeneous firms*.

Besides our contribution to the advance purchase literature, this article adds to the

discussion of the effects of price-discrimination on oligopolistic markets more generally. While there exists a vast literature on the effects of price-discrimination under monopoly (see Varian, 1989, for a survey), the corresponding literature on oligopoly is less extensive and mostly restricted to settings where consumers can be discriminated according to some observable characteristic.[2]

Thisse and Vives' (1988) seminal work on spatial competition has shown that price-discrimination emerges as a characteristic feature of oligopolistic markets. Their setting is a Hotelling model where firms can charge different prices to consumers located at different locations. Our model can be interpreted as a Hotelling model where consumers learn their locations over time and firms can vary their price across periods. While in our setting, price-discrimination is inter-temporal rather than spatial and second-degree rather than third-degree, we share Thisse and Vives' (1988) finding that price-discrimination harms firms and benefits consumers.

Holmes (1989) provides the important insight that, in oligopolistic markets, welfare predictions are complicated by the fact that the influence of price discrimination on total output depends on the cross-price elasticities of demand. In our setting, consumers have unit demands and parameters are chosen such that, in equilibrium, all consumers participate in the market. This allows us to abstract from output effects and concentrate our welfare analysis on the allocational distortions that arise from differences in the firm's cost of supply and the potential mismatch between consumer preferences and product characteristics. Armstrong and Vickers (2001) show that, in the framework of Holmes (1989), in spite of welfare-predictions being ambiguous, third-degree price discrimination has a positive effect on profits and a negative effect on consumer surplus. This finding is diametrically opposed to the results we obtain for our setting with individual demand uncertainty and inter-temporal, second-degree price-discrimination, which demonstrates

[2] A notable exception is the seminal article by Armstrong and Vickers (2001), who provide a framework for modeling oligopolistic competition in "utility space" that can be used to address price-discrimination with unobserved consumer heterogeneity.

that the desirability of price-discrimination may depend crucially not only on the market's characteristics but also on *what kind of* price-discrimination is considered.

Corts (1998) uses a model of third-degree price discrimination in a vertically differentiated market to argue that when one firm's strong market segment represents the weak market segment of its rival, then prices can be lower than under uniform pricing in *both* segments. He concludes, however, that "conditions on demand that generate [such all–out competition] remain elusive." Our theory identifies individual demand uncertainty as a natural cause of all-out-competition and shows that Corts' (1998) insights extend to settings of second-degree price-discrimination where consumers may choose (contingent on the firms' pricing) to which market segment they belong.

Finally, more loosely related is the literature on price-discrimination based on purchase history. Villas-Boas (1999) and Fudenberg and Tirole (2000) show that when firms can "poach" each others' customers with targeted discounts, equilibrium prices are lower than when prices cannot condition on purchase history. Chen (2008) and Carroni (2016) introduce firm asymmetries into similar settings and find that customer poaching may become one-directional, i.e. from the weak to the strong firm. This contrast with our result that advance selling allows a new-coming firm to extend its market share at the expense of the industry leader and might be explained by the fact that with history dependent pricing consumer are poached ex post whereas with advance purchase discounts consumer are poached ex ante, i.e. before their preferences become revealed.

2 Model

We consider a market in which two firms, an industry leader L and a newcomer N, sell two horizontally differentiated products $i \in \{L, N\}$. Consumers can make a purchase in two periods; an advance purchase period and the consumption period. Both firms are active in the market in both periods, i.e. we consider a situation in which the newcomer

has already joined the market.

There exists a unit mass of consumers with unit demand who differ in their privately known choosiness $\sigma \in [0,1]$. A consumer of type σ obtains utility $s + \frac{\sigma}{2}$ from his preferred product. His utility from consuming the non-preferred product is $s - \frac{\sigma}{2}$. For more choosy consumers the difference in the products' characteristics thus weighs more heavily. To obtain closed form-solutions, we assume that σ is distributed uniformly in $[0,1]$.

The industry leader is more *prominent* amongst consumers than the newcomer. In particular, we assume that a fraction $\rho \in (\frac{1}{2}, 1)$ of consumers prefer product L and a fraction $1-\rho$ prefer product N.[3] Firms have constant unit costs of production. We normalize the newcomer's cost to zero, and denote the industry leader's cost by c. Our main results are driven by the firms' difference in prominence and remain valid when $c = 0$, i.e. they do not rely on differences in marginal costs. We allow for the more general case $c \geq 0$ to investigate the possibility of welfare gains when the newcomer has access to a more efficient technology of supply.[4] To simplify the analysis, we abstract from discounting of future payoffs.

Strategies. Each firm $i \in \{L, N\}$ commits to a price schedule $(p_i, P_i) \in \mathbb{R}^2_+$ prior to the advance purchase period.[5] p_i denotes the price of good $i \in \{L, N\}$ in the advance purchase period and P_i is the price in the consumption period. Our notation accounts for the fact that in equilibrium firms will choose $p_i < P_i$, i.e. they will offer an advance purchase discount $d_i = \frac{P_i - p_i}{P_i} \in (0,1)$. We choose s such that in equilibrium, all consumers buy exactly one of the two products, and consume the product they purchased even when it turns out to be their non-preferred product.

[3] While prominence constitutes an exogenous preference parameter in our setting, it may be endogenized in a repeated version of our model, where a firm's prominence today depends on its market share in the past. Further discussion of this issue is postponed until Section 5.

[4] The case where the industry leader has a cost advantage can be accommodated into our analysis but does not lead to further insights.

[5] While in some markets commitment to an introductory and a regular price is explicit (e.g. newspapers) in others commitment arises implicitly from the repeated nature of transactions (e.g. transport tickets).

Information. The key difference between the advance purchase period and the consumption period is the consumers' information about their individual preferences. Consumers are aware of their choosiness σ at all times, but in the advance purchase period, they have no information about the identity of their preferred product other than their knowledge of the distribution ρ of aggregate preferences. In the consumption period, each consumer learns the identity of his preferred product.

