Fighting for Lemons: The Encouragement Effect in Dynamic Contests with Private Information*

Juan Beccuti† Marc Möller‡

November 3, 2022

Abstract

In a common value environment with multi-stage competition, losing a stage conveys positive news about a rival's estimation of a contested prize, capable of balancing the discouraging effect of falling behind. We show that, due to this *encouragement effect*, aggregate incentives under private information are greater than under public information and may even exceed the static competition benchmark. Moreover, laggards can become *more* motivated than leaders, leading to long-lasting fights. Our results have implications for the duration of R&D races, the desirability of feedback in promotion tournaments and procurement contests, and the campaign spending and selective efficiency of presidential primaries.

Keywords: Dynamic contests, private information, discouragement effect, information design.

JEL: C72, D72, D82.

*We are grateful to Andrea Prat, Giacomo Calzolari, Mikhail Drugov, Ian Jewitt, Jorge Lemus, Meg Meyer, Marco Serena, and participants of the European Summer Symposium in Economic Theory for helpful comments and suggestions. Special thanks go to Igor Letina for the fruitful title suggestion. Financial Support from Swiss National Science Foundation Grant no. 207874 is gratefully acknowledged.

†Department of Economics, University of Bern. Email: juan.beccuti@vwi.unibe.ch

‡Department of Economics, University of Bern. Email: marc.moeller@vwi.unibe.ch

1 Introduction

Competition for a scarce resource, such as a patent, political candidacy, promotion, procurement contract, or, generally, a *prize*, is frequently of a dynamic nature, thus featuring the possibility that competitors have taken the lead or have fallen behind. There are concerns that in such situations, incentives are undermined by the so-called *discouragement effect* (Konrad, 2009): As followers feel discouraged by the costs of catching up, leaders can allow themselves to lower their efforts, resulting in a reduction of incentives on aggregate.[1] Evidence supporting these concerns has been reported not only for experimental settings (Mago et al., 2013) and sports (Malueg and Yates, 2010, Iqbal and Krumer, 2019), but also by an influential recent study of online innovation contests (Lemus and Marshall, 2021).

The consequences of the discouragement effect are far reaching and have been noted for a broad variety of settings. In R&D races, an early breakthrough may mute the investment-incentives of rival firms and lead to a slow-down of innovation (Fudenberg et al., 1983; Harris and Vickers, 1987; Judd et al., 2012). In promotion tournaments, workers can become demotivated by the achievements of their co-workers, putting under scrutiny the wide-spread use of interim performance evaluations (Klein and Schmutzler, 2017) and feedback policies (Gershkov and Perry, 2009; Aoyagi, 2010; Ederer, 2010; Goltsman and Mukherjee, 2011). In presidential primaries, overall campaign spending is reduced and early voting in non-representative districts can become decisive for the overall outcome of the election (Klumpp and Polborn, 2006). Finally, in sports competitions, performance differences accumulated during earlier stages may lead to a deterioration of suspense (Chan et al., 2009).

In this article, we argue that, besides their direct effect on a contest's score, stage-wins or -losses may have an indirect, *informational* effect, with an opposing, and so far

[1] This effect has also been denoted as *momentum effect*, to distinguish it from the purely *static* discouragement resulting from differences in contestants' abilities (Drugov and Ryvkin, 2022).

overlooked influence on incentives. Our starting point is the observation that, while in many of the aforementioned applications stage-wins or -losses are observable, contestants typically cannot observe each others' efforts and may be privately informed about the, arguably, common value of the contested prize.[2] In such situations, a stage-loss (-win) represents good (bad) news about the contest's prize, because the likelihood of a loss (win) is increasing (decreasing) in the opponent's effort which correlates with his private information. For example, in an R&D race, an early breakthrough may be the consequence of a large but unobservable investment by a rival company whose market-research has revealed a profitable future for the contested innovation. In the presence of private information, it is therefore no longer clear whether dynamic contests are subject to a discouragement effect or whether losers of early stages are actually encouraged to exert *larger* efforts than their rivals. Moreover, the implications of dynamic competition for *aggregate* incentives are ambiguous, because the discouragement effect might be mitigated or even overcome by an *encouragement effect* arising from the contestants' ability to learn about their rival's information.

To shed light on these issues, in Section 2, we propose a stylized model of dynamic competition with private information. In our model, two homogeneous contestants compete in a best-of-three contest by exerting costly efforts in three sequential battles.[3] We allow for a generic class of mappings between efforts and battle outcomes, including the frequently employed Tullock (1980) success function as a special case. The winner of the overall contest obtains a prize whose common value is uncertain, either one or zero. At the start of the contest, contestants receive private, independent, and identically distributed signals, that are informative about the contest's prize, and whose realizations can be either good or bad. In each battle, contestants learn whether they win or lose but cannot

[2]Our model allows for an equivalent formulation where private information concerns the costs of a common input of competition rather than the contest's prize.

[3]Klein and Schmutzler (2017) provide an incentive-based rationale for why competition may take the format of a best-of-N contest akin to our model.

observe their rival's effort.

Our model owes its tractability to the assumption that the underlying information structure is partially conclusive. In particular, we assume that, while conditional on the prize being zero, either a good or a bad signal can be observed, both contestants will receive a good signal when the prize is one.[4] It follows that contestants will conclude from the observation of a bad signal that the contest's prize must be zero and that it is optimal to refrain from exerting effort. The characterization of a Perfect Bayesian equilibrium—a challenging task for models combining dynamic competition with private information—is thus simplified to the description of the contestants' effort choices, conditional on the receipt of a good signal.

After introducing the *static competition* and the *public information* benchmarks in Section 2, we show in Section 3.1 that, in accordance with the aforementioned intuition, the gap between the leader's and the follower's efforts in the Perfect Bayesian equilibrium is reduced relative to the public information benchmark. Moreover, when the battles' rate of rent dissipation is sufficiently low, the follower's effort may even exceed the leader's, making it *less* likely that the contest is decided after two rather than three battles. In empirical studies, contests that are decided within a few battles have been interpreted as evidence for the discouragement effect. Our finding, that in the presence of private information, the discouragement effect might be overcome by the encouragement effect means that long-lasting fights will be frequently observed and need not be an indication for the absence of discouragement.[5]

Our two main results are presented in Section 3.2 and are concerned with the effect of private information on aggregate incentives. First, we show that expected aggregate effort

[4]This information structure is called the "bad news" model in the literature on strategic experimentation (e.g. Keller and Rady, 2015; Bonatti and Hörner, 2017).

[5]Ferrall and Smith (1999) argue that in basketball-, hockey-, and baseball-playoffs "a simple model in which players do not give up [...] best explains the outcome of the championship series." Similarly, Zizzo (2002) denotes the lack of evidence for discouragement in experimental patent race data as "a puzzle from the perspective of patent race theory."

in the Perfect Bayesian equilibrium is strictly larger than in the public information benchmark. This holds independently of the signals' *informativeness*, given by the likelihood of receiving a bad signal when the contest's prize is zero. Our finding contrasts with the linkage principle from auction theory (Milgrom and Weber, 1982) which states, loosely speaking, that in a common value setting, private information creates information rents thereby reducing an auctioneer's expected revenue.[6] Although, in our framework, private information causes a winner's curse, the encouragement effect leads to an overall gain in aggregate incentives. The reason is that, in a dynamic contest, private information helps to level the playing field when contestants have established a lead over others. Naturally, because efforts are costly, a direct implication of this result is that private information is harmful from the contestants' perspective, i.e. asymmetric information leads to "fighting for lemons".

Second, we show that aggregate incentives in the Perfect Bayesian equilibrium can be even higher than in the static benchmark. This happens when the encouragement effect is strong relative to the discouragement effect, which is the case when the battles' rate of rent dissipation is low. Our result shows that the common wisdom, that incentives are reduced by the dynamic nature of competition, needs not hold in contests that are subject to private information. Dynamic competition can improve upon static competition because the encouragement that contestants derive from learning can overcome the discouragement that arises from intermediate performance evaluations, even in terms of *aggregate* incentives.

In Section 4 we relate our work to a nascent literature on information design in contests by characterizing the contest's optimal (partly-conclusive) information structure.[7] Our analysis reveals a dichotomy of optimal contest designs with the optimal information

[6] While the linkage principle requires that the auction is won by the highest bidder, our contest is noisy and can be won by a player other than the one with the highest effort. Another setting where this happens and the linkage principle fails are multi-unit auctions (Perry and Reny, 1999).

[7] The existing literature on information design has mostly restricted attention to static contests. For a detailed discussion of this literature see Section 4.

structure selecting between two modes of competition. Either, contestants are induced to fight hard to establish themselves as leaders of the competition and leaders are likely to become final winners. Or, incentives are relatively weak at the start of the contest but fighting is likely to last until the very end. Our theory can thus be used to characterize the circumstances under which competition can be expected to be fierce but short or mild but long-lasting.

In Section 5, we extend our model by introducing heterogeneity in the contestants' valuation of the contest's prize. A valid concern is that the encouragement effect, although beneficial for aggregate incentives, may have a negative impact on a contest's selective efficiency. In particular, because a low-valuing contestant is more likely to be lagging behind, narrowing the gap between a leader's and a follower's effort may have the adverse effect of reducing the likelihood with which the high-valuing contestant can claim the contest's prize. We argue that this intuition is incomplete and show that, instead, private information can have a *positive* effect not only on aggregate incentives but also on a contest's selective efficiency.

The robustness of our results is discussed in Section 6 where we argue that our main insights extend to contests with more general information structures and longer horizons. All formal proofs can be found in the Appendix.

Related literature

The discouragement effect has made its first appearance in the literature on R&D races, where it can take the particularly severe form of ϵ-preemption (Fudenberg et al., 1983): Even the smallest innovation advantage can obstruct the investment of rival firms. The seminal model of Harris and Vickers (1987) takes the format of a best-of-N contest and its battle-components are strategically equivalent to a Tullock contest when investments are lump-sum (Baye and Hoppe, 2003). Our results thus apply and they suggest that, due to the inherently uncertain value of innovation, the dynamic nature of R&D-competition is not an obstacle but may in fact promote investment on aggregate, because firms' become encouraged by the success of their rivals. This finding resonates well with the idea of Choi (1991) that a rival's success may improve a firm's belief in the feasibility of a contested innovation (see also Malueg and Tsutsui, 1997 and Bimpikis et al., 2019). An important difference is that in our setting, information is private rather than public, which means that the observation of progress augments the investment incentives of lagging firms while reducing the investment incentives of leading firms. Our results thus suggest that, in an R&D setting, private information induces a closer but also longer race for innovation.[8]

Our theory combines a dynamic contest framework with private information and it thereby contributes to two, mostly separate branches of the literature. The first branch investigates the role of information in static contests, where a different form of discouragement arises from potential differences in players' prize-valuations or abilities. While for private-value environments, asymmetric information is found to have a positive effect on aggregate incentives (Morath and Münster, 2008; Dubey, 2013; Wasser, 2013; Fu et al., 2014; Serena, 2021), in common-value settings, more akin to ours, private information typically has a negative or no effect (Hurley and Shogren, 1998; Wärneryd, 2003; Einy et al., 2017). Our analysis of the static competition benchmark in Section 2 shows that, in our model, private information has an influence on aggregate effort only when the contest is dynamic, thus identifying contestants' learning as the origin of the identified incentive-gains.

The second branch of the literature characterizes incentives for various types of dynamic contests with perfect information. Konrad and Kovenock (2009) provide the seminal analysis of a best-of-N contest, with individual battles modeled as all-pay auctions, where the rate of rent-dissipation and hence the discouragement effect are extreme. For more moderate rates of rent dissipation, the characterization of equilibrium in a best-of-N

[8]Further effects of private information include the possibility of homogeneous investment-behavior by heterogeneous firms (Moscarini and Squintani, 2010) and of an information-backlash due to firms' ability to learn from a better informed rival (Awaya and Krishna, 2021).

contest has proven rather elusive. Ferrall and Smith (1999) determine a mixed-strategy equilibrium when battles take the form of an additive tournament with normally distributed noise and show, numerically, that the players' likelihood to provide positive effort falls towards zero when the contest reaches an asymmetric state. For standard Tullock-battles, a characterization of equilibrium for a best-of-N contest has been obtained by Klumpp and Polborn (2006). They take the predicted discouragement as an argument in favor of the sequential format of US presidential primaries, where efforts consist of wasteful campaign spending.[9] We contribute to this literature by providing a characterization of equilibrium for generic tournaments with multiplicative noise, including the Tullock specification as a special case.

