The Distribution of Talent across Contests*

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Abstract

Do the contests with the largest prizes attract the most-able contestants? To what extent do contestants avoid competition? We show that the distribution of abilities is crucial in determining contest choice. Complete sorting exists only when the proportion of high-ability contestants is small. As this proportion increases, high-ability contestants shy away from competition and sorting decreases, making reverse sorting a possibility. We test our theoretical predictions with a large panel data set containing contest choice over twenty years. We use exogenous variation in the participation of highly-able competitors to provide evidence for the relationship among prizes, competition, and sorting.

JEL *classification*: L20, M52, D02
Keywords: Contests, competition, sorting, incentives.

Competition is a defining feature of most economic and social environments. Contestants of differing ability compete for valuable but limited resources by exerting effort. In many cases, contestants choose from a variety of potential contests. For example, architects choose design competitions; pharmaceutical companies select R&D contests; athletes pick sports tournaments; and college graduates apply for positions in firms. In these settings, contests typically differ in the way in which (relative) performance is rewarded.

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†We thank seminar participants at Bocconi, Collegio Carlo Alberto, CREST, LSE, McGill, Stanford GSB, St. Gallen, and Toronto for valuable comments and suggestions. Our work benefited from discussions with Winfried Aufenanger, former coach of the German national marathon team and current organizer of the Kassel city marathon. We are especially grateful to Ken Young from the Association of Road Racing Statisticians for providing us with the data.

Rewarding contestants according to their relative performance is motivated by two objectives: the provision of incentives to exert effort and the attraction of the most-able participants. Lazear and Rosen (1981) were the first to consider rank-order tournaments as a way to provide incentives. Since then, a large theoretical literature has been developed, determining the optimal design of such tournaments.[1] A common theme in this literature is that contestants exert greater efforts when prizes are larger and more concentrated towards the highest ranks.[2] Ehrenberg and Bognanno (1990), using data on golf contests, and Eriksson (1999) and Bognanno (2001), studying labor tournaments, have provided empirical evidence for these incentive effects.

While the relationship between prizes and effort seems to be well understood, comparatively little is known about their influence on contest selection.[3] For other incentive schemes, which use absolute rather than relative performance evaluation, selection effects have been found to be as important as incentive effects. Lazear (2000) documents a 44-percent increase in productivity for a firm switching from salaries to piece rates and attributes half of this increase to selection effects. High-ability workers find firms offering piece rates more attractive than firms offering salaries. In the context of tournaments, it remains an open question whether selection effects play a similarly important role.

Contest selection is complicated by its multidimensional and interdependent nature. Contests may differ, not only in the size, but also the number of their prizes, making prize-concentration and its effect on competition an important consideration for contest choice. Moreover, a contestant's set of opponents is endogenously determined through his rivals' participation decisions, creating the possibility of multiple equilibria. Existing models of contest choice have either assumed that each contest awards a single prize (Leuven et al. 2010) or that all contestants are homogeneous (Azmat and Möller 2009, Konrad and Kovenock, 2012). In this paper, we relax both of these assumptions by proposing a simple, illustrative model of contest selection with multiple prizes and heterogeneous contestants, featuring a unique equilibrium. We show that the contests with the largest prizes attract the highest number of talented contestants only when talent is relatively scarce. In contrast, when talent exists in abundance, the contests with the least concentrated prize allocation become most attractive. To the best of our knowledge, our model is the first to provide this tight link between the distributions of prizes and talent within

[1]For an extensive survey, see Konrad (2009).

[2]Exceptions to this rule exist when contestants are risk-averse (Krishna and Morgan, 1998) or effort costs are sufficiently convex (Moldovanu and Sela, 2001).

[3]A notable exception are models in which contest selection is independent of effort considerations either because effort choices are absent (Damiano, Li, and Suen 2010, 2012) or because contests (and hence effort costs) are approximately identical (Morgan, Sisak, and Várdy, 2015).

and across contests.

In our setting, two types of contestants (high- and low-ability), choose between two types of contests (high- and low-type). High-ability contestants have lower (constant) marginal costs of effort than low-ability contestants. Contests differ in their prize structure. High-type contests are characterized by high prizes and high prize concentration, whereas low-type contests are characterized by low-prizes and low-prize concentration. More specifically, high-type contests offer a small number of large prizes, whereas low-type contests offer a large number of small prizes. We show that the probability with which a high-ability contestant participates in a high-type, rather than a low-type, contest is decreasing in the overall proportion of high-ability contestants. When high-ability contestants become sufficiently numerous, sorting is reversed, that is, high-ability contestants are more likely to enter low-type contests than high-type contests.

At first glance, the possibility of reverse sorting seems counterintuitive since, in this case, contestants are attracted by contests with smaller prizes *and* stronger opponents. The intuition is that low-type contests become more attractive since they mitigate competition by spreading out their prize budget. As a consequence, the contestants' effort costs are lower in low-type contests than in high-type contests. The interaction between effort choices and contest selection underlines the importance of incorporating *both* elements into models of tournament theory.

Empirically testing for selection effects is often difficult, if not impossible. In a labor-market setting, for example, it is difficult to establish firm and worker types, and, quite often, measuring individual performance is complicated or confounded by a number of factors. It is also difficult to obtain information about the full range of workers' outside options, as well as their counterfactual earnings. Moreover, an exogenous shock to the pool of talent, allowing for the study of its effects on sorting, rarely exists.

In this paper, we take advantage of an unusually clean opportunity to investigate the extent of sorting across contests in a sports setting. Using extensive panel data, we examine the contest choices of professional marathon runners. Our setup contains all the relevant ingredients needed to test the predictions of our model. Individual performance is readily available, together with complete information on contest and runner characteristics. This allows us to abstract from a number of identification problems present in other types of data.

There are two important features that make marathons the ideal setting in which to study contest choice. First, five Major marathons (Berlin, Boston, Chicago, London, and New York) have a special status comparable to the Grand Slam tournaments in tennis.

They offer much higher prizes than other marathons and, on average, allocate a considerably greater proportion of their prize money to the winner. This allows us to identify a runner's decision between competing in a Major or a Minor marathon, as a choice between high-type and low-type contests. Second, highly-talented East-African runners, mainly from Kenya and Ethiopia, dominate the sport of marathon running. This dominance is striking and unparalleled in other sports. For example, according to the International Association of Athletics Federations' (IAAF) Top List, the 50 fastest male marathon runners in 2012 were exclusively from Kenya or Ethiopia. This endows us with a proxy of the contestants' abilities (runners' origin), which, unlike performance measures (finishing times), is independent of effort and prize considerations. More importantly, it allows us to use exogenous variation in local economic conditions to predict the participation of high-ability runners (Brückner and Ciccone, 2011).

We find that the likelihood that a high-ability runner will participate in a given marathon is increasing in the race's prize budget but decreasing in the expected number of high-ability opponents. The participation of one additional high-ability opponent must be compensated by a $\$2,583$ increase in the contest's average prize to keep the race equally attractive to high-ability runners. In line with our model, we find that the concentration of a race's prize structure has a positive effect on participation when opposition is expected to be weak, but has a negative effect when opposition is expected to be strong. This is important since it establishes that selection and incentive effects are either aligned or opposed, depending on the overall competitiveness of the environment.