Equilibrium. Our analysis will focus on equilibria in which both firms sell their product in both periods. For such a *price-discrimination equilibrium* to exist the following parametric assumption is necessary:

Assumption 1. *If the newcomer has a cost advantage, i.e. if $c > 0$, then it is moderate, i.e. $c < \frac{1}{2}$, and it is compensated by a sufficiently large prominence-advantage of the industry leader, i.e. $\rho > \underline{\rho} \equiv \frac{1}{2(1-c)} \in (\frac{1}{2}, 1)$.*

Note that $\lim_{c\to 0} \underline{\rho} = \frac{1}{2}$, i.e. in the absence of cost-differences Assumption 1 becomes obsolete. When Assumption 1 is violated, the newcomer's cost advantage is so large that, in equilibrium, only the newcomer's product is sold in the advance purchase market. Motivated by the stylized facts described in the Introduction, we abstract from this possibility and focus on situations in which both firms engage in advance selling.

Relation to Hotelling model. Our model is closely related to a Hotelling model and a familiarity with this standard model of horizontal product differentiation can be helpful to understand the intuition for some of our results. To understand this analogy, consider a linear city model with firm L located at 0 and firm N located at 1, where a mass $\rho \in (\frac{1}{2}, 1)$ of consumers are uniformly distributed across the left half interval $[0, \frac{1}{2}]$ and a mass $1-\rho$ is uniformly distributed across the right half interval $[\frac{1}{2}, 1]$ (see Figure 1). While the aggregate distribution of consumers is common knowledge, each individual consumer knows only his distance $d = \frac{1}{2}\sigma$ from the center of the city but does not know on which side he is located. If a consumer's net utility from consuming a firm's product is $s + \frac{1}{2}$

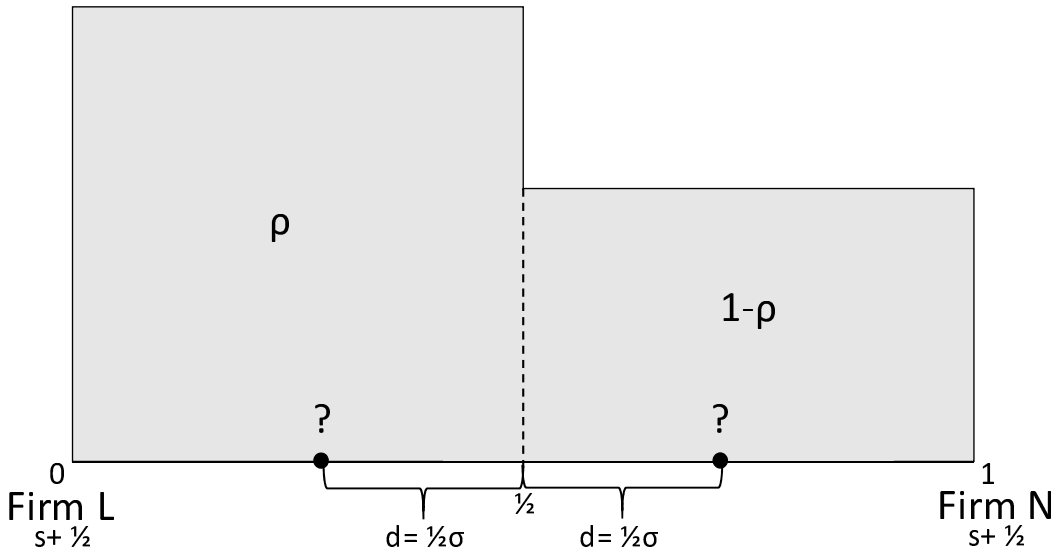

Figure 1: Analogy with Hotelling model: Consumers are uncertain whether they are located to the left or to the right of the city-center. They know their individual distance from the center and that on aggregate a mass $\rho \in (\frac{1}{2}, 1)$ of consumers are located on the side of the industry leader L while a mass $1 - \rho$ of consumers are located on the side of the newcomer N.

minus the distance between the firm's and the consumer's location then the consumer receives utility $s + \frac{\sigma}{2}$ from the closest product and utility $s - \frac{\sigma}{2}$ from the most distant product. Our model is thus analogous to a Hotelling model with the features described above and the added complication that there are two potential periods of purchase with consumers learning over time whether they are located on the left or the right of the city's center.

3 Benchmark: Uniform pricing

As a benchmark, we first consider the case where inter-temporal price discrimination is ruled out by the requirement that prices must be constant across periods, i.e. $p_i = P_i$. In the absence of an advance purchase discount, consumers will postpone their purchase until the consumption period, independently of their choosiness. Purchasing in advance may only result in suboptimal consumption without the benefit of a reduced price.

To derive the uniform pricing equilibrium (P_N^U, P_L^U), suppose that the industry leader sets a higher price than the newcomer, i.e. $P_L^U > P_N^U$.[6] When N is the cheaper product, a consumer whose preferred product is N buys product N independently of σ. In contrast, a consumer whose preferred product is L, buys product L if and only if

$$s + \frac{\sigma}{2} - P_L \geq s - \frac{\sigma}{2} - P_N, \qquad \text{or} \qquad \sigma \geq \hat{\sigma} \equiv P_L - P_N. \tag{1}$$

The mass of consumers who buy from the industry leader is thus given by $\rho(1-\hat{\sigma})$. The newcomer sells to all consumers who prefer product N and to those consumers who prefer product L and have choosiness lower than $\hat{\sigma}$, i.e. demand for the newcomer's product is $1-\rho+\rho\hat{\sigma}$. Profits are thus given by

$$\Pi_L = (P_L - c)\rho(1-\hat{\sigma}) \tag{2}$$

$$\Pi_N = P_N(1-\rho+\rho\hat{\sigma}), \tag{3}$$

and solving the corresponding first-order conditions yields the equilibrium prices

$$P_L^U = \frac{2}{3}c + \frac{1+\rho}{3\rho} \qquad \text{and} \qquad P_N^U = \frac{1}{3}c + \frac{2-\rho}{3\rho}. \tag{4}$$

The following proposition compares the newcomer and the industry leader with respect to their equilibrium prices, markets shares, and profits. It also offers comparative statics concerning changes in the firms' prominence-gap.