The few articles that combine dynamic contests and incomplete information belong to a growing literature about the desirability of intermediate performance feedback in labor tournaments (Gershkov and Perry, 2009; Aoyagi, 2010; Goltsman and Mukherjee, 2011; Ederer, 2010) or cryptocurrency mining protocols (Ely et al., 2021). Feedback can induce fierce competition when the contest is close, but has a discouraging effect when large performance differences are revealed. As our static competition benchmark is strategically equivalent to a situation where players compete sequentially without knowledge of the individual battles' outcomes, our theory contributes to this literature. In particular, our results imply that, in the presence of private information about the contest's prize (e.g. the value of becoming promoted), intermediate performance feedback is detrimental when contestants are very poorly or very well informed but can improve incentives when information is of moderate quality.

Finally, on a more abstract level, our results resonate well with the general idea that, in

[9] By introducing multiplicative biases into a best-of-three version of the Klumpp and Polborn (2006) model, Barbieri and Serena (forthcoming) show that aggregate effort can be increased by favoring the loser of battle one, thereby extending the logic of leveling the playing field from a static to a dynamic setting. While we share with Barbieri and Serena the finding that, in battle two, efforts are maximal when winning probabilities are equalized, in our setting, maximization of effort on aggregate requires the playing field to be "unleveled". Private information acts differently than a multiplicative bias because it influences the players' valuations rather than their probabilities of winning.

strategic common-value settings, dynamics and private information, although each detrimental on their own, can be beneficial in combination. For example, in a preemption game, where players aim to be the first to invest but only when investment is lucrative, private information can be welfare improving by counteracting the players motive to invest earlier than in the social optimum (Hopenhayn and Squintani, 2011; Bobtcheff et al., 2021). Similarly, in a strategic experimentation setting, where players can learn from the experimentation of others, private information can mitigate the players' free-riding problem (Heidhues et al., 2015; Dong, 2016; Klein and Wagner, 2022). In our dynamic contest framework the incentive-improving role of private information derives from the fact that battle outcomes induce competitors to update their beliefs in opposite directions which helps to level the playing field.

2 Model

We consider two homogeneous, risk-neutral players engaged in a dynamic contest for a single prize of common value.[10] The prize can take two values, $V \in \{0, 1\}$, and we denote by $\omega \in (0, 1)$ the likelihood that $V = 0$ and by $\mathbb{E}[V] = 1 - \omega$ the expected prize.[11] The contest consists of three identical, consecutive battles and the prize is awarded to the first player achieving a total number of two battle victories.[12] In each battle $t \in \{1, 2, 3\}$, the two players $i \in \{1, 2\}$ choose an effort $e_{it} \geq 0$ simultaneously. A player's payoff equals his prize winnings minus his effort costs aggregated over all battles, i.e. we abstract from discounting. The costs of effort are identical across players and battles and are assumed to be linear, i.e. $C(e_{it}) = e_{it}$. Linearity facilitates comparison with an auction setting and is a natural assumption in contexts where "efforts" consist of financial outlays, such as investments in an R&D race. More importantly, we show below that with linear costs,

[10] The possibility of heterogeneity in prize valuations is introduced in Section 5.

[11] While $V = 1$ is just a normalization, the assumption that the prize may have zero value greatly simplifies our analysis, as will become clear below.

[12] A discussion of the effects of extending the contest to more than three battles is postponed until Section 6.

expected aggregate effort is independent of the players' information in both, the static competition benchmark and the public information benchmark. This allows us to focus on the effect of information on incentives that arises from the players' learning rather than from the curvature of their cost functions.

Competition. We model competition as a multiplicative tournament. More specifically, we assume that each battle t is won by the player with the highest performance (with ties broken randomly) and player i's performance in battle t is given by the product of his effort e_{it} and an individual noise component $x_{it} > 0$. Individual noise is distributed identically and independently across battles and players. Denoting by $H(.)$ the cumulative distribution function of the ratio of individual noise $y_t = \frac{x_{jt}}{x_{it}}$, player i's probability of winning battle t is thus given by $H(\frac{e_{it}}{e_{jt}})$. As equilibrium will be fully determined by the distribution of the ratio of individual noise, we make assumptions directly on the corresponding probability density $h = H'$.[13] Note that from symmetry it follows that $H(y) = 1 - H(\frac{1}{y})$ and differentiating both sides leads to $yh(y) = \frac{1}{y}h(\frac{1}{y})$. The function $yh(y)$, which will play an important role in our analysis of incentives, must thus have a minimum or a maximum at $y = 1$. To guarantee that $y = 1$ constitutes a global maximum and that a pure-strategy equilibrium exists we make the following assumption.

Assumption 1. *The density h of the ratio of individual noise is continuously differentiable and strictly decreasing and the function $yh(y)$ is unimodal with* $\lim_{y\to 0} yh(y) = 0$.

Unimodality is a common assumption in models where performance is additive in effort and noise (e.g. Lazear and Rosen, 1981).[14] A family of densities that satisfy our

[13]Note that two different individual noise distributions, f and $\tilde{f}$, can give rise to the same ratio distribution h, even when f and $\tilde{f}$ differ in their "shape". For example, the distribution of $\frac{x_1}{x_2}$ is given by $h(\frac{x_1}{x_2}) = \frac{1}{(1+\frac{x_1}{x_2})^2}$ when x_1, x_2 are distributed according to $f(x_i) = \exp(-x_i)$ *and* when x_1, x_2 are distributed according to $\tilde{f}(x_i) = \frac{1}{x_i^2}\exp(-\frac{1}{x_i})$, although f is monotone decreasing whereas $\tilde{f}$ has a unique positive mode. It it therefore sensible to consider h as the primitive of our model and to make assumptions about the shape of h rather than the shape of f.

[14]Hodges and Lehmann (1954) show that the distribution of the difference of two unimodal noise distributions must itself be unimodal. Using this result, a straight forward logarithmic transformation shows that $yh(y)$ must be unimodal when the underlying distribution of individual noise is unimodal.

distributional assumptions is given by

$$h_{(d,r)}(y) = \frac{r\Gamma(2\frac{d}{r})}{\Gamma(\frac{d}{r})^2} \frac{y^{-d-1}}{(1+y^{-r})^{2\frac{d}{r}}}, \quad d \in (0,1], \quad r > 0. \tag{1}$$

For $d = r$, these ratio distributions generate the generalized Tullock contest success function $H_r(\frac{e_1}{e_2}) = \frac{e_1^r}{e_1^r + e_2^r}$ (Jia, 2008). They arise when individual noise follows a generalized Gamma distribution (Malik, 1967). Assumption 1 thus not only allows individual noise to follow an exponential ($r = d = 1$) or Weibull distribution ($r = d < 1$), as in the Tullock model, but also accommodates distributions such as the Chi ($r = 2, d < 1$), Chi-squared ($r = 1, d < 1$), or folded-normal ($r = 2, d = 1$), to name just a few. Besides its ability to accommodate the frequently employed Tullock model as a special case, an important advantage of a multiplicative tournament is that a player with a zero effort cannot win against a player with a positive effort, which simplifies Bayesian updating in the dynamic contest considerably. While this property seems a realistic feature of many settings (e.g. innovation requiring investment), it distinguishes our framework from those models where effort and noise are substitutes rather than complements (e.g. Lazear and Rosen, 1981).

Information. Our model captures situations in which contestants have private information about the common value of a contested prize and may learn about their rival's information via the observation of intermediate outcomes. In particular, we assume that after each battle, players observe the identity of the battle's winner, while neither individual performances nor the rival's effort are observable. For example, in procurement contests, such as the Small Business Innovation Research program of the U.S. Department of Defense, organizers often inform firms about the identity of their preferred supplier(s) at intermediate phases, while the scores of firms' proposals as well as the time it took to prepare them remains undisclosed (Bhattacharya, 2021). Similarly, in promotion tournaments, intermediate performance feedback often takes the form of an ordinal rather than cardinal ranking and individual efforts are commonly considered as unobservable (Meyer, 1991).

To model the players' private information about the contest's prize, we assume that prior to the first battle, each player i obtains a private signal, $s_i \in \{B, G\}$, that is informative about the value of V. Signals are independent draws from the same conditional probability distribution $\text{Prob}(s_i|V)$ specified by Table 1. The parameter $\sigma \in (0, 1)$ mea-

$\text{Prob}(s_i\|V)$	$V = 0$	$V = 1$
$s_i = B$	σ	0
$s_i = G$	$1 - \sigma$	1

Table 1: Information structure.

sures the informativeness of the players' signals. In particular, for $\sigma \to 1$ players become perfectly informed about the value of the prize, whereas for $\sigma \to 0$ signals become completely uninformative. Note that implicit in this formulation is the assumption that a "bad" signal $s_i = B$ is conclusive, as it can only be received when $V = 0$. For example, workers competing for a promotion may learn that the position will be filled with an outsider. This assumption, together with the fact that, in this state of the world, the prize has *zero* value, greatly simplifies the analysis because it implies that efforts must be zero upon the observation of a bad signal.[15] Based on the players' prior ω and the above signal structure, parametrized by σ, two variables will play an important role for our analysis. In particular, we let

$$\beta_1 \equiv \text{Prob}(s_j = G|s_i = G) = \frac{1 - \omega + \omega(1 - \sigma)^2}{1 - \omega + \omega(1 - \sigma)} \tag{2}$$

denote a player's belief that, conditional on having received a good signal, the rival's signal is also good, and we let

$$V^G \equiv \mathbb{E}[V|s_1 = s_2 = G] = \frac{1 - \omega}{1 - \omega + \omega(1 - \sigma)^2} \tag{3}$$

be the contest's expected prize, conditional on both signals being good.

[15] Our results are robust to pre-play communication if we assume that signals are non-verifiable, because players have an incentive to report a bad signal, independently of their true signal, making all communication uninformative. With verifiable signals, private information would unravel, because only players with a good signal have an incentive to conceal.

Equilibrium. Our setting constitutes a dynamic Bayesian game, with players' "types" given by their signals. We use Perfect Bayesian equilibrium as our solution concept and focus our analysis on symmetric equilibria in pure strategies. In our model, a symmetric, pure-strategy Perfect Bayesian equilibrium – in the remainder simply denoted as "an equilibrium" – is fully characterized by a vector of efforts $(e_1^*, e_L^*, e_F^*, e_3^*)$ which players exert conditional on having observed a good signal. Here e_1^* and e_3^* denote a player's efforts during the first and the third battle, respectively, whereas e_L^* and e_F^* denote a player's effort in the second battle depending on whether the player has become the leader (L) or follower (F). Note that effort in the third battle is independent of the sequencing of battle-outcomes (win-loss, loss-win) because in equilibrium a player with a good signal will conclude that his rival's signal is good, given that both parties were capable of winning one of the previous battles.

Benchmarking

We now discuss two variations of our model that will serve as benchmarks. In the *static competition benchmark*, all battles take place simultaneously rather than sequentially, ruling out the possibility that players may learn about their rival's signal.[16] In the *public information benchmark* all signals are public rather than private, making learning obsolete.

The following lemma determines expected effort, aggregated over all players and battles, for these two benchmarks. It shows that, in both benchmarks, expected aggregate effort is independent of the contest's information structure and can be fully characterized by the intensity of competition in the contest's individual battle components.

Note, for this purpose, that if the contest consisted of a single battle, and players

[16]In applications, an alternative *no feedback* benchmark can be relevant, where battles are sequential but learning is ruled out because battle outcomes are unobservable. We show in the Appendix that expected aggregate effort in the no feedback benchmark is the same as under static competition. In particular, whether battle 3 takes place for sure or only when battles 1 and 2 result in a draw, has no influence on aggregate incentives.

had homogeneous expectations about the contest's prize, say $\mathbb{E}[V]$, then in equilibrium, efforts would be given by $e^* = \arg\max_{e\geq 0} H(\frac{e}{e^*})\mathbb{E}[V] - e = h(1)\mathbb{E}[V]$ and each player would expect the payoff $U^* = [\frac{1}{2} - h(1)]\mathbb{E}[V]$.[17] Hence we can determine a battle's *rate of rent dissipation* as

$$R \equiv \frac{\mathbb{E}[V] - 2U^*}{\mathbb{E}[V]} = 2h(1). \tag{4}$$

Rent dissipation is high when the impact of effort on performance is strong relative to the impact of noise, which is the case when the distribution of the ratio of individual noise is rather concentrated around $y = 1$. For example, in the Tullock model, where $h(1) = \frac{r}{4}$, rent dissipation is linearly increasing in the Tullock parameter $r > 0$, which provides an inverse measure of the contest's noisiness. It is instructive to express expected aggregate effort in terms of R rather than the underlying primitive $h(1)$:

Lemma 1 (Benchmarks)**.** *In the static competition benchmark, expected aggregate effort is $E^S = \min\{\frac{3}{2}R, 1\} \cdot \mathbb{E}[V]$. In the public information benchmark, expected aggregate effort is strictly lower, i.e. $E^P = [R+(1-R)^2 h(\frac{1-R}{1+R})] \cdot \mathbb{E}[V] < E^S$. In both benchmarks, aggregate incentives thus depend on the individual battle's rate of rent dissipation R but not on the informativeness, σ, of the players' signals.*

An important implication of this result is that in our framework, any effect of σ on aggregate incentives must be due to the players' learning. Lemma 1 is a consequence of the linearity of players' effort cost functions. It justifies this assumption given our objective to understand the effect of learning on incentives.