Our paper uses a simple theoretical framework to illustrate that complete sorting exists in tournaments only when the proportion of high-ability participants is sufficiently small.[4] Our empirical findings constitute first evidence for tournament selection effects in a real setting.[5] In line with our main theoretical result, we find that, when the overall ability distribution shifts upwards, potential participants become more likely to avoid competition. In particular, when the proportion of talented contestants increases by ten percent, the likelihood with which any one of them participates in a Major rather than a Minor race falls by around seven percent. These results suggest that, depending on the ability distribution and prize structure, contestants avoid one another to the extent that

[4] From a theoretical perspective, assortative matching in the labor market has been extensively studied in non-tournament settings (see, for example, Eeckhout and Kircher, 2011; Shimer and Smith, 2000).

[5] Sorting has been the focus of several recent empirical studies in settings such as the labor market (Bagger and Lentz, 2012; Lise et al., 2016; Lopes de Melo, 2013) or school choice (Urquiola, 2005). Experimental studies have also considered sorting across single-prize tournaments (Leuven et al., 2011) and the choice between tournaments and alternative incentive schemes (Dohmen and Falk, 2011; Eriksson et al., 2009).

reverse sorting becomes a possibility.

Our results have the following implications for contest design: When contest choice is endogenous, selection effects cannot be neglected, and the optimal prize allocation depends crucially on the distribution of abilities among potential contestants. This holds true, regardless of whether the objective is to maximize aggregate output or the winner's performance since prizes affect both the quality of the field and the incentives to exert effort. More importantly, selection effects can be diametrically opposed to incentive effects, and the positive influence of concentrated prize allocations on efforts may be more than compensated by their negative influence on the self-selection of talented contestants.

1 Theoretical Framework

We present a simple theoretical framework to illustrate the effect of changes in the ability distribution on the level of sorting across contests. The model demonstrates that the provision of strong incentives increases participation of talented contestants, but that talent crowds out talent. The model makes precise how these two factors interact, resulting, first, in a negative relation between the frequency of high abilities and the level of sorting and, second, in the possible existence of reverse sorting.

1.1 *Model*

We assume a continuum of contests and a continuum of risk-neutral players.[6] Contests allow for the same number $N+1$ of participants. The integer N indicates a player's number of opponents. We let $N \geq 2$ to guarantee that in each contest the number of players is larger than the number of prizes. In order to balance the number of players with the number of available contest slots, we assume that there exists a mass 1 of players and a mass $\frac{1}{N+1}$ of contests.

There are two types of contests, *high-type* contests and *low-type* contests. A contest of type $j \in \{l, h\}$ offers $M_j \in \{1, 2, \ldots, N\}$ prizes, identical in size, $b_j > 0$.[7] High-type con-

[6]In such a setting, with many contests and a large number of players, a single player's action has no effect on the optimal contest choice of the remaining players. This rules out coordination issues, dominant in settings with a small number of contests and players (Amegashie and Wu, 2004), and guarantees the uniqueness of equilibrium. The implications of risk aversion are discussed at the end of the section.

[7]The assumption that, within a given contest, all prizes are identical makes the model tractable. A general description of competition for the case of heterogeneous players and non-identical prizes is still missing. Bulow and Levin (2006), Cohen and Sela (2008), Xiao (2016), and Olszewsky and Siegel (2016) are first steps in this direction. We discuss the effect of skewed prize distributions on contest choice after stating our main thoeretical result.

tests award fewer ($M_h < M_l$) but larger ($b_h > b_l$) prizes than low-type contests.[8] In other words, high-type contests are characterized by high prizes and high prize concentration, whereas low-type contests are characterized by low-prizes and low-prize concentration. Note that we do not make any restrictions with respect to the comparison of the contests' overall prize budgets. However, for the purpose of the subsequent comparative statics analysis, we define an increase in a prize structure's concentration to be an increase in b_j accompanied by a decrease in M_j, keeping the overall prize budget $M_j b_j$ unchanged. Apart from the differences in their prize structures, high-type and low-type contests are assumed to be identical. For simplicity, we assume that both types exist in equal proportions.[9]

There are two types of players, low-ability players and high-ability players, $i \in \{L, H\}$. A high-ability player's (constant) marginal cost of effort, $c_H > 0$, is strictly smaller than a low-ability player's marginal cost, $c_L > c_H$. To abbreviate notation, we define $c \equiv \frac{c_H}{c_L} \in (0, 1)$. The crucial parameter of the model is the proportion of high-ability players, denoted by y. We focus on the case in which high-ability players are in the minority, $y \in (0, \frac{1}{2})$. This assumption guarantees that, if they desire, all high-ability players can enter a high-type contest.

The model has two stages. In the first stage, players choose (simultaneously) which (type of) contest to enter, and in the second stage, they compete by exerting effort (simultaneously).[10] At the *entry stage*, players form expectations about their opponents' abilities based on their knowledge of the overall distribution of types and the equilibrium strategies. At the *competition stage*, players observe their opponents' abilities and, given the contest's prize structure, then simultaneously make their effort choices.[11]

We model competition as a perfectly discriminating contest, where prizes are awarded to the players who exert the highest levels of effort (and ties are broken randomly).[12] This follows an extensive literature on contest design (see, for example, Clark and Riis (1998)

[8]This assumption makes contest choice non-trivial. If, instead, one type of contest offered fewer and *smaller* prizes, then, neglecting potential differences in opposition, all contestants would prefer the other type of contest. In a labor tournament setting, Yun (1997) shows that first-best efforts and efficient self-selection can be achieved when workers are offered the choice between a tournament with many large prizes and a tournament with few small prizes.

[9]We have verified that our results remain qualitatively unchanged when this assumption is relaxed. The corresponding comparative statics are discussed at the end of this section.

[10]If players would choose contests sequentially and could observe who entered previously, they would have an even stronger incentive to avoid contests with strong opponents. Hence, our assumption of simultaneous entry is the most conservative with respect to the possibility of reverse sorting.

[11]Abstracting from effort choices and instead assuming that performance is determined by a player's ability plus noise would neglect the fact that low-type contests might be attractive due to their mitigating effect on competition.

[12]Alternatively, competition could have a stochastic element–i.e., winning could depend on efforts *and* random factors. For a discussion of this case, see footnote 16.

and Moldovanu and Sela (2001, 2006)). In terms of payoffs, a player of type i who exerts effort $e \geq 0$ in a contest of type j will receive utility $U_i^j = b_j - c_i e$ if he wins one of the M_j prizes, and $U_i^j = -c_i e$ otherwise.

Since, at the competition stage, players can guarantee themselves a payoff of zero by exerting no effort, and players are assumed to have a zero outside option, at the entry stage, no player will choose not to participate in any contest at all.[13] This means that if a fraction $q_i \in [0, 1]$ of type i players enters high-type contests, then the remaining fraction $1 - q_i$ will enter low-type contests. The players' behavior at the entry stage can, therefore, be completely described by the fractions of low-ability (q_L) and high-ability (q_H) players that enter high-type contests.[14]

The distribution of players across contests can be characterized as exhibiting: *complete sorting* when all high-ability players enter high prize contests, $q_H = 1$; *partial sorting* when a larger number of high-ability players enter high-type contests than low-type contests, $q_H > \frac{1}{2}$; and *reverse sorting* when the opposite is the case, $q_H < \frac{1}{2}$.