Proposition 1 (Benchmark: Uniform pricing)**.** *If prices cannot vary across periods, equilibrium prices are given by (4) and the following holds:*

1. *The newcomer sets a lower price, serves a smaller fraction of the market, and earns a smaller profit than the industry leader.*

2. *An increase in the industry leader's prominence, ρ, leads to lower prices.*

[6] Assuming the opposite leads to a contradiction with the prices that solve the corresponding first-order conditions.

To understand the effects of changes in the industry leader's prominence in the uniform pricing benchmark, note that, due to the static nature of competition, ρ can be interpreted as the size of the "contested market". By offering a lower price than the industry leader, the newcomer not only serves its small "home market" of size $1-\rho$ but also attracts customers from the industry leader's market of larger size $\rho > 1-\rho$. As the size of the contested market increases, competition intensifies, leading to a reduction in firms' prices and profits.

In light of our subsequent analysis, it is worth emphasizing that, under uniform pricing, the newcomer has a "competitive disadvantage" relative to the industry leader, both in terms of profits and market share. In particular, under uniform pricing, a potential cost advantage of the newcomer cannot translate into an advantage in market share or profit, because it will be more than compensated by the industry leader's advantage in prominence.

4 Price-discrimination equilibrium

In this section, we determine the prices that firms must charge in a price-discrimination equilibrium, where both products are sold in the consumption and the advance purchase stage. To specify profits, we first need to establish how consumers allocate across firms and stages. For this purpose, consider a consumer purchasing product L in the first period. Because the consumer will prefer product L with probability ρ and product N with probability $1-\rho$, his expected utility is given by

$$U(\sigma|1,L) = s + \rho\frac{\sigma}{2} - (1-\rho)\frac{\sigma}{2} - p_L. \tag{5}$$

If the consumer waits, he will buy his preferred product. The reason is that waiting comes at a cost, i.e. a forgone discount, and no consumer would be willing to incur this cost unless he is planning to employ the acquired information about his individual preference.

Postponing his purchase until period 2, the consumer thus expects the utility

$$U(\sigma|2) = s + \frac{\sigma}{2} - \rho P_L - (1-\rho)P_N \tag{6}$$

because he will end up purchasing product L with probability ρ and product N with probability $1-\rho$. Therefore, a consumer with choosiness σ prefers postponing his purchase until the consumption period over buying product L in advance if and only if $U(\sigma|2) \geq U(\sigma|1, L)$ or equivalently

$$\sigma \geq \sigma_{WL} \equiv P_N + \frac{\rho P_L - p_L}{1-\rho}. \tag{7}$$

Analogously, the consumer prefers waiting over buying product N in advance if and only if $U(\sigma|2) \geq U(\sigma|1, N) = s + (1-\rho)\frac{\sigma}{2} - \rho\frac{\sigma}{2} - p_N$ or equivalently

$$\sigma \geq \sigma_{WN} \equiv P_L + \frac{(1-\rho)P_N - p_N}{\rho}. \tag{8}$$

The thresholds σ_{WL} and σ_{WN} determine the set of consumers who purchase in advance. More precisely, a consumer of type σ buys in the first period if and only if $\sigma \in [0, \sigma_W)$ where we define $\sigma_W \equiv \max\{\sigma_{WL}, \sigma_{WN}\}$. If $\sigma \in [\sigma_W, 1]$, the consumer waits for the second period to guarantee the purchase of his preferred product.

It remains to determine the consumers' product choice in the advance purchase market. We assume (and later confirm) that $p_L > p_N$. A consumer buying in the first period therefore faces a tradeoff between buying the cheaper product (N) and the product he is more likely to prefer (L). Buying product L is optimal for the consumer if the gain in expected consumption value, $\rho\frac{\sigma}{2} - (1-\rho)\frac{\sigma}{2} - [(1-\rho)\frac{\sigma}{2} - \rho\frac{\sigma}{2}]$, is greater than the price difference, $p_L - p_N$, that is, if and only if

$$\sigma \geq \bar{\sigma} \equiv \frac{p_L - p_N}{2\rho - 1}. \tag{9}$$

Assumption 1 guarantees (see proof of Proposition 2) that, in equilibrium, $0 < \bar{\sigma} < \sigma_{WN} < \sigma_{WL} < 1$ which means that $\sigma_W = \sigma_{WL}$ and consumers' purchase decisions are thus as

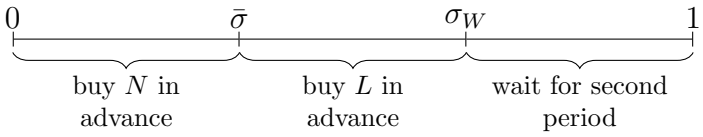

Figure 2: Consumers' purchase decisions in dependence of their choosiness σ.

depicted in Figure 2: Types $\sigma \in [0, \bar{\sigma})$ buy product N in advance; Types $\sigma \in [\bar{\sigma}, \sigma_W)$ buy product L in advance; Types $\sigma \in [\sigma_W, 1]$ wait and buy their preferred product in the consumption period. The profits of firm L and N are then given by

$$\Pi_L = (p_L - c)[\sigma_W - \bar{\sigma}] + (P_L - c)\rho[1 - \sigma_W] \tag{10}$$

$$\Pi_N = p_N\bar{\sigma} + P_N(1 - \rho)[1 - \sigma_W]. \tag{11}$$

Solving the corresponding system of first order conditions leads to the following candidate $\{(p_L^*, P_L^*), (p_N^*, P_N^*)\}$ for a price-discrimination equilibrium:[7]

$$p_L^* = P_L^U - (\frac{1}{2} + 2\rho)\frac{2(1-\rho)}{3\rho}, \qquad p_N^* = P_N^U - (1+\rho)\frac{2(1-\rho)}{3\rho}, \tag{12}$$

$$P_L^* = P_L^U - \frac{2(1-\rho)}{3\rho}, \qquad P_N^* = P_N^U - \frac{2(1-\rho)}{3\rho}, \tag{13}$$

where P_L^U and P_N^U are the equilibrium prices from the uniform pricing benchmark analyzed in Section 3. From these expressions it becomes clear that a potential cost advantage of the newcomer has no effect on firms' pricing, beyond the differences already present in the uniform-pricing benchmark. This emphasizes that the key to understand potential differences between a newcomer's and an industry leader's *dynamic* pricing is the firms' heterogeneity with respect to their prominence amongst consumers. In the Appendix we prove the following:

[7]Existence of such an equilibrium has been shown by Möller and Watanabe (2016) for the case of homogeneous firms. By the continuity of firms' profit functions, the existence of a price-discrimination equilibrium is guaranteed as long as firms are not too heterogeneous. As (12) and (13) are the unique solution of a system of linear first order conditions, these prices constitute the *unique* price discrimination equilibrium. Equilibria in which one of the firms sells exclusively in only one period are ruled out by our parametric assumptions. To show that uniform pricing cannot constitute an equilibrium we can apply a similar logic as the proof of Proposition 2 in Möller and Watanabe (2016).