Finally, note that aggregate incentives in the static competition benchmark are given by E^S, no matter whether signals are private or public. By showing that $E^S > E^P$, Lemma 1 thus proves the existence of a discouragement effect. In the absence of private

[17] This payoff is positive because it follows from Assumption 1 that the function $H(y) - yh(y)$ is strictly increasing, converges to zero for $y \to 0$, and equals $\frac{1}{2} - h(1)$ for $y = 1$.

information about the contest's prize, giving players the opportunity to observe individual battle outcomes leads to a reduction in aggregate incentives.

3 Equilibrium characterization

This section characterizes the unique symmetric pure-strategy Perfect Bayesian equilibrium of the dynamic contest with private information. Existence of such an equilibrium is guaranteed by the following:

Lemma 2 (Equilibrium existence)**.** *A symmetric pure-strategy Perfect Bayesian equilibrium exists and it is unique when players are sufficiently informed or uninformed, i.e. when σ is sufficiently close to* 0 *or* 1*. For the ratio distribution $h_r(y) = \frac{ry^{r-1}}{(1+y^r)^2}$ generating the Tullock contest success function with parameter $r \leq 1$, existence and uniqueness are guaranteed for all σ.*

In light of Lemma 2, our subsequent analysis determines the unique candidate for a symmetric pure-strategy Perfect Bayesian equilibrium. Note, for this purpose, that the analysis of battle 3 is straight forward, because in the last battle players must have symmetric beliefs about the contest's prize. This is because, in equilibrium battle 3 can only be reached when players' signals coincide. If signals differ, then the player with the bad signal and zero effort will lose battles 1 and 2 against the player with the good signal and positive effort, making battle 3 obsolete. In analogy to our single battle analysis in Section 2, players' efforts (conditional on observing a good signal) are thus given by

$$e_3^* = h(1)V^G \tag{5}$$

and the continuation value of reaching the last battle is given by

$$U_3 = [\frac{1}{2} - h(1)]V^G > 0. \tag{6}$$

We proceed in two steps. Section 3.1 determines effort levels in battle 2 with a focus on the difference between the leader's and the follower's incentives. Section 3.2 analyzes incentives in battle 1 and derives the implications for aggregate effort.

3.1 The encouragement effect

In battle 2, a player $i \in \{1, 2\}$ with signal $s_i = G$ updates his belief β_1 about the rival's signal $s_j \in \{B, G\}$ based on whether he has become the contest's leader or follower by winning or losing the previous battle, respectively.

If player i has lost battle 1 with effort $e_1 > 0$, he will conclude that his opponent has observed a good signal. Had his opponent observed a bad signal he would have exerted zero effort and could not have defeated him. Hence, the follower will update his belief in battle 2 from β_1 upwards to

$$\beta_F^* \equiv \text{Prob}(s_j = G | s_i = G, i \text{ lost}) = 1 > \beta_1, \tag{7}$$

i.e. losing the first battle represents "good news". If instead, player i has won battle 1 with effort $e_1 > 0$, then he does not know whether he was simply lucky or whether his opponent failed to provide effort upon observation of a bad signal. More specifically, assuming his opponent employed the equilibrium strategy of exerting effort $e_1^* > 0$ upon observation of a good signal and zero effort after observation of a bad signal, player i would have won the first battle with probability $H(\frac{e_1}{e_1^*})$ in the case where $s_j = s_i = G$ and with certainty in the case where $s_j = B \neq G = s_i$. Hence the leader will update his belief in battle 2 from β_1 downwards to

$$\beta_L(e_1) \equiv \text{Prob}(s_j = G | s_i = G, i \text{ won}) = \frac{\beta_1 H(\frac{e_1}{e_1^*})}{\beta_1 H(\frac{e_1}{e_1^*}) + 1 - \beta_1} < \beta_1, \tag{8}$$

i.e. winning the first battle represents "bad news". Note that, generally, the leader's updated belief in battle 2 depends on the effort e_1 he exerted in battle 1.[18] However, its

[18] The fact that a deviation from e_1^* to $e_1 \neq e_1^*$ influences the informativeness of the first battle's outcome will be taken into account in the determination of the equilibrium effort level e_1^* in Section 3.2.

equilibrium value $\beta_L^* \equiv \beta_L(e_1^*) = \frac{1-\omega+\omega(1-\sigma)^2}{1-\omega+\omega(1-\sigma)^2+2\omega\sigma(1-\sigma)}$ is determined entirely by the values of ω and σ.

In equilibrium, effort choices (e_L^*, e_F^*) must satisfy:

$$e_L^* \in \arg\max_{e_L \geq 0} \beta_L^* \left[U_3 + H(\frac{e_L}{e_F^*})(V^G - U_3) \right] - e_L \tag{9}$$

$$e_F^* \in \arg\max_{e_F \geq 0} H(\frac{e_F}{e_L^*}) U_3 - e_F. \tag{10}$$

By Assumption 1, the above objectives are concave and the corresponding first order conditions lead to the equilibrium values

$$e_F^* = \frac{1+R}{2} \beta_L^* h(\beta_L^* \frac{1+R}{1-R}) V^G \tag{11}$$

$$e_L^* = \frac{1}{2} \frac{[\beta_L^*(1+R)]^2}{1-R} h(\beta_L^* \frac{1+R}{1-R}) V^G. \tag{12}$$

In the Appendix we prove the following result:

Proposition 1 (Encouragement Effect.). *Private information increases the probability that a follower catches up with a leader, i.e.* $\frac{e_L^*}{e_F^*}$ *has U-shape with a minimum at* $\sigma = \hat{\sigma}$ *and* $\lim_{\sigma \to 0} \frac{e_L^*}{e_F^*} = \lim_{\sigma \to 1} \frac{e_L^*}{e_F^*} = \frac{1+R}{1-R} > 1$*, where*

$$\hat{\sigma}(\omega) \equiv \frac{1 - \sqrt{1-\omega}}{\omega} \in (0, 1). \tag{13}$$

For low rates of rent dissipation, private information can make the follower more likely to win the second battle than the leader, i.e. if $R < \hat{R}$ *then* $\frac{e_L^*}{e_F^*} < 1$ *for all* $\sigma \in (\sigma_-, \sigma_+)$ *where*

$$\hat{R}(\omega) \equiv \frac{2 - \omega - 2\sqrt{1-\omega}}{\omega} \in (0, 1), \tag{14}$$

$$\sigma_\pm \equiv \frac{1+R}{2} \pm \sqrt{(\frac{1+R}{2})^2 - \frac{R}{\omega}} \in (0, 1), \tag{15}$$

and the sum of the leader's and the follower's expected effort is maximized when private information "levels the playing field", i.e. when $\sigma \in \{\sigma_-, \sigma_+\}$ *so that* $\frac{e_L^*}{e_F^*} = 1$.

The intuition for Proposition 1 can be obtained from considering the ratio of the leader's and the follower's equilibrium efforts, which determines their probabilities of winning the second battle. From (11) and (12) and given that $\beta_F^* = 1$, this ratio can be written as follows:

$$\frac{e_L^*}{e_F^*} = \frac{\beta_L^*}{\beta_F^*}\frac{1+R}{1-R}. \tag{16}$$

In the limit where players are symmetrically informed or uninformed, i.e. for $\sigma \to 1$ or $\sigma \to 0$, it holds that $\frac{\beta_L^*}{\beta_F^*} \to 1$. The leader then exerts a higher effort than the follower and the contest is more likely to end after two rather than three battles. The follower is discouraged from exerting effort and, intuitively, his discouragement is increasing in the battle's rate of rent dissipation, $R \in (0, 1)$. In contrast, in the presence of private information, i.e. for $\sigma \in (0, 1)$, the leader and the follower update their beliefs about their rival's signal in opposite directions because winning represents bad news whereas losing represents good news, i.e. $\frac{\beta_L^*}{\beta_F^*} < 1$. The follower is then encouraged to exert effort and Proposition 1 describes how this informational encouragement counteracts and sometimes even overcomes the discouraging effect of falling behind. The encouragement effect is stronger than the discouragement effect, making the follower more likely to win the second battle than the leader, if and only if rent dissipation is low ($R < \hat{R}$) and players' information is sufficiently asymmetric ($\sigma_- < \sigma < \sigma_+$). Note that the threshold $\hat{R}(\omega)$ is strictly increasing in the players' prior, ω, and $\lim_{\omega \to 1} \hat{R}(\omega) = 1$. This means that when players are rather pessimistic about the contest's prize, asymmetric information can induce followers to exert more effort than leaders independently of the battles' degree of rent dissipation. Also note that we recover the familiar result from the literature on static contests, that incentives in battle 2 are maximized when private information "levels the playing field". The sum of players' efforts is largest when the leader and the follower have the same incentive to exert effort. This happens when the informativeness of the players' signals is such that the encouragement effect and the discouragement effect are equalized, i.e. when $R < \hat{R}$ and $\sigma \in \{\sigma_-, \sigma_+\}$.

Summarizing, the results in this section suggest that in the presence of private information, dynamic competition does not have to suffer from a discouragement effect. Followers might be as motivated or, in fact, more motivated than leaders. An important implication for *R&D* races is that long-standing concerns about ϵ-preemption (Fudenberg et al., 1983) and suboptimal investments (Harris and Vickers, 1987) might not be warranted. Moreover, our theory identifies a mechanism—the positive updating of beliefs following a rival's success—that might help to understand why in innovative industries, the firm with the initial breakthrough sometimes fails to become the dominant supplier.[19]

3.2 Aggregate incentives

Our results in the previous section suggest that in dynamic contests, private information may have a *positive* effect on incentives. Private information increases the likelihood that the contest's final battle is reached, giving players the opportunity to exert additional efforts. Moreover, for low rates of rent dissipation, private information can level the playing field in an intermediate battle between a leader and a follower, thereby maximizing the sum of their efforts. To understand the effect of private information on incentives *on aggregate*, we now complete our characterization of equilibrium by determining the players' effort choice e_1^* in the contest's opening battle. We then compare aggregate incentives in the equilibrium with the benchmarks of public information and static competition.

In battle 1, a player with a good signal believes that with probability β_1 the rival observed a good signal, and hence the rival's effort is e_1^*, whereas with probability $1-\beta_1$ the rival observed a bad signal, and hence the rival's effort is zero. Denoting the continuation values of the leader and the follower, *conditional on the rival's signal* $s \in \{G, B\}$, by U_L^s

[19]Examples abound in the pharmaceutical industry. For instance, cholesterol lowering compounds, so-called statins, were first discovered by the Japanese company Sankyo, but it was Merck who invested heavily into the development of the best-selling drug Zocor with earnings over one billion US$ (Endo, 2010).

and U_F^s, respectively, the players' equilibrium effort in battle 1 must therefore satisfy:

$$e_1^* \in \arg\max_{e_1>0} \beta_1 \left\{ H(\frac{e_1}{e_1^*})U_L^G(e_1) + [1 - H(\frac{e_1}{e_1^*})]U_F^G \right\} + (1-\beta_1)U_L^B(e_1) - e_1. \tag{17}$$

Here we have used the fact that, conditional on the rival's signal being bad, a player exerting a positive effort $e_1 > 0$ must win battle 1 with certainty.[20] Moreover, it is important to note that the continuation values of becoming the leader, depend on the player's effort choice e_1 through its influence on the leader's belief $\beta_L(e_1)$ in (8). A deviation to $e_1 \neq e_1^*$ changes the player's belief about the rival's signal after winning battle 1 and will thus induce him to adjust his effort e_L in battle 2 optimally. In the proof of Lemma 2 we can thus employ the envelope theorem to show that the first-order condition corresponding to (17) takes the following simple form:

$$e_1^* = \beta_1 h(1)[U_L^G - U_F^G]. \tag{18}$$

Equation (18) shows that when players choose their efforts in battle 1, they evaluate continuation values conditional on their rival having received a good signal, thereby correctly anticipating the winner's curse that the contest's prize is zero when the rival's signal is bad. From

$$U_L^G - U_F^G = H(\frac{e_L^*}{e_F^*})V^G - (e_L^* - e_F^*), \tag{19}$$

we see that incentives in battle 1 derive from the fact that an early success leads to the opportunity to secure overall victory already in the intermediate battle, which happens with probability $H(\frac{e_L^*}{e_F^*})$. It comes at the expense of the future effort differential $e_L^* - e_F^*$ because the effort the player will choose in the subsequent battle when he has become the leader may differ from the effort he will choose when he has become the follower.