An equilibrium distribution of talent (q_H, q_L) has to satisfy two conditions: an *optimality condition* and a *feasibility condition*. The optimality conditions requires that no player must be able to increase his payoff by entering another (type of) contest. This means that if players of the same type i enter both types of contests, $q_i \in (0, 1)$, then these players must expect equal payoffs. In addition, if all players of type i enter the same type of contest–i.e., $q_i \in \{0, 1\}$–then their expected payoff must not be higher in the other type of contest. The feasibility condition requires that the number of players who participate in a given type of contest must equal the number of available slots in contests of this type:

$$yq_H + (1 - y)q_L = y(1 - q_H) + (1 - y)(1 - q_L) = \frac{1}{2}. \tag{1}$$

The novelty of the model outlined above is that it allows for the study of contest selection in a setting with multiple prizes as well as heterogeneous contestants. While heterogeneity is a pre-requisite for the study of sorting, allowing for multiple prizes is important since the choice between a more competitive environment (with few prizes) and

[13] We assume that players participate when indifferent between participation and non-participation. We show that low-ability players expect a zero payoff from participation since their expected prize winnings are compensated exactly by their effort costs. A zero outside option, thus, means that, apart from prizes, participation must offer alternative sources of utility that are independent of the choice of contest and offset the potential benefits from non-participation. Adding a performance-independent payment (wage, attendance pay) to the contests' payoff structure has no effect on our results.

[14] Alternatively, q_i can be interpreted as the probability with which a player of type i enters a high-type contest. Since we consider a continuum of players, both interpretations are equivalent.

a less competitive environment (with many prizes) constitutes one of the key dimensions of the contest choice problem. In previous work, Azmat and Möller (2009) and Konrad and Kovenock (2012) consider a group of homogeneous contestants choosing between contests with differing prize structures. Due to the absence of ability differences, sorting could not be analyzed in these models. In a setting with two single-prize contests of varying size, Leuven et al. (2010) study sorting by allowing for two types of contestants. They share our finding that reverse sorting is a possibility but reverse sorting arises for a different reason and often in conjunction with positive sorting (multiple equilibria). In their setting, reverse sorting can be an equilibrium only if by deviating to the low-prize contest high-ability contestants would encounter a higher number of opponents. In our setting reverse sorting arises even when contestants face the same number of opponents in each contest and is due to the mitigating effect of low prize concentrations on competition.

Our analysis proceeds by backward induction and consists of two steps. In Section 1.2, drawing on a recent result by Siegel (2009), we characterize a player's expected payoff from participating in a contest with a given set of opponents. The main result necessary for the subsequent analysis, which is the focus of our study, is that a player's expected payoff is positive (and equal to $b_j(1-c)$) if and only if the player has high ability and the number of high-ability opponents is strictly smaller than the number of prizes M_j. In Section 1.3, we use these payoffs to derive our main theoretical results on the players' individual contest choice and the equilibrium distribution of talent across contests. All proofs are given in the Appendix.

1.2 *Competition*

In this section, we derive a player's expected payoff at the competition stage–that is, for a given set of prizes and opponents. In making their effort choice, players trade off a higher chance of winning against an increase in their costs of effort. The characterization of equilibrium effort strategies has proven difficult in general, even for the case in which all prizes are identical (Baye et al. (1996), Clark and Riis (1998), and Barut and Kovenock (1998)). Players use mixed strategies due to the all–pay auction character of competition. Because of the potential presence of identical players, multiple equilibria may exist. These equilibria differ with respect to the set of players who are *active*–that is, who provide effort with positive probability. In equilibrium, all active players win a prize with positive probability. More-able players are more likely to win a prize since they exert higher efforts in the sense of first-order stochastic dominance.

Siegel (2009) shows that for a large class of "generic contests," all equilibria are payoff-equivalent. More specifically, the players' expected payoffs depend on their abilities and the contest's prize structure, but not on the particular equilibrium that is played. In our setting, a contest with M_j prizes is generic if the player with the $M_j + 1$'s lowest marginal cost of effort has marginal costs that are different from any other player's. In what follows, we use a perturbation argument that allows us to employ Siegel's results.

For this purpose, suppose that there exist arbitrarily small differences in the marginal costs of effort for players of the same type $i \in \{L, H\}$.[15] Under this additional assumption, the main result of Siegel (2009) implies that, in a contest with M_j prizes, a player's expected payoff, in any equilibrium, is given by $\max(0, b_j(1-\gamma))$, where γ denotes the ratio of the player's marginal cost over the $M_j + 1$'s lowest marginal cost of all players in the contest. Therefore, by taking the limit, we get the following:

Lemma 1 *Suppose that $N_H \in \{0, 1, \dots, N+1\}$ high-ability players and $N+1-N_H$ low-ability players participate in a contest offering M_j prizes of size b_j. A player's expected payoff is $b_j(1-c)$ if the player has high ability, and the number of high-ability opponents is strictly smaller than the number of prizes. Otherwise, his payoff is zero.*

Note that for low-ability players, (expected) prize winnings are exactly offset by the (expected) costs of effort.[16] High-ability players enjoy a comparative advantage due to their lower marginal cost of effort and, therefore, obtain a positive payoff. This comparative advantage disappears when the number of high-ability players, N_H, exceeds the number of prizes, M_j. In this case, all players expect a zero payoff, independent of their ability.

1.3 *Contest Choice*

In this section, we first consider how a player's preferences over contests depend on the contests' prize structure and the expected opposition. In a second step, we determine the equilibrium allocation of talent across contests.

1.3.1 *Individual preferences*

The analysis in the previous section showed that low-ability players expect the same (zero) payoff, independent of the type of contest they enter. Hence, low-ability players

[15]The argument is made precise in the proof of Lemma 1 contained in the Appendix.

[16] This is a consequence of contests being perfectly discriminating. If contests involved a random element, then the expected payoffs of low-ability players would depend on prizes, but this dependence would still be weaker than it is for high-ability players. Since sorting can be expected to be strongest when ability matters most, the absence of randomness is the most conservative assumption with respect to our finding that sorting may be reversed. For a detailed study of the relationship between a contest's prize structure and its randomness, see Azmat and Möller (2009).

are indifferent between the two types of contests, and we can concentrate our analysis on the preferences of high-ability players. The expected payoff of a high-ability player does depend on the specific features of the contest he enters. In the preceding section, we demonstrated that in a contest offering M_j prizes of size b_j, a high-ability player expects a positive payoff equal to $b_j(1-c)$ if the number of high-ability opponents is smaller than M_j and a zero payoff otherwise.

At the time of entry, the number of high-ability opponents in a particular type of contest is uncertain. Hence, from the viewpoint of the entry stage, the player's preferences will depend on the likelihood p_j with which an opponent has high ability. According to Lemma 1, the probability with which a high-ability player obtains a positive payoff is identical to the probability with which at most M_j-1 of his N opponents have high-ability. It is given by $F(M_j-1;N,p_j)$ with F denoting the cumulative binomial distribution function

$$F(K;N,p) \equiv \sum_{k=0}^{K} f(k;N,p) \equiv \sum_{k=0}^{K} \binom{N}{k} p^k (1-p)^{N-k} \tag{2}$$

measuring the likelihood of observing at most K "successes" within N independent binomial draws with success-probability p. A high-ability player's expected payoff from entering the contest is

$$E[U_H] = b_j(1-c)F(M_j-1;N,p_j). \tag{3}$$

It depends on the contest's prize structure, represented by M_j and b_j, and the expected opposition, given by the likelihood p_j of meeting high- rather than low-ability opponents. Note that, at this stage, the variable p_j is treated as exogenous. The determination of its equilibrium value follows below. In the Appendix, we prove the following result:

Proposition 1 *A high-ability player's expected payoff from entering a contest is increasing in the size b_j of its prizes but decreasing in the probability p_j with which opponents have high ability. Payoffs are increasing in the concentration of the contest's prize structure when opposition is weak ($p_j < \bar{p}_j$) but decreasing when opposition is strong ($p_j > \bar{p}_j$).*

The first part of Proposition 1 is intuitive and follows easily from (2) and (3). The last part of Proposition 1 considers the effect of a decrease in the number of prizes, accompanied by an increase in the size of the prize. As can be seen from the proof contained in the Appendix, the particular value taken by the threshold $\bar{p}_j$ depends on the specific changes in M_j and b_j. Intuitively, when the probability of meeting high-ability opponents is small, high-ability players prefer a more concentrated prize structure due to their comparative advantage over low-ability players. In contrast, when the probability of meeting high-ability opponents is large, high-ability players prefer a less concentrated prize structure due to its mitigating effect on competition and the resulting decrease in effort costs.