Proposition 2 (Price-discrimination equilibrium)**.** *In a price-discrimination equilibrium, firms charge prices* $p_N^* < P_N^*$ *and* $p_L^* < P_L^*$ *given by (12) and (13) and the following holds:*

1. *The newcomer sets lower prices than the industry leader in every period and offers a larger advance purchase discount, i.e.* $d_N^* = \frac{P_N^* - p_N^*}{P_N^*} > \frac{P_L^* - p_L^*}{P_L^*} = d_L^*$.

2. *The newcomer's market share and profit are larger than the industry leader's when the newcomer has a cost advantage* $c > 0$ *and* $\rho \in (\underline{\rho}, \bar{\rho})$ *where* $\bar{\rho} \equiv \min\{c + \frac{1}{2}, \frac{3c+2+\sqrt{(9c+8)c}}{4(c+1)}\} \in (\underline{\rho}, 1)$.

3. *An increase in the industry leader's prominence,* ρ*, or marginal cost,* c*, leads to higher prices and lower discounts.*

To understand the differences between the newcomer's and the industry leader's pricing described in part 1 of Proposition 2, note first that

$$P_N^* - p_N^* = \rho\frac{2(1-\rho)}{3\rho} < (2\rho - \frac{1}{2})\frac{2(1-\rho)}{3\rho} = P_L^* - p_L^*. \tag{14}$$

In absolute terms, the newcomer's discount is smaller than the industry leader's. Hence it is the fact that the newcomer sets a lower consumption price $P_N^* < P_L^*$ which produces the result that $d_N^* > d_L^*$. It is reassuring to see that both features are in line with the stylized facts about train-ticket pricing presented in the Introduction (see Table 1).

It is important to note from part 2 of Proposition 2 that advance selling can give the newcomer a competitive advantage over the industry leader, both in terms of market share and profit. The area of the parameter space where the newcomer obtains a higher profit than the industry leader and serves a greater fraction of the market is depicted in Figure 3. This is a first hint at the fact that advance selling benefits new-coming firms and we will come back to this point in our comparison of the price-discrimination equilibrium with the uniform pricing benchmark contained in the subsequent section.

Finally, part 3 of Proposition 2 contains comparative statics with respect to the marginal costs, c, and prominence, ρ, of the industry leader. We can see from (12)

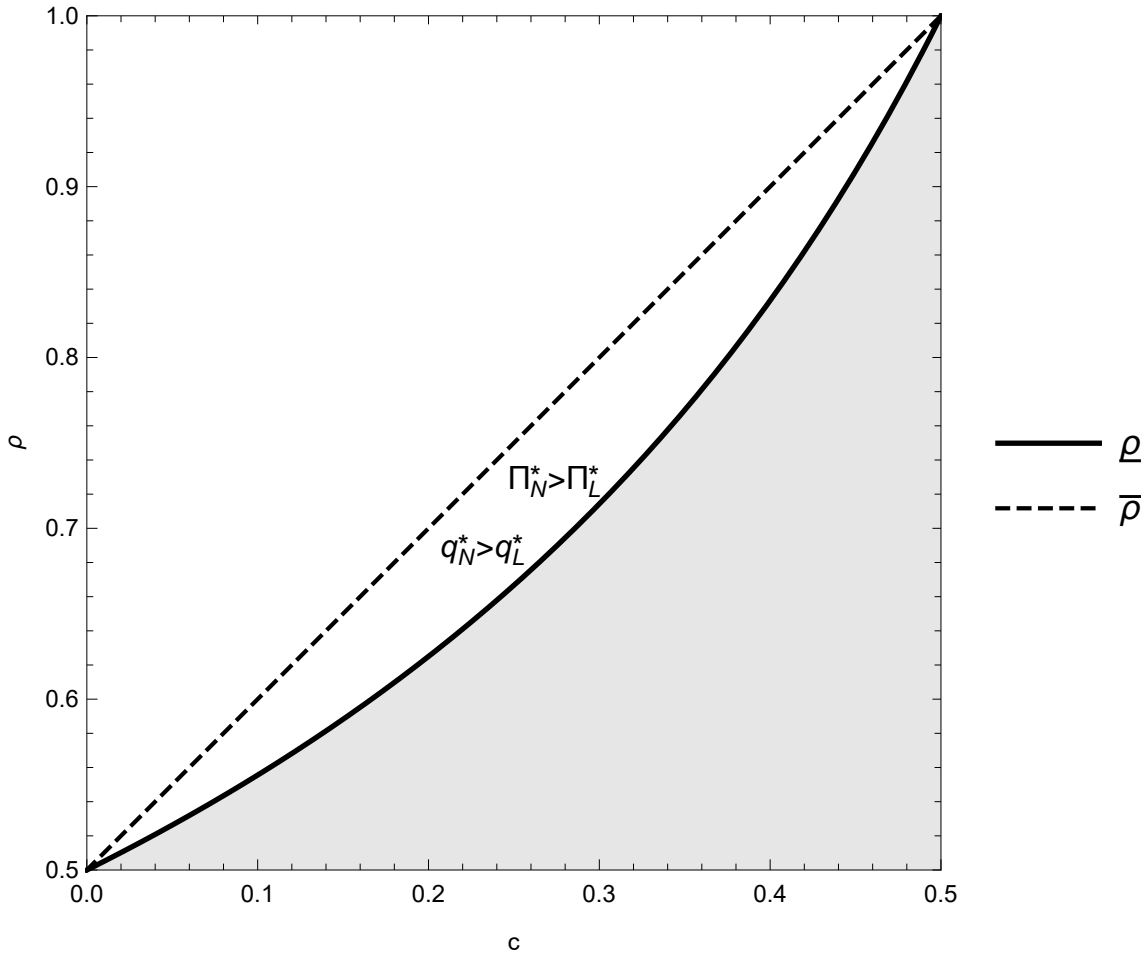

Figure 3: The newcomer's competitive advantage due to advance selling. When advance purchase discounts are feasible, the newcomer's profit and market share can be larger than the industry leader's, which happens when $\rho \in (\underline{\rho}, \bar{\rho})$. In the shaded area, Assumption 1 is violated and no price-discrimination equilibrium exists. For $\rho < \underline{\rho}$, advance selling is practiced exclusively by the newcomer.