Having completed our characterization of equilibrium efforts $(e_1^*, e_L^*, e_F^*, e_3^*)$ we are now ready to consider aggregate incentives, i.e. the expected sum of efforts aggregated over all

[20]The possibility of a deviation to $e_1 = 0$ must be checked separately, because in that case a player will lose battle 1 against a rival with a bad signal with probability $\frac{1}{2}$. See the proof of Lemma 2 for details.

battles and all players. We already know from Proposition 1 that the sum of the leader's and follower's effort is maximized when σ levels the playing field and expected aggregate effort in battle 2 can be written as

$$E_2^* = \text{Prop}(s_1 = s_2 = G)(e_L^* + e_F^*) + \text{Prop}(s_1 \neq s_2)e_L^* = \mathbb{E}[V]\frac{e_L^*}{e_F^*}h(\frac{e_L^*}{e_F^*}). \tag{20}$$

The fact that E_2^* is maximal when $e_L^* = e_F^*$ follows directly from the unimodality of the function $yh(y)$.

It remains to consider expected aggregate effort in battles 1 and 3. The expected sum of players' efforts in battle 1 is $E_1^* = 2\text{Prob}(s_1 = G)e_1^*$. As battle 3 is reached only when both players observe a good signal and when the follower wins the second battle, the expected sum of efforts in battle 3 is given by $E_3^* = 2\text{Prob}(s_1 = s_2 = G)[1 - H(\frac{e_L^*}{e_F^*})]e_3^*$. Substituting efforts the expected sum of efforts in battles 1 and 3 can be expressed as follows:

$$E_1^* + E_3^* = R \cdot \mathbb{E}[V]\left\{1 - \frac{1}{V^G}(e_L^* - e_F^*)\right\}. \tag{21}$$

Note that effort aggregated over battles 1 and 3 is entirely determined by the difference between the leader's and the follower's effort in the intermediate battle. Intuitively, a change in those efforts affects the likelihood that the last battle is reached $[1 - H(\frac{e_L^*}{e_F^*})]$ by the same absolute amount as it influences the likelihood $H(\frac{e_L^*}{e_F^*})$ that securing leadership in battle 1 results in overall victory already in battle 2. In other words, any potential gain in aggregate effort that is due to a higher likelihood of reaching the final battle is exactly compensated by the loss in incentives from a reduction of the benefits of becoming the contest's leader. $E_1^* + E_3^*$ thus consists of a constant term and the contribution of the effort differential $e_L^* - e_F^*$ to battle 1 incentives. A decrease in $e_L^* - e_F^*$ increases the sum of efforts in battle 1 and 3 by making it less costly to become the leader.

In summary, we thus obtain the result, familiar from the literature on static contests, that leveling the playing field is beneficial for efforts in the intermediate battle where the endogenous asymmetry in the dynamic contest's score creates a wedge in player's incentives. It is important to note, however, that in our dynamic setting, leveling the playing field in the intermediate battle has the additional effect of increasing efforts aggregated over the remaining battles, as long as it comes in the form of a reduction in the leader's effort cost differential $e^*_L - e^*_F$.

In the remainder of this section, we compare aggregate incentives in the Perfect Bayesian equilibrium, $E^*(\sigma) = E^*_1 + E^*_2 + E^*_3$, with the two benchmarks of public information, E^P, and static competition, E^S, characterized in Lemma 1.

3.2.1 Comparison with public information benchmark

When comparing aggregate incentives, $E^*(\sigma)$ with the benchmark value, E^P, note first that the former converge to the latter in the limits where the signals' privacy ceases to play a role. In particular, for $\sigma \to 0$ or $\sigma \to 1$, signals become perfectly uninformative or perfectly informative, respectively, and it does not matter whether they are observed privately or publicly. As in the public information benchmark, aggregate incentives, E^P, are independent of σ, it therefore holds that $\lim_{\sigma\to 0} E^*(\sigma) = \lim_{\sigma\to 1} E^*(\sigma) = E^P$. In the Appendix we prove the following:

Proposition 2 (Private vs. Public Information)**.** *In the dynamic contest with private signals, aggregate incentives are higher than in the public information benchmark. In particular, if $R \geq \hat{R}(\omega)$ then $E^*(\sigma) > E^P$ for all $\sigma \in (0, 1)$, while if $R < \hat{R}(\omega)$ then $E^*(\sigma) > E^P$ for all $\sigma \in (0, \sigma_-) \cup (\sigma_+, 1)$.*

When the rate of rent dissipation is high, private information cannot level the playing field in battle 2 fully. In this case, the disadvantage of requiring two rather than only one battle wins for overall victory is too large to be offset by the good news about the contest's prize associated with a loss in battle 1. Private information can then reduce the difference between e^*_L and e^*_F but will never induce the follower to exert as much effort as the leader. Because private information can only make the playing field more leveled, aggregate effort is larger than in the public information benchmark, independently of the signals' precision.

In contrast, if the rate of rent dissipation is low, then the encouragement effect can overcome the discouragement effect and private signals can make the follower exert a higher effort than the leader. In this case, information can happen to be "too asymmetric" and only sufficiently informative or sufficiently uninformative signals are guaranteed to increase aggregate incentives above the public information benchmark.

3.2.2 Comparison with static competition benchmark

Based on the insight of Proposition 2 that private information can be employed to increase aggregate incentives, a natural question to ask if whether the corresponding gain in incentives due to encouragement, $E^*(\sigma) - E^P > 0$ can be sufficient to overcome the loss in incentives, $E^S - E^P > 0$, that is due to the discouragement effect. Our next result determines the conditions under which the answer to this question is affirmative.

Proposition 3 (Dynamic vs. Static Competition)**.** *When the rate of rent dissipation is low, aggregate incentives can be strictly higher in the dynamic contest with private signals than in the static benchmark. Formally, if $R < \hat{R}(\omega)$, then there exist a $\sigma^* \in (0, 1)$ such that $E^*(\sigma^*) > E^S$.*

Proposition 3 contrasts with the common wisdom that the dynamic nature of competition must be harmful for incentives (e.g. Klumpp and Polborn, 2006). Our benchmark analysis in Section 2 has confirmed that this intuition applies to our setting when players are symmetrically informed about the contest's prize. However, as shown by Proposition 3, dynamics need not necessarily be harmful for incentives when players are endowed with private information. When signals are private rather than public, the discouragement effect can be overcome by the encouragement effect, leading not only to an increase in the incentives of a follower beyond the incentives of a leader but to an increase in *aggregate* incentives beyond the static benchmark.

The proof of Proposition 3 shows that, when the follower can be induced to exert higher effort than the leader for *some* level σ of the signals' informativeness, then at its maximized value, aggregate effort must be higher than in the static benchmark. To understand this result, remember from Proposition 1 that for low rates of rent dissipation, the discouragement effect can be compensated by the encouragement effect. If σ is chosen such that the leader's and the follower's efforts become equal, then the contest is equally likely to be decided after two or three battles. Hence, the only difference between winning or losing the first battle is the corresponding change in the contest's intermediate score. Incentives at every stage of the dynamic contest then become equal to static incentives, leading to $E^*(\sigma) = E^S$. Decreasing the leader's effort below the follower's effort by a small margin has a zero first-order effect on E_2^* (because E_2^* is maximized when $e_L^* = e_F^*$) but a positive first-order effect on $E_1^* + E_3^*$, making aggregate incentives in the dynamic contest strictly larger than in the static benchmark. Intuitively, the static contest can be improved upon, because in the dynamic contest, additional incentives can be created by establishing an appropriate link between battle outcomes and the players' learning of their rival's signal. In particular, if $e_F^* > e_L^*$, then winning battle 1 not only establishes a lead, but allows the winning player to maintain the belief that his rival's signal might be bad, thereby reducing his future effort cost.

On a more abstract level, Proposition 3 is reminiscent of Milgrom and Weber's (1982) result that in a common value setting with affiliated signals, expected revenue is (weakly) higher in a dynamic, English auction than in a static, first-price auction. Note, however, that in our environment, both auction formats generate the same expected revenue given by $\mathbb{E}[V]$.[21] This shows that, while, generally, dynamic formats benefit both auction- and

[21]Given our informational assumptions, player i will bid zero upon observation of $s_i = B$ and t upon observation of $s_i = G$. Using arguments familiar from the all-pay auction literature (Baye et al., 1996) it is straightforward to show that in the unique symmetric equilibrium of a first-price sealed bid auction t is distributed according to the cdf $F(t) = \frac{\omega(1-\sigma)\sigma t}{1-\omega-[1-\omega+\omega(1-\sigma)^2]t}$ and expected revenue equals $\mathbb{E}[V] = 1 - \omega$. Because expected revenue in an English auction is weakly higher due to the linkage principle, expected revenues in both auctions formats have to be identical.

contest-designers through enabling players to learn about each others' private information, they do so for different reasons. While in dynamic auctions, learning mitigates the winner's curse that induces bidders to shade their bids when bidding is static, in dynamic contests learning reduces the future effort costs of early winners, thereby raising the players' incentives to establish a lead.

4 Information design

In this section, we characterize the signal quality that maximizes aggregate incentives in dependence of the contest's rate of rent dissipation, R, and the contestants' prior, ω. Our analysis shows that, when information can be used as a design variable, a contest designer will use it to fine-tune the likelihood with which initial leaders will become final winners, thereby resolving a basic trade-off between short but fierce and mild but long-lasting fights.

While our results contribute to a nascent but growing literature on information design in contests (discussed below), we should emphasize that our assumption of partially conclusive signals poses a restriction on the set of posteriors that can be induced.[22] More specifically, besides the usual requirement of Bayes plausibility, in our model posteriors satisfy $\text{Prob}(V = 1|s_i = B) = 0$.[23] In our setting an "information structure" is therefore fully determined by the posterior $\text{Prob}(V = 1|s_i = G) = \frac{1-\omega}{1-\omega+\omega(1-\sigma)}$, parametrized by the signal quality σ, and our following result characterizes the information structure that maximizes aggregate incentives within the set of all such partially conclusive information structures. Importantly, this set contains fully informative ($\sigma = 1$) and fully uninformative ($\sigma = 0$) information structures as extreme cases, which means that with respect to

[22] An alternative simplification of the information design problem can be achieved by assuming information to be verifiable and to focus on the designer's choice between disclosure and concealment (e.g. Serena, 2021).

[23] In the seminal contribution of Kamenica and Gentzkow (2011) and related articles, the optimal information structure turns out to be partially conclusive, i.e. the restriction to such structures might be less restrictive than it appears.

our subsequent claims about the optimality of a partially revealing information structure, our assumption is without loss of generality.

Proposition 4 (Information design)**.** *In the dynamic contest with private signals, the signal quality σ^* that maximizes aggregate incentives, $E^*(\sigma)$, depends on the contest's rate of rent dissipation, $R = 2h(1)$ and the contestants' prior $\omega = Prob(V = 0)$ as follows:*

- *If $R \geq \hat{R}(\omega)$ then $E^*(\sigma)$ has inverted U-shape and the optimal signal is $\sigma^* = \hat{\sigma}$ as defined in (13). Optimal signals induce the contest to be more likely to be decided after two rather than three battles, i.e. $\frac{e_L^*}{e_F^*} > 1$. More pessimistic priors require more accurate information, i.e. $\hat{\sigma}(\omega)$ is strictly increasing with $\lim_{\omega\to 0} \hat{\sigma}(\omega) = \frac{1}{2}$ and $\lim_{\omega\to 1} \hat{\sigma}(\omega) = 1$.*

- *If $R < \hat{R}(\omega)$ then $E^*(\sigma)$ is strictly increasing in $(0, \sigma_-]$ and strictly decreasing in $[\sigma_+, 1)$ and the optimal signal quality satisfies $\sigma^* \in (\sigma_-, \sigma_+)$. Optimal signals induce the contest to be more likely to be decided after three rather than two battles, i.e. $\frac{e_L^*}{e_F^*} < 1$.*

The threshold $\hat{R}(\omega)$ is strictly increasing with $\lim_{\omega\to 0} \hat{R}(\omega) = 0$ and $\lim_{\omega\to 1} \hat{R}(\omega) = 1$.

Proposition 4 is illustrated in Figure 3. The figure reveals a dichotomy of optimal contest designs. For high rates of rent dissipation and relatively optimistic priors, optimal signals create fierce but short fighting. The incentive maximizing contest is characterized by a large likelihood ($> 50\%$) to be decided within only two battles and high initial efforts e_1^* to become the contest's leader. In contrast, for low rates of rent dissipation and relatively pessimistic priors, optimal signals lead to longer lasting but less fierce fighting.