To summarize, while prizes are predicted to have a positive effect on a player's decision to enter a particular contest, the effect of (expected) opposition is negative. Moreover, opposition has not only a level effect, but also an interactive effect with the concentration of the contest's prize structure.

1.3.2 *Sorting*

Having described the players' individual preferences, we now determine their equilibrium allocation across the two types of contests. Our analysis proceeds as follows. For a given allocation (q_H, q_L), we determine the likelihood p_j of meeting high-ability opponents in a contest of type $j \in \{l, h\}$, which allows us to calculate the players' expected payoffs in both types of contest. We then verify whether the optimality and feasibility conditions outlined above are satisfied. The indifference of low-ability players implies that optimality needs to be checked only for high-ability players and that feasibility is guaranteed by the low-ability players' willingness to fill any slot that has remained idle.

For a given allocation (q_H, q_L), the number of high-ability players who choose a high-type contest is given by yq_H. Since there are $\frac{1}{N+1}$ contests, and both types of contests exist in equal proportion, there are $\frac{1}{2(N+1)}$ high-type contests, each offering $N+1$ slots. The likelihood with which a slot in a high-type contest is filled with a high-ability opponent can be calculated by dividing the number of high-ability players who choose a high-type contest, yq_H, by the overall number of slots available in the high-type contests, $\frac{1}{2}$. It is given by $p_h = 2yq_H$. Similarly, the likelihood with which a slot in a low-type contest is filled by a high-ability opponent is given by $p_l = 2y(1 - q_H)$.

To check optimality for high-ability players, we need to consider the difference between their expected payoffs from entering a high-type versus a low-type contest. From (3) this difference is proportional to

$$\Delta \equiv b_h F(M_h - 1; N, p_h) - b_l F(M_l - 1; N, p_l). \tag{4}$$

High-ability players strictly prefer a high-type (low-type) contest when $\Delta > 0$ ($\Delta < 0$) and are indifferent when $\Delta = 0$. In the Appendix, we prove the following result:

Proposition 2 *There exists a unique equilibrium allocation* (q_H^*, q_L^*) *of abilities that depends on the proportion* y *of high abilities in the population of players. In particular, there exist critical values* $\bar{y} \in (0, \frac{1}{2})$ *and* $\bar{\bar{y}} \in (\bar{y}, \frac{1}{2}]$ *such that the following hold:*

1. *For* $y \leq \bar{y}$*, sorting is complete,* $q_H^* = 1$*. All high-ability players enter high-type contests.*

2. *For* $\bar{y} < y < \bar{\bar{y}}$*, sorting is only partial,* $q_H^* \in (\frac{1}{2}, 1)$*. High-type contests attract a greater number of high-ability players than low-type contests. Moreover, talent crowds out talent–i.e.,* q_H^* *is strictly decreasing in* y*.*

3. *For* $\bar{\bar{y}} \leq y$*, sorting is reversed,* $q_H^* \leq \frac{1}{2}$*. Low-type contests attract a greater number of high-ability players than high-type contests.*

An increase in the high-type contests' prize budget $M_h b_h$ *relative to the low-type contests' prize budget* $M_l b_l$ *leads to a higher level of sorting by increasing* q_H^* *and* $\bar{y}$*.*

The intuition for this result is as follows. High-type contests offer high prizes, while low-type contests mitigate competition by spreading out their prize budget. From the viewpoint of a high-ability player, effort considerations become more important as the likelihood of meeting high-ability rivals increases, and his comparative advantage over low-ability players plays a smaller role. When high abilities become sufficiently frequent, the mitigation of competition outweighs all else, such that high-ability players prefer low-type contests over high-type contests, even though prizes are smaller and rivals are more able in the former than in the latter. This contrasts with the common intuition that, in equilibrium, contest choices should be driven by a trade-off between high prizes and strong opposition, versus low-prizes and weak opposition. The possibility of reverse sorting, therefore, emphasizes the need for including effort considerations in models of contest choice.

For the general case, we cannot rule out that $\bar{\bar{y}} = \frac{1}{2}$. To show that within our range of parameters $y \in (0, \frac{1}{2})$, reverse sorting is indeed a possibility, we provide an example in which $\bar{\bar{y}}$ is *strictly* smaller than $\frac{1}{2}$.

Example: Reverse sorting between one-prize and two-prize contests. Consider the special case in which both types of contests have the same total prize budget B. Let high-type contests award their entire budget to the player with the highest effort–i.e., $M_h = 1$ and $b_h = B$. Let low-type contests offer two identical prizes instead–i.e., $M_l = 2$ and $b_l = \frac{B}{2}$. In the proof of Proposition 2, we show for the general case that Δ is strictly decreasing in

q_H. This is intuitive since an increase in q_H raises the expected opposition in a high-type contest while lowering the expected opposition in a low-type contest. Hence, $\bar{\bar{y}} < \frac{1}{2}$ if and only if $\Delta(q_H = \frac{1}{2}) < 0$ for some $y < \frac{1}{2}$. For the special case under consideration, substitution of M_j and b_j into (4) leads to

$$\Delta(q = \frac{1}{2}) = \frac{B}{2}(1-y)^{N-1}(1-(N+1)y). \tag{5}$$

This shows that reverse sorting between one-prize and two-prize contests of identical budgets exists when $y > \frac{1}{N+1}$. For example, when contests allow for 20 participants, then sorting would already be reversed when more than five percent of the players in the population of potential participants have high-ability. ■

We expect that our results will hold quite generally. In our model, the main driver of the results is that low prize concentration mitigates competition, leading to a reduction in effort costs. This element of the model is not unique to our setting. Indeed, it has been established that low prize concentration (in form of multiple rather than single prizes) can lead to an increase in (aggregate) efforts only in exceptional cases, for example, when effort costs are sufficiently convex (Moldovanu and Sela, 2001), or when the number of contestants is small and contestants are sufficiently risk averse (Krishna and Morgan, 1998) or heterogeneous (Szymanski and Valletti, 2005). It is therefore likely that Proposition 2 will hold in alternative contest setups.