and (13) that marginal costs affect dynamic pricing equally in both periods through their influence on prices in the uniform pricing benchmark. Hence discounts are independent of c in absolute terms and, because prices in the benchmark are increasing in c, relative discounts must be decreasing. Given the analogy between our setting with uniform pricing and a Hotelling model outlined in Section 2, these findings seem rather intuitive.

Figure 4 illustrates the comparative statics regarding the industry leader's prominence by depicting regular and advance selling prices as a function of ρ for a fixed value of c. As

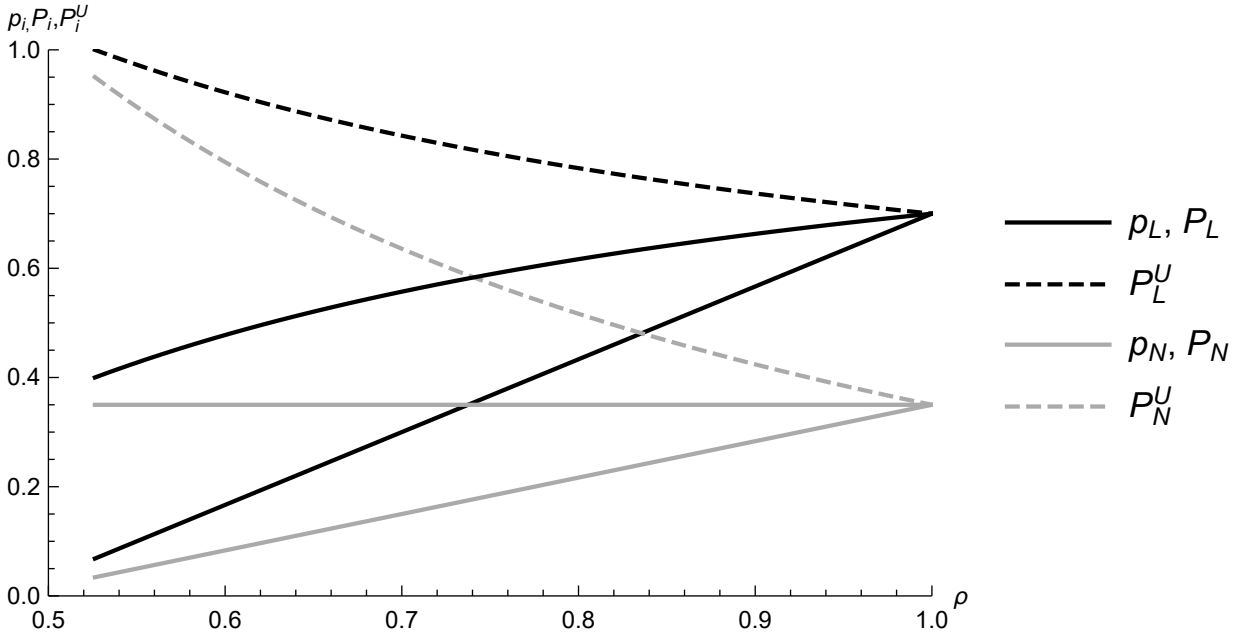

Figure 4: Equilibrium price schedules (p_N^*, P_N^*) and (p_L^*, P_L^*) (solid) of the newcomer (gray) and the industry leader (black), respectively, in dependence of the leader's prominence ρ. Comparison is with the equilibrium prices P_N^U and P_L^U in the uniform pricing benchmark (dashed). Parameter values are $c = \frac{1}{20}$ and existence of the price-discrimination equilibrium requires that $\rho > \frac{1}{2(1-c)} = \frac{10}{19}$.

can be seen from the figure, prices are increasing in the industry leader's prominence and advance purchase discounts are decreasing, in absolute and hence relative terms. This is intuitive, because an increase in the leader's prominence makes consumers less uncertain about the identity of their preferred product. Hence products appear more differentiated already during the advance purchase period and price-competition becomes mitigated. Moreover, when products are less homogeneous ex ante, the newcomer has less incentive to divert competition to the advance purchase market by use of a discount. In the limit, where $\rho \to 1$, pricing converges to the uniform pricing benchmark in both periods.

Figure 4 emphasizes that, when inter-temporal price-discrimination is feasible, the industry leader's prominence plays a markedly different role than in the uniform pricing benchmark. In particular, while in the price-discrimination equilibrium, prices are increasing in ρ, under uniform pricing, prices are decreasing. In the presence of inter-temporal

price-discrimination, an increase in ρ mitigates competition by making products appear more differentiated at the advance purchase stage. In contrast, under uniform pricing, an increase in ρ intensifies competition by augmenting the size of the contested market at the consumption stage. Although our model is, arguably, stylized in that it considers the industry leader's prominence as exogenous, Propositions 1 and 2 identify prominence as a key determinant of the differences between competition with and without advance purchase discounts.

5 The effects of a ban on advance purchase pricing

From a policy maker's perspective, price discrimination is often under scrutiny. In markets, where individuals know their preferences, price discrimination can have adverse effects, both for consumers and for entering firms. In this section, we compare the outcome of the price-discrimination equilibrium in Section 4 with the uniform pricing benchmark in Section 3. This allows us to examine the effects that price discrimination has on markets subject to individual demand uncertainty. We will argue, that for such markets, the effects of price-discrimination are markedly different. In particular, our main results show that the firms' ability to price-discriminate inter-temporally can be beneficial both for new-coming firms and for consumers.