Note that, independently of the contest's rate of rent dissipation and the contestants' prior, dynamic incentives are maximized when private information is neither perfectly informative nor perfectly uninformative. This is a direct implication of Proposition 2

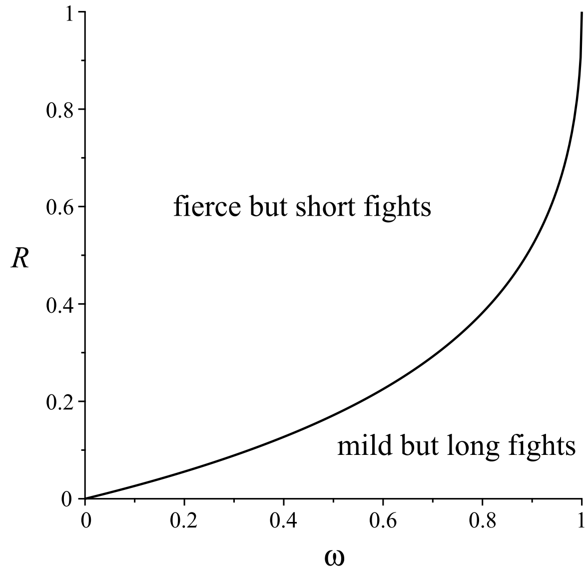

Figure 1: **Information Design**: The incentive maximizing signal quality σ^* induces either fierce but short or mild but long-lasting fights, depending on the individual battles' rate of rent dissipation $R = 2h(1)$ and the contestants' prior $\omega = \text{Prob}(V = 0)$. The diagram depicts the critical value $\hat{R}(\omega) = \frac{2-\omega-2\sqrt{1-\omega}}{\omega}$.

which has shown that private information is beneficial for aggregate incentives. As for $\sigma \to 0$ and $\sigma \to 1$, the distinction between public and private information becomes obsolete, aggregate incentives must be maximized by some $\sigma^* \in (0, 1)$, i.e. the optimal information structure is *partially revealing*. The reason for this result is that for the encouragement effect to exist, contestants must be able to update their beliefs about the contest's prize in opposite directions, depending on their observation of a battle-win or battle-loss, respectively, which is impossible when information is symmetric.

Proposition 4 adds to an ongoing discussion about the potential optimality of partially revealing information structures in contests. While the seminal article by Zhang and Zhou (2016) has mainly advocated fully informative (or fully uninformative) information structures, more recent articles have rationalized the use of information structures that are only partially revealing (Chen, 2021; Clark and Kundu, 2021a; Clark and Kundu, 2021b; Kuang et al., 2019; Melo-Ponce, 2020). Our setting distinguishes itself from most existing work by allowing asymmetric information to be two-sided and the contest to be dynamic. Probably most closely related is Antsygina and Teteryatnikova (2022) who consider a two-player static all-pay auction with linear costs where both players' valuations are binary and ex ante uncertain. They allow for information technologies that send messages to players privately or publicly and show that the optimal information structure features private signals and induces symmetric beliefs. This structure reveals the state whenever both players valuations are identical but employs noisy and correlated signals when valuations differ. Intuitively, the designer tries to make players believe that their valuations are likely to be equal, because effort is largest when valuations are identical. As in our setting, information is thus used to "level the playing field", but the incentive-deteriorating heterogeneity emerges from exogenous differences in prize valuations rather than endogenous differences in intermediate scores.

5 Selective efficiency

In this section, we extend our analysis to allow for possible differences in the contestants' prize valuations. This enables us to consider the effect of private information on the contest's selective efficiency, i.e. the probability with which the contest's prize is allocated to the highest valuing contestant. Selective efficiency is a valid concern in light of Proposition 1 which has shown that private information encourages contestants who lag behind. Because a low-valuation contestant is more likely to be lagging behind than a high-valuation contestant, private information may therefore have an adverse effect on selective efficiency. However, as we show in this section, this reasoning is incomplete, as it neglects the fact that the intensity with which private information improves a contestant's incentives also depends on the contestant's valuation. More specifically, we identify conditions under which private information can have a positive effect on selective efficiency. This allows us to conclude that the gain in aggregate incentives from private information identified by Proposition 2 does not necessarily come at the cost of a reduction in selective efficiency.

Selective efficiency is especially relevant in promotional contests where, besides the provision of incentives, the selection of the most "able" candidate constitutes an important objective. In such settings, "ability" is commonly interpreted as the inverse of a contestant's constant marginal cost of effort. In this section, we thus introduce heterogeneity by allowing contestants to differ in their marginal costs of effort but it should be noted, that, because prizes and costs enter linearly into our model, our approach is equivalent to allowing for differences in the contestants' valuation of the contest's prize.

We thus extend our model by assuming that costs of effort are $C^i(e_{it}) = c^i e_{it}$ for $i \in \{l, h\}$ and that one contestant has a lower marginal cost than the other, i.e. we let $\frac{c^h}{c^l} \equiv \gamma > 1$. A super-index will be used throughout the analysis to denote the contestants' cost-types. To keep our model tractable, we assume that contestants observe whether they are the low-cost contestant l or the high-cost contestant h only *after* they have competed once by exerting effort in the first battle.[24] In some applications, such as promotion tournaments, where workers are ignorant of their abilities relative to their rivals initially, this assumption may be a reasonable starting point. In other settings, where abilities are known right from the start, our subsequent results remain valid when ability differences are sufficiently small.

Selective efficiency, i.e. the probability that the low-cost (high-ability) contestant wins the contest is given by

$$S \equiv \frac{1}{2} \cdot [H(\frac{e_L^l}{e_F^h}) + H(\frac{e_F^h}{e_L^l})H(\frac{e_3^l}{e_3^h})] + \frac{1}{2} \cdot H(\frac{e_F^l}{e_L^h})H(\frac{e_3^l}{e_3^h}). \tag{22}$$

The two terms represent the cases where the low-cost type has won or lost the first battle, respectively. Both cases are equally likely because, given our assumptions, contestants

[24] When contestants are heterogeneous in battle 1 they will exert differing efforts, so that equilibrium beliefs in battle 2 will depend on past efforts, which means that the model can no longer be solved recursively.

will exert identical efforts in the first battle.

Efforts and expected payoffs in battle 3 are straightforward to calculate and given by

$$e_3^l = \gamma \cdot \frac{V^G}{c^l} h(\gamma) > \frac{V^G}{c^l} h(\gamma) = e_3^h, \tag{23}$$

$$U_3^l = [H(\gamma) - \gamma h(\gamma)]V^G > [H(\frac{1}{\gamma}) - \gamma h(\gamma)]V^G = U_3^h. \tag{24}$$

In the second battle, we have to distinguish between two cases. If the low-cost contestant has become the leader, equilibrium efforts must solve

$$e_L^l \in \arg\max_{e \geq 0} U_3^l + \beta_L^*(V^G - U_3^l)H(\frac{e}{e_F^h}) - c^l e \tag{25}$$

$$e_F^h \in \arg\max_{e \geq 0} U_3^h H(\frac{e}{e_L^l}) - c^h e \tag{26}$$

and it follows that

$$\frac{e_L^l}{e_F^h} = \gamma \beta_L^* \frac{V^G - U_3^l}{U_3^h}. \tag{27}$$

Similarly, if the high-cost contestant has become the leader, we get

$$\frac{e_L^h}{e_F^l} = \frac{1}{\gamma} \beta_L^* \frac{V^G - U_3^h}{U_3^l}. \tag{28}$$

Substitution of (24), (27), and (28) into (22) gives a closed form expression for selective efficiency $S(\gamma, \sigma)$ in dependence of the contestants' cost differential γ and their signals' informativeness σ:

$$S(\gamma, \sigma) = \frac{1}{2}[H(\gamma) + H\left(\beta_L^*(\sigma)\gamma \frac{V^G - U_3^l}{U_3^h}\right) H(\frac{1}{\gamma})] + \frac{1}{2} H\left(\frac{\gamma}{\beta_L^*(\sigma)} \frac{U_3^l}{V^G - U_3^h}\right) H(\gamma). \tag{29}$$

We obtain the following result:

Proposition 5 (Selective Efficiency)**.** *Private information can improve the contest's selective efficiency. Formally, for all* $\gamma > 1$ *there exist* $\sigma^{min}, \sigma^{max} \in (0, 1)$ *such that for all* $\sigma \in (0, \sigma^{max}) \cup (\sigma^{min}, 1)$*, selective efficiency* $S(\gamma, \sigma)$ *is strictly larger than in the public information benchmark.*

To understand the intuition for this result consider the effect of a marginal reduction in the leader's equilibrium belief β_L^* starting from its public information benchmark value $\beta_L^* = 1$, which can be achieved via the introduction of private information. Lowering the leader's equilibrium belief raises the likelihood with which the second battle is won by the lagging contestant. This decreases selective efficiency when the low-cost contestant is in the lead but increases selective efficiency when the low-cost contestant has fallen behind. From (29), the resulting change in selective efficiency is $\Delta S = \Delta S^+ - \Delta S^-$ where

$$\Delta S^+ = \frac{1}{2}\gamma\frac{U_3^l}{V^G - U_3^h}h(\gamma\frac{U_3^l}{V^G - U_3^h})H(\gamma) > 0, \tag{30}$$

$$\Delta S^- = \frac{1}{2}\gamma\frac{V^G - U_3^l}{U_3^h}h(\gamma\frac{V^G - U_3^l}{U_3^h})H(\frac{1}{\gamma}) > 0. \tag{31}$$

In the proof of Proposition 5 we show that $\Delta S^+ > \Delta S^-$. Intuitively, the encouragement effect is stronger for a lagging low-cost contestant than for a lagging high-cost contestant. When the low-cost contestant is equally likely to be lagging as the high-cost contestant, which happens when abilities are initially unknown or when differences in abilities are small, the overall effect is an increase in selective efficiency.

6 Discussion and conclusion

In this article, we have identified the encouragement effect as a novel aspect of dynamic competition with private information. Before we summarize our main message and its implications, a discussion of the model's main assumptions is in order. While our model has put few restrictions on the "shape of competition" by allowing for rather generic "contest success functions", the assumed information structure and the focus on a best-of-three contest deserve some commenting. We have relaxed both assumptions in variations of our model and were able to show that our basic insight, that learning improves incentives in dynamic contests, is not an artifact of the partial conclusiveness of our signals nor of our contest's short and finite horizon. Here we only summarize our findings while details are available on request.

Non-conclusive signals. To lend tractability to our model, we have assumed that a bad signal is conclusive, in that it allows players to conclude that the contest's prize has no value. As an alternative, one could consider a model with "good news" where players can conclude from the observation of a good signal that the contest's prize must be valuable. In strategic experimentation settings, good news models (e.g. Keller et al., 2005) produce different investment and learning dynamics than bad news models (e.g. Keller and Rady, 2015; Bonatti and Hörner, 2017). Although a "bad news" model appears to be the most conservative in light of our result that private information *improves* incentives under dynamic competition, a thorough investigation of dynamic incentives in a contest with good news is important. Similar to strategic experimentation settings, good news may lead to novel equilibrium features, whose analysis is beyond the scope of the present paper and is thus left for future research. Nevertheless, to check the robustness of our finding that private information improves incentives, we have analyzed a model with symmetric signals where $\text{Prob}(s_i = B|V = 0) = \text{Prob}(s_i = G|V = 1) \in (0, 1)$. Assuming efforts to be binary with costs $C(0) = 0$ and $C(1) = c$, we were able to confirm that private information induces an upwards shift of the cost-interval for which players are induced to exert effort conditional on the observation of a good signal. This is reassuring as it shows that learning improves incentives also when signals are non-conclusive.

Longer or infinite horizons. While our analysis has focused on a best-of-three contest, where the gap between the leader and the follower can take only one value, we know from empirical studies that the discouragement effect can become more pronounced when this gap is widened. As a consequence, one would expect the discouragement effect to have a heavier toll on incentives in contests with longer horizons. Given that, in equilibrium, players will conclude that their rival's signal is good as soon as they have been defeated in a single battle, our model maintains its tractability when extended to more than three battles. Surprisingly, in a best-of-five Tullock contest, the effect of private information on aggregate incentives turns out to be even more positive than in a best-of-three contest.

More specifically, we have confirmed that the relative gain in incentives due to information being private rather than public is larger in a best-of-five contest than in a best-of-three contest, *independently* of the signals' informativeness. Finally, we have also analyzed a potentially infinite contest where the prize is awarded once a player has established a two-battle lead. Assuming future payoffs to be discounted with discount factor $\delta \in (0, 1)$, and denoting by e_0^* players' equilibrium efforts when their score is equal and by e_{+1}^*, e_{-1}^* players' efforts when one player has taken a one-battle lead, it is straight forward to show that $\frac{e_{+1}^*}{e_{-1}^*} = \frac{\beta_{+1}^*}{\beta_{-1}^*} \frac{1+R}{1-R}$ and $e_0^* = \delta h(1)[H(\frac{e_{+1}^*}{e_{-1}^*})V^G - (e_{+1}^* - e_{-1}^*)]$. This means that the main equations of our model, i.e. (16) and (18), remain largely unchanged which indicates that our results are not driven by the assumption that the contest's horizon is finite.