Proposition 2 is also robust with respect to other features of our setup. First, it remains valid when players are risk averse rather than risk neutral. To see this, note that from the viewpoint of a high-ability player, each contest can be understood as a lottery with two possible outcomes. A high payoff is obtained when the number of high-ability participants fails to exceed the number of prizes, and a low payoff is obtained otherwise. For $q_H > \frac{1}{2}$, the high payoff, though smaller, is more likely to be obtained in low-type contests than in high-type contests. Hence, low-type contests constitute the less-risky lottery. Risk aversion gives high-ability players an additional incentive to choose a low-type rather than a high-type contest.[17] Therefore, we consider our assumption of risk neutrality as the most conservative with respect to the possibility of reverse sorting.[18]

Second, consider the effect of relaxing our assumption that both types of contests exist in equal proportions. Suppose, for example, that there exists a larger number of high-type

[17]This is in line with Dohmen and Falk's (2011) experimental finding that subjects who choose a tournament rather than a fixed payment have a lower degree of risk aversion.

[18]Note that this discussion ignores that risk aversion may also influence the way in which players compete. It has been shown, for example, that risk aversion decreases the effort of low-ability contestants but increases the effort of high-ability contestants in single-prize contests (Fibich et al., 2006).

than low-type contests. In this case, the likelihood of meeting a high-ability opponent in a high-type contest is lower than $2yq_H$, and the likelihood of meeting a high-ability player in a low-type contest is higher than $2y(1-q_H)$, for any given value of q_H. This makes high-type contests more attractive relative to low-type contests, leading to a (weak) upward shift in the equilibrium value of q_H^*. The thresholds $\bar{y}$ and $\bar{\bar{y}}$ shift to the right. The results in Proposition 2 change quantitatively but remain qualitatively unchanged.

Finally, suppose that contests offer decreasing rather than identical prizes. If high-type contests offer a steeper prize allocation than low-type contests then contest-types differ in the same way as before, although differences are less pronounced. In particular, contests with steeper prize allocations offer higher prizes to top-performers while contests with flatter prize allocations can be expected to mitigate competition.[19] We therefore believe that our results would extend to settings with heterogeneous prizes.

1.3.3 *Coordination*

Our model can be used to shed light on the influence of coordination on the allocation of talent across contests. For this purpose, assume that, rather than being non-cooperative, the contest choice of *all* high-ability contestants is the task of a common coordinator.[20] The coordinator influences the allocation of high-ability contestants by choosing the fraction $q_H \in [0,1]$ entering high-type contests.[21] The coordinator's objective is to maximize the sum of all high-ability contestants' (expected) payoffs:

$$E\left[\sum U_H\right] = (1-c)\left[q_H b_h F(M_h-1;N,p_h) + (1-q_H) b_l F(M_l-1;N,p_l)\right]. \tag{6}$$

The coordinated solution q_H^C must satisfy the first order condition

$$\Delta_C = \Delta + 2y(1-c)\left[q_H b_h \frac{\partial F(M_h-1;N,p_h)}{\partial p} - (1-q_H) b_l \frac{\partial F(M_l-1;N,p_l)}{\partial p}\right] \geq 0. \tag{7}$$

Here Δ denotes the term defined in (4), determining the non-cooperative equilibrium q_H^*. The term in square brackets measures the externalities of a high-ability contestant's contest choice on all other high-ability contestants. Since F is decreasing in p, a contestant's

[19] Although this seems reasonable, confirming it would require a model of competition with heterogeneous players and heterogeneous prizes.

[20] Assuming full coordination allows us to consider sorting in a setting which is diametrically opposed to our benchmark case of non-cooperative contest choice. We expect all partially coordinated outcomes to lie in between these two polar cases.

[21] While contest-type choices are coordinated, we continue to assume that, within each type, contests are picked randomly and, once contestants have entered a certain contest, they choose their efforts non-cooperatively.

switch from low-type to high-type contests, decreases the payoff of the q_H contestants in high-type contests by $(1-c)b_h$ while increasing the payoff of the $1-q_H$ contestants in low-type contests by $(1-c)b_l$. The difference between the coordinated and the non-cooperative solution is that the coordinator internalizes these externalities whereas they are neglected when contestants choose individually.

The internalization of contest-choice externalities may prevent the coordinator's objective function from being concave, thereby complicating the characterization of the coordinated solution q_H^C along the lines of Proposition 2. However, the coordinator's objective is, in fact, concave when the number of high-ability contestants is sufficiently low, which is when coordination is most likely to play a role. This allows us to obtain the following:

Proposition 3 *Suppose that $y < \frac{M_h}{2N}$. If (non-cooperative) sorting is positive, coordination decreases the fraction of high-talent players participating in high-type contests, i.e. $q_H^* \geq \frac{1}{2} \Rightarrow q_H^C \leq q_H^*$ with strict inequality for $q_H^* < 1$.*

Proposition 3 shows that the coordinated solution q_H^C serves as a lower bound for the non-cooperative equilibrium q_H^*.[22] This is intuitive since, due to the externalities described above, the coordinator has an incentive to spread high-ability players across contests. Moreover, even with coordination, the two major forces -high prizes versus low effort costs- determining contest choice in the non-cooperative setting are still present. We therefore expect the coordinated solution to share the properties of the non-cooperative equilibrium outlined in Proposition 2. In particular, the negative dependence of sorting on the number of high-ability contestants should continue to exist in the presence of coordination.[23]

2 Empirical Framework

Our theoretical framework makes precise how a contest's attractiveness to high-ability runners depends on its prize structure and how the overall number of high-ability runners influences their sorting across the two types of contests. Thus, testing the model's predictions requires variation in the distribution of abilities and variation in prize structures across contests. In this section, we test our model's predictions using a large panel

[22] Since $q_H^* < 1 \Leftrightarrow b_h F(M_h - 1, 2\bar{y}) < b_l$ (see proof of Proposition 2), we can always choose $\frac{b_l}{b_h}$ such that $\bar{y} < \frac{M_h}{2N}$, i.e. there indeed exist parameters for which q_H^C is *strictly* smaller than q_H^*.

[23] We have confirmed this numerically. Details are available on request.

dataset of international city marathons and professional marathon runners, which spans more than 20 years.[24]

Beyond the common advantages of sports data recognized in the literature, two important factors make marathons the ideal setting to test our theory.[25] First, a fairly homogeneous group of high-ability runners can be identified by their (East-African) origin rather than by performance measures–such as finishing time–that may be endogenous to the prize budget. Second, there are five races (Boston, Berlin, Chicago, London, and New York), which, for historical reasons, have a special status in running, comparable to the "Grand-Slam" tournaments in tennis. These races offer considerably higher and more-concentrated prizes than others.

The dominance of East-African marathon runners is most striking. In 2009, for example, 88 of the 100 fastest (male) marathon runners were from either Kenya or Ethiopia.[26] This dominance, unparalleled in other sports, has been explained by genetic, social, nutritional, and geographical factors (Finn, 2012). It allows us to overcome the usual identification problem of measuring ability using past performance, which, unlike origin, may depend on prize and effort considerations. Another advantage is that this group of high-ability runners is fairly homogeneous, as postulated by our model, and exhibits a good deal of variation in marathon participation, thereby enabling our analysis of sorting. The dominance of East-African runners became apparent in the 1970s, when a handful of East-African runners participated in international marathons, winning by great margins. Their success sparked a professional running culture in their home countries making marathon running a way to escape poverty. Certain minimum standards, however, must be met to make travel abroad worthwhile and, as a consequence, the participation of East- Africans in international races is still restricted to the most-talented.[27] Marathon running, in general, has become more competitive (see Figure 1). While in the early 1980s, the fastest runners had a comparative advantage of around six percent (eight minutes), this advantage had decreased to less than two percent (two minutes) by the late 2000s. This change

[24]We are not the first to use sports data to test the predictions of contest theory, although this literature has focused mainly on incentive effects; see Ehrenberg and Bognanno (1990) and Brown (2011) on golf; Becker and Huselid (1992) on auto racing; and Lynch and Zax (2000) on running.