We separate our analysis by discussing first the effects that advance purchase pricing has on firms before turning to consumer surplus and welfare considerations at the end of the section. Figure 3 already hinted at the possibility that advance purchase pricing may improve a newcomer's position by showing that, in contrast to the uniform pricing benchmark, the newcomer's market share and profit can be larger than the industry leader's. Our next result shows that the ability to sell in advance is beneficial for new-coming firms more generally. In particular, in the Appendix we prove the following:

Proposition 3 (Prices, market shares, and profits)**.** *Relative to the uniform pricing bench-*

mark, the firms' ability to offer advance purchase discounts decreases prices for both the industry leader and the newcomer, increases the newcomer's market share, and reduces the difference between the leader's and the newcomer's profits.

Proposition 3 shows that advance selling allows the newcomer to extend its market share. Intuitively, by inducing consumers to buy in advance, the newcomer can increase his market share because the consumers' preference-bias in favor of the industry leader becomes relaxed due to the presence of individual demand uncertainty. Demand uncertainty makes products appear more homogeneous, which intensifies price-competition leading to lower prices in both periods. Moreover, Proposition 3 shows that advance selling benefits the newcomer in terms of its relative profitability with respect to the industry leader. Both, market share and relative profitability, are key determinants of a new-coming firm's market position in the long-run, deciding eventually whether a newcomer can catch-up with an industry's leader. We thus conclude from Proposition 3 that, from a policy perspective, advance selling should be considered as a remedy for market-concentration.

It is important to note that, while in our model, prominence is exogenous, an increase in the newcomer's market share may have an additional positive effect on the newcomer's prominence in the future. Endogenizing prominence in this way constitutes a relevant direction for future research, as is may provide further insights into the long-run dynamics of competition in advance purchase markets. Given that advance purchase pricing increases the newcomer's market share and hence its future prominence, we conjecture that in a repeated version of our model, the benefits of advance selling for the newcomer would be even more pronounced.

Finally, we turn our attention to the effects of advance selling on consumer surplus and welfare. From Proposition 3 it is immediate that consumer surplus must increase. If both, advance purchase prices and regular prices are below the uniform pricing benchmarks, then consumers must be better off. The reason is that consumers are free to choose

their timing of purchase and those who accept the risk of a suboptimal purchase in the advance market do so because it is more than compensated by the existence of an advance purchase discount. In our setting with individual demand uncertainty, inter-temporal price discrimination creates "all-out-competition" in the sense of Corts (1998), and consumer unambiguously benefit.

Regarding welfare, the firms' ability to price-discriminate inter-temporally has two opposing effects. First, the increase in the newcomer's market share from

$$q_N^U = 1 - \rho + \rho\hat{\sigma}^U = \frac{2 - (1-c)\rho}{3} \tag{15}$$

under uniform pricing to

$$q_N^* = \bar{\sigma}^* + (1-\rho)(1-\sigma_W^*) = \frac{-2(c+1)\rho^2 + (3c+5)\rho - 2}{2(3\rho - 1)} > q_N^U \tag{16}$$

under price-discrimination, reduces the costs of supply when the newcomer is more efficient than the industry leader. Second, advance selling bears the risk of suboptimal purchases, leading to a reduction in realized benefits. More specifically, relative to the optimal match between consumers and products, those consumers who purchase from firm N in advance lose $s + \frac{\sigma}{2} - [s + (1-\rho)\frac{\sigma}{2} - \rho\frac{\sigma}{2}] = \rho\sigma$ in expected benefit, whereas those consumers who purchase from firm L in advance lose $s + \frac{\sigma}{2} - [s + \rho\frac{\sigma}{2} - (1-\rho)\frac{\sigma}{2}] = (1-\rho)\sigma$. In the uniform pricing benchmark, a mismatch between consumers and products arises only from the fact that all consumers with low choosiness choose the cheaper product N although a fraction ρ of them prefer product L, inducing a welfare loss of size $\rho[s + \frac{\sigma}{2} - (s - \frac{\sigma}{2})] = \rho\sigma$. The overall effect of inter-temporal price discrimination on welfare thus depends on the comparison of the cost-savings from a potentially more efficient supply with the loss in expected benefits from sub-optimal product-choices:

$$W^* - W^U = (q_N^* - q_N^U)c - \int_0^{\bar{\sigma}^*} \rho\sigma d\sigma - \int_{\bar{\sigma}^*}^{\sigma_W^*} (1-\rho)\sigma d\sigma + \int_0^{\hat{\sigma}^U} \rho\sigma d\sigma. \tag{17}$$

It turns out that the first effect is dominated by the second. In particular, we have the following:

Proposition 4 (Welfare and consumer surplus)**.** *In comparison to the uniform pricing benchmark, the firms' ability to offer advance purchase discounts increases consumer surplus but leads to a loss in welfare.*

It is important to note that in our setting, the firms' pricing has no effect on the market's total supply. In particular, we have assumed s to be sufficiently large, such that, in equilibrium, all consumers participate in the market. From the assumption of unit demands it thus follows that total output is the same, no matter whether firms are able to price-discriminate inter-temporally or not. Proposition 4 is thus concerned with the effects of price-discrimination on the market's efficiency, in the absence of quantity considerations. Holmes (1989) has shown that, in oligopolistic markets, the welfare effects of price discrimination depend on quantity effects through the cross-price elasticities of demand and can therefore be ambiguous. By focusing on the purely *allocative* consequence of price-discrimination, we are able to highlight a drawback of price-discrimination that arises in markets subject to individual demand uncertainty. In line with Möller and Watanabe (2016), we thus find that advance selling induces an efficiency loss. By allowing firms to differ in their costs, our theory shows that this efficiency loss cannot be overcome by potential reductions in the costs of supply. Advance selling is detrimental for the market's efficiency, in spite of the fact that it allows a more efficient newcomer to extend its market share relative to an industry's leader.