We are thus confident to conclude, that in a dynamic contest, private information about the contest's prize, has a positive effect on aggregate incentives. Under private information, the discouraging effect of falling behind is offset by the encouraging effect of learning about a rival's information. As an important consequence, the common wisdom that dynamics have to be harmful for incentives, may not be correct. In the presence of private information, aggregate incentives in a dynamic contest can be even greater than in the static benchmark. Our results contrast with the existing literature on dynamic contests that has mostly abstracted from the potential privacy of information. They shed new light on a broad variety of applications, by showing, for example: that lagging firms can be more motivated to invest in an R&D race than leading firms; that wasteful campaign spending can be reduced if presidential primaries were held simultaneously rather than sequentially; and that feedback policies in labor tournaments can have a positive effect not only on workers' incentives but also on the likelihood of promoting the most able candidate.

Appendix

Proof of Lemma 1. In this proof we derive the equilibria of the static competition and public information benchmarks. We also consider an alternative benchmark where competition is dynamic but battle outcomes are unobservable, i.e. there exists no feedback.

Static competition benchmark: In the following, we derive the unique symmetric equilibrium of the static competition benchmark. Suppose that conditional on having received a good signal, players exert effort $e^S > 0$ in each battle. From the viewpoint of a player with a good signal, the contest's prize can have non-zero value only when the rival's signal is good as well, which happens with probability β_1 given by (2). Moreover, in any particular battle, a player's effort influences his chance of winning only if there is a draw in the remaining two battles, which, given symmetry, happens with probability $\frac{1}{2}$. In equilibrium e^S must therefore solve

$$e^S \in \arg\max_{e \geq 0} \frac{1}{2}\beta_1 H(\frac{e}{e^S})V^G - e, \tag{32}$$

with V^G given by (3). Taking the first order condition of (32) and setting $e = e^S$ gives $e^S = \frac{1}{2}\beta_1 V^G h(1)$ as the unique candidate for a symmetric pure-strategy equilibrium. Summing efforts over both players and all battles, and multiplying with the probability that $s_i = G$ gives $3h(1)\mathbb{E}[V]$ as the corresponding expected aggregate effort. Note that the corresponding equilibrium payoff of each player is $\frac{1}{2}\beta_1 V^G[1 - 3h(1)]$, i.e. existence of a pure-strategy equilibrium in the static contest requires $h(1) < \frac{1}{3}$. For larger values of $h(1)$, equilibrium must be in mixed strategies and, in analogy to Klumpp and Polborn (2006), equilibrium features full rent-dissipation, i.e. expected aggregate effort must equal the expected prize $\mathbb{E}[V]$. In summary, we thus have

$$E^S = \min\{3h(1), 1\} \cdot \mathbb{E}[V]. \tag{33}$$

If signals were observed publicly rather than privately, then, conditional on $s_1 = s_2 = G$ both players would exert efforts $e = \frac{1}{2}V^G h(1)$ and it follows from the fact that Prob$(s_1 =$

$s_2 = G) = \text{Prob}(s_i = G)\beta_1$ that expected aggregate effort would be the same as in (33).

Public information benchmark: In the following, we characterize the unique pure-strategy Subgame Perfect equilibrium of the public information benchmark. We use $(e_1^P, e_L^P, e_F^P, e_3^P)$ to denote players' effort levels conditional on $s_1 = s_2 = G$. Our characterization can restrict attention to first order conditions because our assumption that h is decreasing guarantees the concavity of players' objectives. Using backwards induction, equilibrium in battle 3 can be described in analogy to our single-battle analysis in Section 2. As players expect the prize to be V^G given by (3) we thus have $e_3^P = h(1)V^G$ and a player's continuation payoff from reaching battle 3 is $U_3 = [\frac{1}{2} - h(1)]V^G$, which is strictly positive due to Assumption 1 (see footnote 17). In battle 2, the follower's valuation of winning is given by U_3 whereas the leader's valuation of winning battle 2 is $V^G - U_3 = [\frac{1}{2} + h(1)]V^G > U_3$. In a Subgame Perfect equilibrium (e_L^P, e_F^P) must therefore solve

$$e_L^P \in \arg\max_{e_L \geq 0} U_3 + (V^G - U_3)H(\frac{e_L}{e_F^P}) - e_L \tag{34}$$

$$e_F^P \in \arg\max_{e_F \geq 0} U_3[1 - H(\frac{e_L^P}{e_F})] - e_F. \tag{35}$$

The first order conditions following from (34) and (35) have a unique solution given by

$$e_F^P = \frac{1+R}{2}h(\frac{1+R}{1-R})V^G \tag{36}$$

$$e_L^P = \frac{1+R}{1-R}e_F^P, \tag{37}$$

with R given by (4). The corresponding continuation payoffs from entering battle 2 as the leader or the follower are

$$U_L^G = U_3 + [H(\frac{1+R}{1-R}) - \frac{1-R}{1+R}h(\frac{1-R}{1+R})](V^G - U_3) > U_3 \tag{38}$$

$$U_F^G = [H(\frac{1-R}{1+R}) - \frac{1-R}{1+R}h(\frac{1-R}{1+R})]U_3 > 0, \tag{39}$$

where the inequalities follow from the fact that $H(\frac{1+R}{1-R}) > H(\frac{1-R}{1+R})$ and because $H(y) > yh(y)$ for all $y > 0$ by Assumption 1 (see footnote 17). Finally, in battle 1 players' have identical valuations of winning, $U_L^G - U_F^G$, and choose their effort to solve

$$e_1^P \in \arg\max_{e_1 \geq 0} U_F^G + H(\frac{e_1}{e_1^P})(U_L^G - U_F^G) - e_1 \tag{40}$$

leading to

$$e_1^P = (U_L^G - U_F^G)h(1) = [H(\frac{1+R}{1-R}) - 2h(1)\frac{1-R}{1+R}h(\frac{1-R}{1+R})]h(1)V^G > 0. \tag{41}$$

The corresponding equilibrium payoff is strictly positive because each player can guarantee himself a payoff of $U_F^G > 0$ by choosing $e_1 = 0$. Aggregating expected efforts over all three battles and both players gives

$$\begin{aligned} E^P &= \text{Prob}(s_1 = s_2 = G)[2e_1^P + e_L^P + e_F^P + 2H(\frac{1-R}{1+R})e_3^P] \\ &= [R + (1-R)^2 h(\frac{1-R}{1+R})] \cdot \mathbb{E}[V]. \end{aligned} \tag{42}$$

Unobservable battle outcomes. Consider a variation of our dynamic contest model with private signals where battle outcomes are unobservable. In the following, we determine the equilibrium effort levels e_1^U, e_2^U, and e_3^U, which players, conditional on having observed a good signal, choose in battles 1, 2, and 3, respectively. Since battle 3 is reached only when both players have won exactly one battle, efforts in battle 3 are the same as in the model with observable battle outcomes, i.e. $e_3^U = e_3^*$, given by (5). In battle 2, a player with a good signal is uncertain whether he has won or lost the first battle. His effort therefore solves

$$e_2^U \in \arg\max_{e_2 \geq 0} \beta_1\{\frac{1}{2}[H(\frac{e_2}{e_2^U})(V^G - U_3) + U_3] + \frac{1}{2}H(\frac{e_2}{e_2^U})U_3\} - e_2, \tag{43}$$

with U_3 denoting the continuation payoff from reaching battle 3. Solving this program leads to the same effort as in the static benchmark, i.e. $e_2^U = e^S$. Finally, in battle 1, a player with a good signal chooses effort to solve

$$e_1^U \in \arg\max_{e_1 \geq 0} \beta_1\{H(\frac{e_1}{e_1^U})U_L^G + [1 - H(\frac{e_1}{e_1^U})]U_F^G\} + (1-\beta_1)U_L^B - e_1, \tag{44}$$

where $U_L^G = \frac{1}{2}V^G + \frac{1}{2}U_3 - e_2^U$, $U_F^G = \frac{1}{2}U_3 - e_2^U$, and $U_L^B = -e_2^U$, denote the continuation payoffs of reaching battle 2 as the leader or follower (without knowing it), conditional on the rival's signal. Solving this program we again find $e_1^U = e^S$. Expected aggregate effort is thus given by

$$E^U = 2\text{Prob}(s_i = G)(e_1^U + e_2^U) + 2\text{Prob}(s_1 = s_2 = G)\frac{1}{2}e_3^U = 3h(1)\mathbb{E}[V]. \quad (45)$$

Comparison with E^S in (33) shows that with unobservable battle outcomes, expected aggregate effort is the same as in the static competition benchmark, as long as the latter allows for a pure-strategy equilibrium, i.e. for all $h(1) \leq \frac{1}{3}$. □

Proof of Lemma 2. We first show that, in battle 1, the first order condition corresponding to the players' objective function (17) takes the simple form in (18). For this purpose, note that we can substitute continuation values $U_L^G(e_1)$ and $U_L^B(e_1)$ to rewrite the term $\beta_1 H(\frac{e_1}{e_1^*})U_L^G(e_1) + (1-\beta_1)U_L^B(e_1)$ as

$$[\beta_1 H(\frac{e_1}{e_1^*}) + 1 - \beta_1]\{\beta_L(e_1)[U_3 + H(\frac{e_L(e_1)}{e_F^*})(V^G - U_3)] - e_L(e_1)\}. \quad (46)$$

The term in parentheses equals the battle 2 objective of a player who deviated in battle 1 by choosing e_1 and happened to become the leader. More precisely, such a player will choose

$$e_L(e_1) \in \arg\max_{e_L} \beta_L(e_1)[U_3 + H(\frac{e_L}{e_F^*})(V^G - U_3)] - e_L \quad (47)$$

in battle 2. Since $e_L(e_1)$ maximizes the above objective, it follows from the envelope theorem that the derivative with respect to e_1 of the term in parentheses in (46) must be zero. Hence, the derivative of the battle 1 objective in (17) with respect to e_1 is given by

$$\beta_1 h(\frac{e_1{}^*}{e_1})\frac{1}{e_1^*}[U_L^G(e_1) - U_F^G] - 1 \quad (48)$$

and evaluation at $e_1 = e_1^*$ leads to the simple first order condition in (18). Together with the analysis contained in Section 3.1, this shows that $(e_1^*, e_L^*, e_F^*, e_3^*)$ defined by (5), (11), (12), and (18), is the unique candidate for a symmetric pure-strategy Perfect Bayesian equilibrium.

A comment is in order concerning the fact that the maximization program in (17) restricts the players' choice to strictly positive effort levels $e_1 > 0$. We now show that a deviation to $e_1 = 0$ is dominated by a deviation to $e_1 = \epsilon$ for $\epsilon > 0$ sufficiently small, which implies that neglecting the possibility of zero effort in (17) comes without loss of generality. Treating the possibility of zero effort separately is necessary because Bayesian updating in the case where $e_1 = 0$ differs from Bayesian updating in the case where $e_1 > 0$. More precisely, consider an equilibrium with $e_1^* > 0$, and suppose a player deviates to $e_1 = 0$. If the deviating player wins the first battle he learns that his rival must have received the signal B, i.e. $\beta_L^0 = 0$. Instead, if the deviating player loses the first battle, he will update his belief to $\beta_F^0 = \frac{\beta_1}{\beta_1 + \frac{1}{2}(1-\beta_1)}$ and then choose an effort $e_F^0 \in \arg\max_{e_F} \beta_F^0 H(\frac{e_F}{e_L^*})U_3 - e_F$. The payoff from a deviation to zero effort in battle 1 is thus given by

$$U_1^0 = \beta_1[H(\frac{e_F^0}{e_L^*})U_3 - e_F^0] - (1-\beta_1)\frac{1}{2}e_F^0. \tag{49}$$

Instead, a deviation to $e_1 = \epsilon$ gives the payoff

$$U_1^\epsilon = \beta_1\{H(\frac{\epsilon}{e_1^*})U_L^G(\epsilon) + [1 - H(\frac{\epsilon}{e_1^*})]U_F^G\} + (1-\beta_1)U_L^B(\epsilon) - \epsilon. \tag{50}$$

After winning battle 1, a player who deviated from an equilibrium $e_1^* > 0$ by exerting only a small effort in battle 1 must be nearly certain that his rival has observed a bad signal. Formally, for $\epsilon \to 0$ it holds that $\beta_L(\epsilon) \to 0$ and thus $e_L(\epsilon) \to 0$. Hence, for $\epsilon \to 0$, it holds that

$$U_1^\epsilon \to \beta_1 U_F^G = \beta_1[H(\frac{e_F^*}{e_L^*})U_3 - e_F^*] \geq U_1^0, \tag{51}$$

and the inequality follows from the fact that $e_F^* \in \arg\max_{e_F} H(\frac{e_F}{e_L^*})U_3 - e_F$. Intuitively, although a player can achieve that a win in battle 1 reveals the rival's signal perfectly by choosing $e_1 = 0$, the player can do even better because when choosing an infinitesimal effort $e_1 = \epsilon$, the rival's signal becomes revealed not only by a win (approximately) but also by a loss in battle 1.