[25]Sports contests share many features with other contests, such as those seen in a labor-market setting. However, unlike in labor tournaments, prizes and performance are easily observed. It is often difficult, if not impossible, to know the pay structure within firms. Moreover, workers' individual performance is seldom observed; nor are there well-defined measures that are recognized across firms, even for those in the same industry or sector.

[26]See Top List of the International Association of Athletic Federations (IAAF) available online at http://www.iaaf.org/statisitics/toplist/index.html.

[27]As a robustness check, we compare performance in years with a greater presence of East-African runners to years in which there are fewer. The quality of performance is not affected. See Section 2.4.

in the ability distribution constitutes a crucial element of our analysis of sorting.

Regarding contests, our model postulates the existence of two types that differ with respect to their prize structure. In the world of running, a clear distinction can be made between the races in Berlin, Boston, Chicago, London, and New York and the remaining races. These five marathons have the longest history and attract the highest number of runners. Their special status has manifested itself in the creation of the World Marathon Majors series in 2006.[28] In the following we will therefore refer to these races as "Major" marathons and denote all remaining races as "Minor" marathons. Most importantly, the Major marathons award much higher prizes and offer considerably more-concentrated prize allocations than other marathons. These features allows us to identify the World Marathon Majors as the high-type contests of our theoretical model.

Apart from the dominance of East-African runners and the special status of the Major marathons, a number of other features of professional marathons make them an appropriate setting for testing the theoretical model. First, the model assumes that players can participate in, at most, one contest. This is consistent with the empirical framework. Marathons are typically clustered into two seasons: spring and autumn. Marathon runners can run more than one race, but to achieve top performance they must allow for a considerable rest period between races. As a consequence, runners typically choose only one race per season.[29]

Second, the model assumes that runners make their race choices simultaneously. In fact, what matters for the analysis is not the precise timing of entry, but that runners face uncertainty regarding the race choice of other runners at the time of their own entry decision. An important feature of marathon running is that runners must choose their races several months in advance in order to achieve peak performance on race day via the exact adjustment of their training plans. Thus, it is reasonable to assume that runners face considerable uncertainty about their prospective opponents when making their race choices.

Third, in the model, players are assumed to be motivated only by prize money. To empirically judge the importance of other factors, such as prestige or the possibility of achieving a personal best, we perform a counterfactual analysis in Section 2.4. In this analysis, we show that, conditional on their effort and that of all other runners, runners most often enter the race in which they maximize their monetary payoff, providing support

[28] Collectively, the group annually attracts more than five million on-course spectators, 250 million television viewers, and 150,000 participants. For more details, see http://worldmarathonmajors.com/US/about/.

[29] In our sample, less than two percent of runners run more than two races per year.

for the model's focus on prizes. In this respect, it is also important to note that in comparison to other sports, very few runners obtain the status of a marketable superstar, so prize money consititutes the dominant source of income for most runners.

Finally, our restriction to two types of contests with few (identical) high prizes or many (identical) low-prizes is certainly a simplification with respect to the more sophisticated prize structures used in marathons. Nevertheless, it provides a good approximation of the runners' main trade-off between a small likelihood of winning a high prize and a large likelihood of winning a low prize.

2.1 *Data Description*

We use data from the Association of Road Racing Statisticians, which contains detailed race and runner information for the largest international marathons. We restrict attention to the 35 marathons that are present in our sample for the entire period from 1986 to 2009.[30] Since a marathon's prize budget and participation are strongly correlated with the number of years that the race has been in existence, these races are among the most important events in the world of road racing.

For each race, we observe the date, location, and the prize distribution. At the runner level, we identify the top (professional) finishers for each race. To maintain a balanced panel and since we are only interested in the race choice of the most-able runners, we restrict our attention to the first 20 finishers in each race (separately by gender). Since marathons award fewer than twenty prizes for each race, our data contain runners who win and runners who do not win a prize. We have information on the runners' gender, nationality, date of birth, finishing time, finishing position, and the prize awarded (if any).[31] Tables 1 and 2 provide the main descriptive statistics for races and runners, respectively.

In Table 1, we show the descriptive statistics separately for Major and Minor races. Table 1 shows that the average prize in a Major marathon is considerably higher than the average prize in a Minor marathon ($17,227 compared to $3,240). Moreover, we see

[30]These are: Beijing, Berlin, Boston, California International, Chicago, Dallas, Detroit, Dublin, Frankfurt, Gold Coast, Grandma's, Hamburg, Honolulu, Houston, Italia, Kosice, London, Los Angeles, Madrid, New York, Ottawa, Paris, Reims, Richmond, San Antonio, Rome, Seoul, Stockholm, Tokyo, Turin, Twin Cities, Valencia, Venice, Vienna, and Warsaw. We exclude the marathons in Rotterdam, Amsterdam, and Fukuoka since no prize-money information was available. We also exclude Dubai because it has existed only a few years.

[31]Some marathons have faster (flatter) race courses than others, but the Association of Road Racing Statisticians has constructed conversion factors to make marathons comparable. We adjust all the finishing times in our dataset using these conversion factors.

that Major marathons award a considerably greater share of their prize budget to the winner than Minor marathons (34 percent compared to 27 percent). A comparison of the Herfindahl concentration index based on the first three prizes reveals that 57 percent of the Major races have a Herfindahl index greater than the average, compared to only 35 percent for Minor races. Hence, in line with our theoretical framework, Major marathons offer higher but more-concentrated prize structures. Further support for the identification of Major races as high-type contests is provided in Section 2.4.

Apart from prizes, there are other stark differences between the two race categories. Major marathons have (overall) around three times more participants than Minor marathons (22,332 compared with 6,838). The two types of races also differ in the quality of the runners they attract. From Table 1, we can see that, on average, over all years, the fraction of high-ability runners has been considerably larger in the Major races. This holds whether we identify high-ability runners by origin or by (course-adjusted) finishing times. For example, 18 percent of the finishers in the Major races were East-African, compared to only 14 percent in the other races. Similarly, 29 percent of runners in the Major races had a finishing time within five percent of the year's best, compared with only eight percent in the Minor races. As a consequence, winning times in Major races are, on average, eight minutes faster, which is equivalent to a 2.6km lead.

Table 2 shows the descriptive statistics of runners. In this table, we compare East-African runners, high-ability Non-East-African runners, and other Non-East-African runners, respectively. High-ability Non-East-African runners are defined as the 100 fastest Non-East-African runners within their gender category, based on their fastest finishing time for a given year.[32] For male runners, we see that East-African runners are comparable to high-ability Non-East-African runners on a number of dimensions, including prize money ($7,676 versus $8,284), finishing times (two hours, 14 minutes versus two hours, 12 minutes), and the number of marathons entered in a given year (1.42 versus 1.44). Compared with other runners, however, these two groups look very different. For female runners, the same patterns hold. East-African runners are comparable with the best Non-East-Africans, lending support to our identification of East-African runners as high-ability contestants; but both groups are noticeably different from other runners. The focus of the analysis will be on these high-ability runners.

[32]Our results are robust with respect to changes in the cut-off point for our definition of "high-ability."

2.2 *Individual Contest Choice*

We are now ready to test the predictions of our model. We start by considering a runner's individual race choice before moving to the equilibrium allocation of talent in the subsequent section.