6 Conclusion

In this paper, we have provided the first tractable model of oligopolistic competition in an advance purchase market with heterogeneous firms. Based on our observation of systematic differences in the dynamic pricing schedules of new-coming and established firms in advance purchase markets, we have first proposed a stylized model of competition with advance purchase discounts, capable of explaining the observed facts. In a second

step, we have employed the model to analyze the "counterfactual" situation in which firms are banned from price-discriminating inter-temporally. The main message that emerges from our theory is that advance purchase pricing is a powerful tool that allows new-coming firms to improve their market position with respect to more prominent incumbents.

Regarding the welfare effects of price-discrimination, our theory extends two important existing results. First, by showing that inter-temporal price-discrimination benefits consumers, we demonstrate that the idea that price-discrimination intensifies competition amongst firms (e.g. Thisse and Vives, 1988; Corts, 1998) is not specific to settings where discrimination is based on observable consumer characteristics but extends to a framework where price-discrimination is of second rather than third degree. Second, by showing that advance purchase pricing leads to a welfare loss in a market with heterogeneous firms, we confirm that the allocative inefficiency due to advance selling (c.f. Möller and Watanabe, 2016; Karle and Möller, 2020) cannot be overcome even though advance selling may increase the market share of the more efficient firm.

Appendix

Proof of Proposition 1. With the equilibrium prices P_L^U and P_N^U given by (4), $P_L^U > P_N^U$ is equivalent to $\rho > \frac{1}{2}$. Substituting the equilibrium prices leads to the market shares of the industry leader and the newcomer:

$$q_L^U = \rho(1-\hat{\sigma}^U) = \frac{1+(1-c)\rho}{3} \qquad \text{and} \qquad q_N^U = 1-\rho+\rho\hat{\sigma}^U = \frac{2-(1-c)\rho}{3}. \tag{18}$$

It thus holds that $q_L^U > q_N^U$ if and only if $\rho > \frac{1}{2(1-c)}$ which is satisfied by Assumption 1. Equilibrium profits are given by

$$\Pi_L^U = \frac{[1+(1-c)\rho]^2}{9\rho} \qquad \text{and} \qquad \Pi_N^U = \frac{[2-(1-c)\rho]^2}{9\rho}, \tag{19}$$

and $\Pi_L^U > \Pi_N^U$ is again equivalent to $\rho > \frac{1}{2(1-c)}$. Finally, the comparative statics follow from

$$\frac{\partial P_L^U}{\partial \rho} = -\frac{1}{3\rho^2} < 0 \qquad \text{and} \qquad \frac{\partial P_N^U}{\partial \rho} = -\frac{2}{3\rho^2} < 0. \tag{20}$$

□

Proof of Proposition 2. We first confirm that at the equilibrium prices $(p_L^*, P_L^*, p_N^*, P_N^*)$ given by (12) and (13), the corresponding equilibrium thresholds

$$\bar{\sigma}^* = \frac{c + 2\rho - 1}{3(2\rho - 1)} \tag{21}$$

$$\sigma_{WN}^* = \frac{c\rho + 1}{3\rho} \tag{22}$$

$$\sigma_{WL}^* = \frac{2 - c}{3} \tag{23}$$

are such that $0 < \bar{\sigma}^* < \sigma_{WN}^* < \sigma_{WL}^* < 1$. For this purpose, note that $\bar{\sigma}^* > 0$ because $\rho \in (\frac{1}{2}, 1)$ and $c \geq 0$. Note next that

$$\sigma_{WN}^* - \bar{\sigma}^* = \frac{(1-\rho)[2(1-c)\rho - 1]}{3\rho(2\rho - 1)} > 0 \tag{24}$$

which is positive by Assumption 1. For the same reason,

$$\sigma_{WL}^* - \sigma_{WN}^* = \frac{2(1-c)\rho - 1}{3\rho} > 0. \tag{25}$$

Finally $\sigma_{WL}^* < 1$ follows from $c \geq 0$. This shows that Assumption 1 is necessary for the existence of a price-discrimination equilibrium, i.e. an equilibrium in which both firms sell in both periods.

In the advance selling period the price difference between the two firms is given by

$$p_L^* - p_N^* = \frac{c}{3} + \frac{(2\rho - 1)}{3}, \tag{26}$$

and in the consumption period it is given by

$$P_L^* - P_N^* = \frac{c}{3} + \frac{(2\rho - 1)}{3\rho}. \tag{27}$$

Both are clearly positive for all $\rho \in (\frac{1}{2}, 1)$ and $c \geq 0$. The difference in the advance purchase discounts is given by

$$d_N^* - d_L^* = \frac{(1-\rho)(c+2\rho-1)}{(c+1)(2c\rho+3\rho-1)}, \tag{28}$$

which is also positive for all $\rho \in (\frac{1}{2}, 1)$ and $c \geq 0$. In equilibrium, the market shares of the industry leader and the newcomer are given by

$$q_L^* = \frac{2(c+1)\rho^2 - 3c\rho + \rho - 1}{3(2\rho-1)} \qquad \text{and} \qquad q_N^* = \frac{-2(c+1)\rho^2 + (3c+5)\rho - 2}{2(3\rho-1)}, \tag{29}$$

from which it follows that

$$q_N^* > q_L^* \;\; \Leftrightarrow \;\; \rho < \rho_1 \equiv \frac{3c+2+\sqrt{c(9c+8)}}{4(c+1)}. \tag{30}$$

The equilibrium profits are given by

$$\Pi_L^* = \frac{c^2\rho(3-2\rho) - 2c\,(2\rho^2+\rho-1) + 14\rho^2 - 13\rho + 3}{9(2\rho-1)} \tag{31}$$

$$\Pi_N^* = \frac{c^2\rho(3-2\rho) - 2c\,(2\rho^2-5\rho+2) + \rho(2\rho-1)}{9(2\rho-1)}, \tag{32}$$

and it holds that $\Pi_N^* > \Pi_L^* \;\; \Leftrightarrow \;\; \rho < \rho_2 \equiv c + \frac{1}{2}$. Hence we have shown that the newcomer has a competitive advantage, both in terms of market share and profits, if and only if $\rho \in (\underline{\rho}, \bar{\rho})$ where $\bar{\rho} \equiv \min\{\rho_1, \rho_2\}$. Note that the interval $(\underline{\rho}, \bar{\rho})$ is non-empty if and only if the newcomer has a cost advantage, that is, if and only if c is strictly positive. Finally, we consider the comparative statics. The effect of ρ on the equilibrium prices is given by