Finally, to prove existence of equilibrium it remains to consider second order conditions. We first consider the case where the distribution of the ratio of noise is given by $h_r = \frac{ry^{-r-1}}{(1+y^{-r})^2}$ generating the generalized Tullock contest success function with parameter r. Nti (1999) shows that in a static Tullock contest a pure strategy equilibrium exists if and only if $r \leq 1 + v^r$ where $v \in (0,1]$ denotes the contestants' ratio of valuations of winning. Our contest is dynamic rather than static, but using continuation values we were able to write each battle in the form of a static Tullock contest. The contestants have identical valuations of winning in battles 1 and 3, i.e. valuations differ only in battle 2 where $v = \frac{U_3}{\beta_L^*(V^G - U_3)}$. v is minimized when signals are public, i.e. for $\beta_L^* = 1$. Note that in contrast to Nti (1999), our contest features imperfect information. However, because contestants exert zero efforts after observing a bad signal, the conditions for a pure strategy Perfect Bayesian equilibrium are just an analogue of the equilibrium conditions in Nti (1999). Since for h_r we find $U_3 = (\frac{1}{2} - \frac{r}{4})V^G$ and $V^G - U_3 = (\frac{1}{2} + \frac{r}{4})V^G$, a pure strategy Perfect Bayesian equilibrium thus exists for all σ if and only if

$$r \leq 1 + (\frac{2-r}{2+r})^r. \tag{52}$$

As this inequality is satisfied for all $r \leq 1$ we have thus shown existence of equilibrium for the family of Tullock contest success functions with parameters $r \leq 1$. The equilibrium is unique and can be determined in closed form as:

$$e_3^* = \frac{rV_G}{4} \tag{53}$$

$$e_L^* = \frac{rV_G}{4}\beta_L^*(2+r)\chi \tag{54}$$

$$e_F^* = \frac{rV_G}{4}(2-r)\chi \tag{55}$$

$$e_1^* = \frac{rV_G}{4}\beta_1\chi\{(\beta_L^*\frac{2+r}{2-r})^r + 1 - \frac{r}{4}[\beta_L^*(2+r) - 2 + r]\} \tag{56}$$

where we abbreviated notation by defining $\chi \equiv \frac{(\beta_L^*)^r(2+r)^r(2-r)^r}{[(\beta_L^*)^r(2+r)^r+(2-r)^r]^2}$.

While for the Tullock family, equilibrium existence is guaranteed for all $\sigma \in [0,1]$, that is, *independently* of the informativeness of the contestants' signals, for general distributions of the ratio of noise, existence is harder to establish. In the remainder of this proof we show that, under the conditions of Assumption 1, an equilibrium exists when contestants' information is "sufficiently public", that is when σ is sufficiently close to 0 or 1.

To see this, first note that the players' objective in battle 3, as well as the leader's and the follower's objectives in battle 2, given by (9) and (10), are globally concave because $h = H'$ is assumed to be strictly decreasing. For the remaining battle 1, the second order condition which guarantees that e_1^* constitutes a maximizer can be obtained by calculating the derivative of (48) with respect to e_1 at $e_1 = e_1^*$. The second order condition requires

$$\frac{h'(1)}{\beta_1 h(1)} + h(1)\frac{dU_L^G(e_1^*)}{de_1} < 0 \tag{57}$$

with $U_L^G - U_F^G > 0$ given by (19) and

$$\begin{aligned} U_L^G(e_1) &= U_3 + H(\frac{e_L(e_1)}{e_F^*})(V^G - U_3) - e_L(e_1), && (58)\\ e_L(e_1) &\in \arg\max_{e_L \geq 0} \beta_L(e_1)[U_3 + (V^G - U_3)H(\frac{e_L}{e_F^*})] - e_L. && (59) \end{aligned}$$

We get

$$\frac{dU_L^G(e_1^*)}{de_1} = [h(\frac{e_L^*}{e_F^*})(V^G - U_3) - 1]\frac{de_L(e_1^*)}{de_1} = \frac{1-\beta_L^*}{\beta_L^*}\frac{de_L(e_1^*)}{de_1} \tag{60}$$

where we have used the fact that e_L^* solves the first order condition $\beta_L^* h(\frac{e_L^*}{e_F^*})(V^G - U_3) = 1$. As $e_L(e_1)$ satisfies an analogue first order condition with β_L^* substituted by $\beta_L(e_1)$, we can employ the Implicit Function Theorem to get

$$\frac{de_L(e_1^*)}{de_1} = -\frac{h(\frac{e_L^*}{e_F^*})e_F^*}{h'(\frac{e_L^*}{e_F^*})\beta_L^*}\frac{d\beta_L(e_1^*)}{de_1} \tag{61}$$

Note from (8) that $\frac{d\beta_L(e_1^*)}{de_1}$ is positive but tends to zero for $\sigma \to 0$ and for $\sigma \to 1$. As $\beta_L^* \to 1$ in both cases, we can thus conclude from $h' < 0$ that the second order condition in (57) must be satisfied when σ is sufficiently close to 0 or 1. □

Proof of Proposition 1. Given that $\beta_F^* = 1$,

$$\frac{e_L^*}{e_F^*} = \frac{1+R}{1-R}\beta_L^* = \frac{1+R}{1-R}\frac{1-\omega+\omega(1-\sigma)^2}{1-\omega+\omega(1-\sigma)^2+2\omega\sigma(1-\sigma)} \tag{62}$$

and it follows that

$$\lim_{\sigma\to 0}\frac{e_L^*}{e_F^*} = \lim_{\sigma\to 1}\frac{e_L^*}{e_F^*} = \frac{e_L^P}{e_F^P} = \frac{1+R}{1-R} > 1. \tag{63}$$

Moreover, the derivative

$$\frac{d}{d\sigma}[\frac{e_L^*}{e_F^*}] = \frac{1+R}{1-R}\frac{2\omega(2\sigma-1-\omega\sigma^2)}{(1-\omega\sigma^2)^2} \tag{64}$$

has a unique root in $(0,1)$ at $\sigma = \hat{\sigma}(\omega)$ defined in (13), is negative for $\sigma \in (0, \hat{\sigma}(\omega))$ and positive for $\sigma \in (\hat{\sigma}(\omega), 1)$. Hence $\frac{e_L^*}{e_F^*}$ has U-shape with a minimum at $\sigma = \hat{\sigma}(\omega)$. Its minimized value is

$$\frac{e_L^*}{e_F^*}|_{\sigma=\hat{\sigma}(\omega)} = \frac{1+R}{1-R}\frac{\sqrt{1-\omega}-(1-\omega)}{1-\sqrt{1-\omega}}. \tag{65}$$

It follows that $\frac{e_L^*}{e_F^*} < 1$ for a non-empty interval (σ_-, σ_+) if and only if

$$\frac{1+R}{1-R}\frac{\sqrt{1-\omega}-(1-\omega)}{1-\sqrt{1-\omega}} < 1 \Leftrightarrow R < \hat{R}(\omega), \tag{66}$$

with $\hat{R}(\omega)$ as defined in (14). The thresholds σ_- and σ_+ solve the equation $\frac{e_L^*}{e_F^*} = 1$ and are given by (15). That the sum of the leader's and the follower's expected effort is maximized when $\sigma \in \{\sigma_-, \sigma_+\}$ follows directly from (20) and the fact that the function $yh(y)$ has a unique maximum at $y = 1$. □

Proof of Proposition 2. To abbreviate notation, define $\rho \equiv \frac{1-R}{1+R}$. Consider

$$E^* = E_1^* + E_3^* + E_2^* = 2(1-\omega)h(1)[1 - \frac{1}{V^G}(e_L^* - e_F^*)] + (1-\omega)\frac{e_L^*}{e_F^*}h(\frac{e_L^*}{e_F^*}). \tag{67}$$

Substitution of e_F^* and e_L^* from (11) and (12) gives

$$E^* = 2(1-\omega)\{h(1) + \frac{\beta_L^*}{\rho}h(\frac{\beta_L^*}{\rho})[\frac{1}{2} - \frac{\beta_L^*-\rho}{1+\rho}h(1)]\}. \tag{68}$$

From $\frac{1-\rho}{1+\rho} = R = 2h(1)$ it follows that $\lim_{\beta_2^* \to 1} E^* = 2(1-\omega)\{h(1) + \rho h(\rho)[\frac{1}{2} - 2h(1)^2]\} = E^P$ where we have used the fact that by symmetry $\frac{1}{\rho}h(\frac{1}{\rho}) = \rho h(\rho)$. Defining the function $g(y) \equiv yh(y)$ and denoting its derivative by g' we can write

$$\frac{1}{2(1-\omega)}\frac{dE^*}{d\beta_L^*} = \frac{1}{\rho}g'(\frac{\beta_L^*}{\rho})[\frac{1}{2} - \frac{\beta_L^* - \rho}{1+\rho}h(1)] - \frac{1}{1+\rho}g(\frac{\beta_L^*}{\rho})h(1). \tag{69}$$

Note that it follows from $\beta_L^* \leq 1$ and from $h(1) < \frac{1}{2}$ that

$$[\frac{1}{2} - \frac{\beta_L^* - \rho}{1+\rho}h(1)] \geq [\frac{1}{2} - \frac{1-\rho}{1+\rho}h(1)] = \frac{1}{2} - 2h(1)^2 > 0. \tag{70}$$

As $g(.)$ is unimodal with a mode at 1, it must therefore hold that $\frac{dE^*}{d\beta_L^*} < 0$ for all $\beta_L^* > \rho$. The Proposition then follows from the fact that $\beta_L^*(\sigma)$ is U-shaped with $\lim_{\sigma \to 0} \beta_L^* = \lim_{\sigma \to 1} \beta_L^* = 1$ and takes its minimum value $\min_{\sigma \in (0,1)} \beta_L^* = \frac{\sqrt{1-\omega}-(1-\omega)}{1-\sqrt{1-\omega}}$ at $\sigma = \hat{\sigma}(\omega)$ and that this minimum value is smaller than ρ if and only if $R < \hat{R}(\omega)$. □

Proof of Proposition 3. Suppose that $R < \hat{R}(\omega)$. Then according to the proof of Proposition 2 it holds that $\frac{dE^*}{d\beta_L^*} < 0$ for all $\beta_L^* > \rho = \frac{1-R}{1+R}$, or equivalently $\frac{e_L^*}{e_F^*} > 1$, i.e. E^* is strictly increasing in σ for all $\sigma \in (0, \sigma_-]$ and strictly decreasing in σ for all $\sigma \in [\sigma_+, 1)$ where $0 < \sigma_- < \sigma_+ < 1$ are the thresholds defined in the proof of Proposition 1. Hence, there must exist a $\sigma^* \in (\sigma_-, \sigma_+)$ such that $E^*(\sigma^*) > \max(E^*(\sigma_-), E^*(\sigma_+))$. Note that, as $R < \hat{R}(\omega)$ implies that $h(1) < \frac{1}{3}$, we can write

$$E^*(\sigma) - E^S = \frac{1}{2}[\frac{\beta_L^*}{\rho}h(\frac{\beta_L^*}{\rho}) - h(1)] - \frac{\beta_L^*}{\rho}h(\frac{\beta_L^*}{\rho})h(1)\frac{\beta_L^* - \rho}{1+\rho}. \tag{71}$$

The result then follows from the fact that $E^*(\sigma_-) = E^*(\sigma_+) = E^S$ which holds because by the definition of the thresholds, at $\sigma = \sigma_-$ and $\sigma = \sigma_+$ it holds that $e_L^* = e_F^*$, or equivalently $\beta_L^* = \rho$. □

Proof of Proposition 4. For $R \geq \hat{R}(\omega)$, $E^*(\sigma)$ inherits its shape from $\beta_L^*(\sigma)$, because $\frac{dE^*}{d\beta_L^*} < 0$ for $\beta_L^* > \rho = \frac{1-R}{1+R} \Leftrightarrow \frac{e_L^*}{e_F^*} > 1$ and because the follower cannot be induced to exert higher effort than the leader, independently of σ, as shown in the proof of Proposition

2. For $R < \hat{R}(\omega)$, the proof of Proposition 3 has shown that $E^*(\sigma)$ must be maximized at a $\sigma^* \in (\sigma_-, \sigma_+)$ and since the thresholds σ_- and σ_+ are defined by the requirement that $e_L^* = e_F^*$, at $\sigma = \sigma^*$ it must hold that $e_F^* > e_L^*$. It thus remains to consider the comparative statics:

$$\frac{d\hat{R}}{d\omega} = \frac{2 - \omega - 2\sqrt{1-\omega}}{\omega^2\sqrt{1-\omega}} > 0 \tag{72}$$

because the nominator is increasing in ω for $\omega \in (0, 1)$ and converges to zero for $\omega \to 0$. For the same reason it holds that

$$\frac{d\hat{\sigma}}{d\omega} = \frac{2 - \omega - 2\sqrt{1-\omega}}{2\omega^2\sqrt{1-\omega}} > 0. \tag{73}$$