To test Proposition 1, we investigate how a runner's expected payoff from a marathon and, hence, his probability of entering depend on the race's characteristics. Letting P_{ijt} denote the probability with which runner i enters race j in time period t, we estimate the following equation:

$$P_{ijt} = \alpha_0 + \alpha_A A_{jt-1} + \alpha_B B_{jt} + \alpha_C C_{jt} + \alpha_{AC}(A_{jt-1} * C_{jt}) + X_i\beta + \varepsilon_{ijt}. \tag{8}$$

The variable A_{jt-1} denotes the level of expected opposition. It is measured as the proportion of high-ability participants among the race's top 20 finishers in the previous year. The variable B_{jt} denotes the marathon's average prize. C_{jt} is a measure of the prize structure's concentration, calculated as the ratio of the first prize over the sum of all prizes. We also include a vector of control variables, X_i, containing the runner's age, nationality, gender, and ranking in the previous year. In addition, we control for whether the race took place on the runner's home turf since that may confer some comparative advantage. We also control for gender-specific time dummies and race fixed-effects. Standard errors are clustered at the runner-year level.

According to Proposition 1, the probability with which a runner enters a race will be increasing in the average prize, B_{jt}, such that $\alpha_B > 0$, and decreasing in expected opposition, $A_{jt-1,}$ such that $\alpha_A < 0$. Moreover, we expect the effect of concentration, C_{jt}, on entry to depend on the level of expected opposition. The model predicts that more-concentrated prize structures are attractive only when there are sufficiently few opponents, and are unattractive otherwise. Therefore, we expect the coefficient on the interaction term $(A_{jt-1} * C_{jt})$ to be negative $(\alpha_{AC} < 0)$. Since Proposition 1 is concerned with the preferences of high-ability contestants, we restrict our attention to the race choice of the top runners.

As our main variable of interest (A_{jt-1}), is based on the past race choices made by a group of top-runners, using the race choice observations for runners from the same group would result in a mechanical bias. This is because their races choices would be influenced by the races' characteristics in an identical way. We would therefore want to separate runners into two groups with identical (high) ability but (potentially) different race choice preferences. We do this by restricting the participation analysis to the high-ability Non-East-African runners and by using the proportion of East-African runners in a race's previous edition as proxy for the expected opposition. We showed in Table 2 that both groups of runners are comparable in their abilities. However, it is likely that there exists enough independent variation in their race choices to give a causal estimation of the effect of expected opposition on race participation. Another advantage of using runners from East-Africa is that it allows us to use exogeneous variation in local conditions as an instrument for expected opposition.[33] We will deal with this issue explicitly in the next section.

In Table 3, we present the results without the interaction between opposition and concentration. Columns 1 and 2 present the baseline regression without and with controls, respectively. Column 3 includes year dummies and year dummies interacted by gender to control for the changing trends in the participation of (East-African) runners in marathons. Column 4 includes race fixed-effects, which allows for race-specific features that are attractive or unattractive to runners. Races tend to take place in the same month each year. We also control for this, as a means to account for seasonal effects. Overall, we find that an increase in expected opposition is associated with a decrease in the entry of a high-ability contestant in a race, and the average prize has a positive effect on entry. The results allow us to determine the "prize" that contestants are willing to pay for a reduction in opposition. We find that a high-ability runner's likelihood of participation is kept unchanged if a reduction in the (expected) number of opponents by one is accompanied by a decrease in the race's average prize by $\$2,583$.[34] This constitutes almost 50 percent of a race's average prize, calculated over all races. With respect to prize concentration, overall, prize concentration has a positive effect on participation once we control for time and race fixed-effects.

Table 4 shows that the results are robust to alternative definitions of high-ability and expected opposition. First, we extend our definition of high-ability Non-East-African runners to include those who finish in the Top 100 during any of the last three years rather than the previous year alone (Column 1). This accounts for the (rare) possibility that during a particular year, a runner with Top 100 potential may have failed to finish a race within the top twenty. Second, we restrict our definition of expected opposition by counting only those East-African opponents whose finishing time was amongst the Top 100 finishing times of the (previous) year (Column 2). Using performance in combination with

[33]Using past finishing times as a measure of expected opposition would not allow for such an instrument and would add measurement error coming from factors such as weather conditions.

[34]A reduction in the number of East-Africans by one is equivalent to a five percentage point decrease in expected opposition since the determination of A_{jt-1} is based on the race's top 20 finishers. Keeping the likelihood of participation constant, therefore, requires a reduction in the race's average prize by $100,000 \cdot 0.05 \cdot \frac{0.0109}{0.0211} = 2,583$ dollars.

the runners' origin allows us to capture the possibility that runners base their expectations about opposition on the speed with which the race was run, while still allowing for a decomposition of runners into two groups as outlined above.

In Table 5, we present the results with the interaction between opposition and prize-concentration. Columns 1 to 4 show that there exists a differential effect of prize concentration on entry, depending on the expected level of opposition. In line with the predictions of Proposition 1, we find that an increase in the prize structure's concentration is associated with an increase in entry if and only if the level of opposition is below a certain threshold. In particular, we find that an increase in the share awarded to the winner makes a race more attractive when expected opposition (i.e., the proportion of East-Africans among the race's top twenty finishers in the previous year) is below 44%.[35] For higher levels of expected opposition, prize concentration has a negative effect on the entry of high-ability runners. This finding provides support for our assertion that, in contests, selection effect can be opposed to incentive effects.

2.2.1 ***Exogenous variation in opposition***

We have shown that participation is negatively related to expected opposition. An important concern, however, is that the main variable of interest, A_{jt-1}, might be correlated with some unobservable characteristics, leading to a biased estimate of α_A. If a race becomes attractive to all high-ability runners, East-African and Non-East-African, for reasons unexplained by our set of observables, it will create a positive correlation between the entry of these runners and the error term. This would translate into an upward-biased estimate of α_A. To deal with this issue, we instrument for expected opposition, A_{jt-1}, using exogenous variation in the entry of East-African runners, that is uncorrelated with the (unobservable) race characteristics. We do this by instrumenting A_{jt-1} with rainfall, as well as commodity prices, in Kenya and Ethiopia in the previous year, $t-1$, including all the second stage controls and time trends. We then construct the interaction of the predicted A_{jt-1} with prize structure concentration. Both rainfall and commodity prices are correlated with the number of East-African runners who compete in a given year but uncorrelated with race characteristics. It is unlikely that these correlations will affect the race choice of Non-East-Africans, except through the effect that they have on the level of expected opposition, A_{jt-1}.

The reasoning behind the two instruments follows a growing literature, mainly in

[35] To determine this threshold we divide the concentration coefficient in Column 4 by the interaction term to get 0.0117/0.0264=0.44.

political economy, which relates rainfall and commodity prices to economic conditions in Sub-Saharan countries. It has been shown that rainfall levels positively affect income per capita (Miguel et al., 2004) and the functioning of democratic institutions (Brückner and Ciccone, 2011) in Sub-Saharan African countries. In addition, Deaton (1999) documents that commodity price downturns cause rapidly worsening economic conditions in Sub-Saharan African economies. Therefore, we expect rainfall and commodity prices to have a positive effect on the international marathon participation of East-African runners. This is intuitive since most East-African runners rely on the support of sponsors, some of which are local businesses or regional government agencies.[36]

We construct international commodity price indices for Kenya and Ethiopia following Deaton (1999) and Brückner and Ciccone (2011). For this purpose, we use the International Monetary Fund monthly price data for exported commodities for the period 1986 to 2009 and the countries' export shares of these commodities taken from Deaton for 1990. The rainfall data, covering the period 1986 to 2009, are taken from the NASA Global Precipitation Climatology Project. The first-stage estimates show that rainfall and commodity prices are, indeed, strongly related to the participation of East-African runners in international marathons. In particular, with the exception of commodity prices in Ethiopia, positive rainfall shocks and commodity price upturns, increase the number of East-African runners competing internationally. The instruments are individually and jointly significant in the first stage (the F-Statistic of their joint significance is 12.22). The first-stage regression is reported in Table 11.