$$\frac{\partial p_L^*}{\partial \rho} = \frac{4}{3} > 0, \qquad \frac{\partial p_N^*}{\partial \rho} = \frac{2}{3} > 0, \tag{33}$$

$$\frac{\partial P_L^*}{\partial \rho} = \frac{1}{3\rho^2} > 0, \qquad \frac{\partial P_N^*}{\partial \rho} = 0, \tag{34}$$

implying that the industry leader's and newcomer's prices (weakly) increase with ρ. With respect to the discounts the effect of ρ is given by

$$\frac{\partial d_L^*}{\partial \rho} = -\frac{2[c(2\rho-1)(2\rho+1) + 6\rho^2 + 4\rho - 1]}{((2c+3)\rho-1)^2} < 0, \tag{35}$$

$$\frac{\partial d_N^*}{\partial \rho} = -\frac{2}{1+c} < 0, \tag{36}$$

that is, discounts decrease with ρ. □

Proof of Proposition 3. The fact that equilibrium prices are lower than in the uniform pricing benchmark follows directly from (12) and (13) given that $\rho \in (\frac{1}{2}, 1)$. The change in the newcomer's market share is given by

$$q_N^* - q_N^U = \frac{4c(1-\rho)\rho}{3(2\rho-1)} > 0. \tag{37}$$

Regarding the differences in profits it holds that

$$\Pi_L^* - \Pi_N^* - (\Pi_L^U - \Pi_N^U) = -\frac{(2\rho-1)(1-\rho)}{3\rho} < 0, \tag{38}$$

implying that advance selling reduces the profit difference between the industry leader and the newcomer. □

Proof of Proposition 4. Because $p_L^* < P_L^* < P_L^U$ and $p_N^* < P_N^* < P_N^U$, every consumer, independently of his choosiness σ must be better off when inter-temporal price-discrimination is feasible. Hence, the possibility of advance selling must lead to an increase in consumer surplus. The difference between welfare under price-discrimination and welfare under uniform pricing is given by

$$W^* - W^U = (q_N^* - q_N^U)c - \int_0^{\bar{\sigma}^*} \rho\sigma d\sigma - \int_{\bar{\sigma}^*}^{\sigma_W^*} (1-\rho)\sigma d\sigma + \int_0^{\hat{\sigma}^U} \rho\sigma d\sigma \tag{39}$$

$$= -\frac{(1-\rho)[4(3-5c^2)\rho^2 - 8\rho + 1]}{18\rho(2\rho-1)}, \tag{40}$$

which, given $\rho \in (\frac{1}{2}, 1)$ is negative if $f(\rho, c) \equiv 4(3-5c^2)\rho^2 - 8\rho + 1 > 0$. Note that for $\rho \in (\underline{\rho}, 1)$ and $c \in (0, \frac{1}{2})$ it holds that

$$f(\underline{\rho}, c) = \frac{2(1-2c)c}{(1-c)^2} > 0 \tag{41}$$

and

$$\frac{\partial f}{\partial \rho} = 8[(3-5c^2)\rho - 1] > 0. \tag{42}$$

Hence, if ρ is large enough for a price discrimination equilibrium to exist, i.e. if $\rho > \underline{\rho}$, then $f(\rho, c) > 0$ and therefore $\Delta W < 0$. □

References

Armstrong, Mark and John Vickers, "Competitive price discrimination," *RAND Journal of Economics*, 2001, *32* (4), 579–605.

Asplund, Marcus, Rickard Eriksson, and Niklas Strand, "Price Discrimination in Oligopoly: Evidence from Regional Newspapers," *The Journal of Industrial Economics*, 2008, *LVI* (2), 333–346.

Carroni, Elias, "Competitive customer poaching with asymmetric firms," *International Journal of Industrial Organization*, 2016, *48*, 173–206.

Chen, Yongmin, "Dynamic price discrimination with asymmertic firms," *Journal of Industrial Economics*, 2008, *56* (4), 729–751.

Corts, Kenneth S., "Third-degree price discrimination in oligopoly : all-out competition and strategic commitment," *Rand Journal of Economics*, 1998, *29* (2), 306–323.

Dana Jr., James D., "Advance-purchase discounts and price discrimination in competitive markets," *Journal of Political Economy*, 1998, *106* (2), 395–422.

Fudenberg, Drew and Jean Tirole, "Customer poaching and brand switching," *RAND Journal of Economics*, 2000, *31* (4), 634–657.

Gale, Ian L., "Price dispersion in a market with advance-purchases," *Review of Industrial Organization*, 1993, *8* (4), 451–464.

_ and Thomas Holmes, "Advance-purchase discounts and monopoly allocation of capacity," *The American Economic Review*, 1993, *83* (1), 135–146.

Holmes, Thomas J., "The effects of third-degree price discrimination in oligopoly," *The American Economic Review*, 1989, *79* (1), 244–250.

Karle, Heiko and Marc Möller, "Selling in Advance To Loss Averse Consumers," *International Economic Review*, 2020, *61* (1), 441–468.

Möller, Marc and Makoto Watanabe, "Advance Purchase Discounts Versus Clearance Sales," *The Economic Journal*, 2010, *120*, 1125–1148.

_ and _, "Competition in the presence of individual demand uncertainty," *RAND Journal of Economics*, 2016, *47* (2), 273–292.

Nocke, Volker and Martin Peitz, "A Theory of Clearance Sales," *The Economic Journal*, 2007, *117* (1988), 964–990.

_ , _ , and Frank Rosar, "Advance-purchase discounts as a price discrimination device," *Journal of Economic Theory*, jan 2011, *146* (1), 141–162.

Thisse, Jacques-Francois and Xavier Vives, "On the strategic choice of spatial price policy," *The American Economic Review*, 1988, *78* (1), 122–137.

Varian, Hal R., "Price discrimination," in R. Schmalensee and R.D. Willig, eds., *The Handbook of Industrial Organization*, Vol. I, Amsterdam: North-Holland, 1989, pp. 527–560.

Villas-Boas, J. Miguel, "Dynamic competition with customer recognition," *The Rand Journal of Economics*, 1999, *30* (4), 604–631.