□

Proof of Proposition 5. Consider $-\frac{\partial S}{\partial \beta_L^*}|_{\beta_L^*=1} = \Delta S^+ - \Delta S^-$ with ΔS^+ and ΔS^- given by (30) and (31), respectively. Note first that $\gamma > 1$ implies that $H(\gamma) > H(\frac{1}{\gamma})$. Remember that the function $yh(y)$ is unimodal with a unique maximum at $y = 1$ and that $yh(y) = \frac{1}{y}h(\frac{1}{y})$. As

$$\frac{V^G - U_3^l}{U_3^h} = \gamma\frac{H(\frac{1}{\gamma}) + \gamma h(\gamma)}{H(\frac{1}{\gamma}) - \gamma h(\gamma)} > 1 \tag{74}$$

it holds that $\gamma\frac{V^G - U_3^l}{U_3^h} > 1$ and it is thus sufficient for $\Delta S^+ > \Delta S^-$ that

$$[\gamma\frac{V^G - U_3^l}{U_3^h}]^{-1} < \gamma\frac{U_3^l}{V^G - U_3^h} < \gamma\frac{V^G - U_3^l}{U_3^h}. \tag{75}$$

The second inequality follows directly from

$$\frac{U_3^l}{V^G - U_3^h} = \frac{H(\gamma) - \gamma h(\gamma)}{H(\gamma) + \gamma h(\gamma)} < 1. \tag{76}$$

For the first inequality note that $\frac{U_3^l}{V^G - U_3^h} > \frac{U_3^h}{V^G - U_3^l}$ if and only if

$$H(\gamma) - \gamma h(\gamma) - [H(\gamma) - \gamma h(\gamma)]^2 > H(\frac{1}{\gamma}) - \gamma h(\gamma) - [H(\frac{1}{\gamma}) - \gamma h(\gamma)]^2. \tag{77}$$

This inequality is satisfied because the terms $H(\gamma) - \gamma h(\gamma)$ and $H(\frac{1}{\gamma}) - \gamma h(\gamma)$ lie between zero and one and the former is closer to $\frac{1}{2}$ than the latter. We have thus shown that $\Delta S^{+} > \Delta S^{-}$, or equivalently, $\frac{\partial S}{\partial \beta_L^*}|_{\beta_L^*=1} < 0$. Using private information to reduce the leader's belief marginally below his belief in the public information benchmark has a positive effect on selective efficiency. $\square$

References

Antsygina, Anastasia and Mariya Teteryatnikova, "Optimal information disclosure in contests with stochastic prize valuations," *Economic Theory*, 2022.

Aoyagi, Masaki, "Information feedback in a dynamic tournament," *Games and Economic Behavior*, 2010, *70* (2), 242–260.

Awaya, Yu and Vijay Krishna, "Startups and Upstarts : Disadvantageous Information in R&D," *Journal of Political Economy*, 2021, *129* (2), 534–569.

Barbieri, Stefano and Marco Serena, "Biasing Dynamic Contests between Ex-Ante Symmetric Players," *Games and Economic Behavior*, (forthcoming).

Baye, M, D Kovenock, and C de Vries, "The All-Pay Auction with Complete Information," *Economic Theory*, 1996, *8* (2), 1–328.

Baye, Michael R. and Heidrun C. Hoppe, "The strategic equivalence of rent-seeking, innovation, and patent-race games," *Games and Economic Behavior*, 2003, *44* (2), 217–226.

Bhattacharya, Vivek, "An Empirical Model of R&D Procurement Contests: An Analysis of the DOD SBIR Program," *Econometrica*, 2021, *89* (5), 2189–2224.

Bimpikis, Kostas, Shayan Ehsani, and Mohamed Mostagir, "Designing dynamic contests," *Operations Research*, 2019, *67* (2), 339–356.

Bobtcheff, Catherine, Raphaël Levy, and Thomas Mariotti, "Negative Results in Science: Blessing or (Winner's) Curse?," *CEPR Discussion Paper*, 2021.

Bonatti, Alessandro and Johannes Hörner, "Learning to disagree in a game of experimentation," *Journal of Economic Theory*, 2017, *169*, 234–269.

Chan, William, Pascal Courty, and Li Hao, "Suspense : Dynamic Incentives in Sports Contests," *The Economic Journal*, 2009, *119* (534), 24–46.

Chen, Zhuoqiong, "Optimal information exchange in contests," *Journal of Mathematical Economics*, 2021, *96*, 102518.

Choi, Jay P, "Dynamic R&D Competition under " Hazard Rate " Uncertainty," *RAND Journal of Economics*, 1991, *22* (4), 596–610.

Clark, Derek J. and Tapas Kundu, "Competitive balance: Information disclosure and discrimination in an asymmetric contest," *Journal of Economic Behavior and Organization*, 2021, *184*, 178–198.

_ and _ , "Partial information disclosure in a contest," *Economics Letters*, 2021, *204*, 109915.

Dong, Miaomiao, "Strategic experimentation with asymmetric information," *Unpublished Manuscript*, 2016.

Drugov, Mikhail and Dmitry Ryvkin, "Hunting for the discouragement effect in contests," *Review of Economic Design*, 2022, (forthcoming).

Dubey, Pradeep, "The role of information in contests," *Economics Letters*, 2013, *120* (2), 160–163.

Ederer, Florian, "Feedback and motivation in dynamic tournaments," *Journal of Economics and Management Strategy*, 2010, *19* (3), 733–769.

Einy, Ezra, Diego Moreno, and Benyamin Shitovitz, "The value of public information in common-value Tullock contests," *Economic Theory*, 2017, *63* (4), 925–942.

Ely, Jeffrey, George Georgiadis, Sina Moghadas Khorasani, and Luis Rayo, "Optimal Feedback in Contests," *CEPR Discussion paper*, 2021.

Endo, Akira, "A historical perspective on the discovery of statins," *Proceedings of the Japan Academy Series B: Physical and Biological Sciences*, 2010, *86* (5), 484–493.

Ferrall, C and A Smith, "A Sequential Game Model of Sports Championship Series : Theory and Estimation," *The Review of Economics and Statistics*, 1999, *81* (4), 704–719.

Fu, Qiang, Qian Jiao, and Jingfeng Lu, "Disclosure policy in a multi-prize all-pay auction with stochastic abilities," *Economics Letters*, 2014, *125* (3), 376–380.

Fudenberg, Drew, Richard Gilbert, Joseph Stiglitz, and Jean Tirole, "Preemption, Leapfrogging, and Competition in Patent Races," *European Economic Review*, 1983, *22*, 3–31.

Gershkov, Alex and Motty Perry, "Tournaments with midterm reviews," *Games and Economic Behavior*, 2009, *66* (1), 162–190.

Goltsman, Maria and Arijit Mukherjee, "Interim performance feedback in multistage tournaments: The optimality of partial disclosure," *Journal of Labor Economics*, 2011, *29* (2), 229–265.

Harris, Christopher and John Vickers, "Racing with uncertainty," *Review of Economic Studies*, 1987, *54* (1), 1–21.

Heidhues, Paul, Sven Rady, and Philipp Strack, "Strategic experimentation with private payoffs," *Journal of Economic Theory*, 2015, *159*, 531–551.

Hodges, J and E. Lehmann, "Matching in Paired Comparisons," *The Annals of Mathematical Statistics*, 1954, *25* (4), 787–791.

Hopenhayn, Hugo A. and Francesco Squintani, "Preemption games with private information," *Review of Economic Studies*, 2011, *78* (2), 667–692.

Hurley, Terrance M. and Jason F. Shogren, "Asymmetric information contests," *European Journal of Political Economy*, 1998, *14* (4), 645–665.

Iqbal, Hamzah and Alex Krumer, "Discouragement effect and intermediate prizes in multi-stage contests: Evidence from Davis Cup," *European Economic Review*, 2019, *118*, 364–381.

Jia, Hao, "A stochastic derivation of the ratio form of contest success functions," *Public Choice*, 2008, *135*, 125–130.

Judd, Kenneth, Karl Schmedders, and Sevin Yeltekin, "Optimal Rules for Patent Races," *International Economic Review*, 2012, *53* (1), 23–52.

Kamenica, Emir and Matthew Gentzkow, "Bayesian persuasion," *American Economic Review*, 2011, *101* (6), 2590–2615.

Keller, Godfrey and Sven Rady, "Breakdowns," *Theoretical Economics*, 2015, *10*, 175–202.

_ , _ , and Martin Cripps, "Strategic experimentation with exponential bandits," *Econometrica*, 2005, *73* (1), 39–68.

Klein, Arnd Heinrich and Armin Schmutzler, "Optimal effort incentives in dynamic tournaments," *Games and Economic Behavior*, 2017, *103*, 199–224.

Klein, Nicolas and Peter Wagner, "Strategic Investment and Learning with Private Information," *Journal of Economic Theory*, 2022, (forthcoming).

Klumpp, Tilman and Mattias K. Polborn, "Primaries and the New Hampshire Effect," *Journal of Public Economics*, 2006, *90* (6-7), 1073–1114.

Konrad, Kai, *Strategy and Dynamics in Contests*, Oxford, UK: Oxford University Press, 2009.

_ **and Dan Kovenock**, "Multi-battle contests," *Games and Economic Behavior*, 2009, *66* (1), 256–274.

Kuang, Zhonghong, Hangcheng Zhao, and Jie Zheng, "Information Design in All-pay Auction Contests," *Unpublished Manuscript*, 2019.

Lazear, Edward P . and Sherwin Rosen, "Rank-Order Tournaments as Optimum Labor Contracts," *The Journal of Political Economy*, 1981, *89* (5), 841–864.

Lemus, Jorge and Guillermo Marshall, "Dynamic tournament design: Evidence from prediction contests," *Journal of Political Economy*, 2021, *129* (2), 383–420.

Mago, Shalom D., Roman M. Sheremeta, and Andrew Yates, "Best-of-three contest experiments: Strategic versus psychological momentum," *International Journal of Industrial Organization*, 2013, *31* (3), 287–296.

Malik, Henrick John, "Exact distribution of the quotient of independent generalized gamma variables," *Canadian Mathematical Bulletin*, 1967, *10*, 463–465.

Malueg, D. and S. Tsutsui, "Dynamic R&D Competition with Learning," *Rand Journal of Economics*, 1997, *28* (4), 751–772.

Malueg, David and Andrew Yates, "Testing Contest Theory : Evidence from Best-of-Three Tennis Matches," *The Review of Economics and Statistics*, 2010, *92* (3), 689–692.

Melo-Ponce, Alejandro, "The Secret Behind The Tortoise and the Hare: Information Design in Contests," *Unpublished Manuscript*, 2020, pp. 1–77.

Meyer, Margaret A., "Learning from coarse information: Biased contests and career profiles," *Review of Economic Studies*, 1991, *58* (1), 15–41.

Milgrom, Paul R. and Robert J. Weber, "A Theory of Auctions and Competitive Bidding," *Econometrica*, 1982, *50* (5), 1089.

Morath, Florian and Johannes Münster, "Private versus complete information in auctions," *Economics Letters*, 2008, *101* (3), 214–216.

Moscarini, Giuseppe and Francesco Squintani, "Competitive experimentation with private information: The survivor's curse," *Journal of Economic Theory*, 2010, *145* (2), 639–660.

Nti, Kofi O, "Rent-Seeking with Asymmetric Valuations," *Public Choice*, 1999, *98* (3), 415–430.

Perry, Motty and Philip Reny, "On the Failure of the Linkage Principle in Multi-Unit Auctions," *Econometrica*, 1999, *67* (4), 895–900.

Serena, Marco, "Harnessing beliefs to optimally disclose contestants' types," *Economic Theory*, 2021.

Tullock, G., "Efficient rent seeking," in J. Buchanan, R. Tollison, and G. Tullcok, eds., *Toward a theory of the rent- seeking society*, College Station, Texas: A&M Press, 1980.

Wärneryd, Karl, "Information in conflicts," *Journal of Economic Theory*, 2003, *110* (1), 121–136.

Wasser, Cédric, "Incomplete information in rent-seeking contests," *Economic Theory*, 2013, *53* (1), 239–268.

Zhang, Jun and Junjie Zhou, "Information Disclosure in Contests: A Bayesian Persuasion Approach," *Economic Journal*, 2016, *126* (597), 2197–2217.

Zizzo, Daniel John, "Racing with uncertainty: A patent race experiment," *International Journal of Industrial Organization*, 2002, *20* (6), 877–902.