In Table 5, Columns 5 and 6, we present the results for the IV estimates including a time trend and year dummies, respectively. Since the predicted A_{jt-1} only varies at an annual frequency we cannot estimate the level effect of expected opposition in Column 6. We therefore focus on the interaction of the instrumented expected opposition with prize structure concentration.

As in the OLS regressions, we find that the effect of concentration on entry depends on the level of (expected) opposition. As opposition increases, prize concentration becomes less attractive. The results are in line with those found using OLS; however, the magnitudes are larger, suggesting that the coefficient on expected opposition is, indeed, biased upwards when using OLS. Separating by gender (Columns 7 and 8), the interaction between expected opposition and prize steepness is slightly stronger for men, but overall

[36]We might be concerned that in years when there are more (fewer) East-African runners, the quality of the marginal runner is lower (higher). We check this by looking at the finishing times of East-African runners in the years when there are many (few) and find that these times are not statistically different from one another.

we observe a similar pattern.

2.3 *Sorting*

While Proposition 1 was concerned with the contestants' individual preferences, Proposition 2 focuses on the equilibrium distribution of players across contests. We now move from the determinants of individual race choice to the analysis of the aggregate distribution of runners across races, using the time-series variation of our dataset.

To test Proposition 2, we analyze whether an increase in the overall number of high-ability contestants leads to a more balanced distribution of talent across contests. More specifically, we test the following equation:

$$S_t^M = \alpha_0 + \alpha_{HA} HA_t + \alpha_B B_t^M + t + \varepsilon_t. \tag{9}$$

The dependent variable, S_t^M, measures the level of sorting. It denotes the proportion of East-African runners who choose to participate in a Major rather than a Minor marathon in period t. For $S_t^M = 1$, sorting is complete–i.e., East-African runners participate exclusively in Major marathons. The main variable of interest, HA_t, is the overall proportion of East-African runners, in period t. According to Proposition 2, sorting should be decreasing in HA_t. The variable B_t^M denotes the proportion of the total prize money that is awarded in the Major marathons. According to Proposition 2, sorting should be increasing in B_t^M. We control for both time trends and for whether the year was an Olympic year. Since marathons can be divided into spring and autumn races, and runners typically choose one from each group, we consider contest choice for a given gender category, per season rather than per year to allow for a richer analysis.

Table 6 shows the estimates for equation (9). Since, in our theoretical model, the number of high-type contests is identical to the number of low-type contests, we first restrict our analysis (Columns 1 to 4) to the top ten races. These races include the five Major marathons, as well as the next five most important races (Hamburg, Honolulu, Frankfurt, Paris, and Rome). In Columns 5 to 8, we consider the runners' allocation across all 35 races. The results are similar for both samples.

We find that an increase in the proportion of high-ability contestants leads to a significant decrease in sorting. More specifically, as the proportion of East-African runners in the top ten races increases by one percent, the share of East-Africans who choose a Major marathon decreases by 0.77 percent without controlling for time trends and 1.28 percent when controlling for time trends. The effect is comparable, when all 35 races are considered. These results constitute evidence for the decrease in sorting, as predicted by

Proposition 2. As expected, we also find evidence for a positive relation between sorting and prize budget differences. In particular, a one-percent increase in the proportion of prize money awarded by the Major races leads to an increase in the share of East-African runners entering a Major race by 1.22 percent for the top ten races and by 0.49 percent for all 35 races. It is reassuring that these effects persist when we control for time trends, gender and differential trends across gender.

We see that in an Olympic year, the proportion of East-African runners entering a Major marathon increases by ten percent. This is intuitive since participation in the Olympics is restricted by country quotas. Due to the large number of talented Kenyan and Ethiopian runners, many of them are unable to run the Olympic marathon, whereas runners of comparable ability but different nationality are able to participate with a higher probability. As a result, the proportion of East-African runners in the Major races, the next-best alternative to the Olympics, is higher in Olympic years.

To understand better the time series behavior of the main variables of interest, in Figure 2 we plot the relationship between sorting (S_t^M) and the overall proportion of East-African runners (HA_t) in each period. It seems that, while the proportion of East-African runners has been increasing over time, sorting has been decreasing. To ensure that our results are not driven by trends, we have always included time trends in all our regressions. In Figure 3, we plot the de-trended variables. From the figure, we see that the deviations from the trend of both variables also exhibit a negative relationship. This variation is the one that identifies our main specification. Moreover, as robustness checks, we now also estimate equation (9) with a quadratic and a cubic time trend function. In Column 2 of Table 7, we show that controlling for more flexible time trends the main results still hold.

To determine whether our results are identified by some time periods more than others, in Column 3 of Table 7, we interact the main variable of interest, the fraction of high ability runners, with time dummies for time periods 1986-1991, 1992-1997, 1998-2003, and 2004-2009. The results show that the effect is identified across all periods, except the first period, where, although the point estimate is highly negative, the standard errors are quite large. Overall, our result that sorting depends negatively on the fraction of high ability runners is consistent over time.

An alternative explanation for the decrease in sorting could be that organizers of Major marathons restrict the number of East-African participants in order to guarantee a diversified field. In order to rule this out, we check the robustness of our results using an alternative proxy for talent. Rather than using origin, we identify a group of high-ability runners in a given season using performances.[37] We identify high-ability runners as those who have (adjusted) finishing times within one percent of the season's fastest finishing time in their gender category. We also look at those finishing within five and ten percent of the fastest time, respectively. It is likely that the changes in the overall number of high-ability runners over the years are, at least in part, a result of the increase in East-African participation. However, this measure of high-ability is less restrictive, especially if the quality and composition of the group of East-African runners are changing over time.

Table 8 shows that our main results still hold when we repeat the analysis for the alternative measure of ability based on rankings. The sorting of high-ability runners into Major races is increasing in the proportion of prize money on offer but decreasing in the overall proportion of high-ability runners. Interestingly, the decrease is stronger the more able the runners under consideration. In particular, a ten-percent increase in the proportion of high-ability runners reduces sorting by 46, seven, or three percent when high-ability refers to runners within one, five, or ten percent of the fastest time, respectively. Thus, it seems as if a contestant's tendency to avoid competition by equally talented opponents is increasing in his ability. Finally, note that in contrast to our estimation based on runners' origin, the Olympic year dummy is no longer significant, which is in line with the reasoning provided above.

2.4 *Robustness*

In this section, we address four relevant concerns: 1) the importance of prize-money for a runner's race choice; 2) the possibility of coordination; 3) the potential endogeneity of prize budgets; and 4) the identification of Major races as high-type contests.

2.4.1 *Do runners choose races based on prizes?*

Based on runner-race characteristics (finishing times, prizes), how important are (expected) prize winnings in a runner's race choice? For example, a runner's race choice might be driven by other (unobservable) factors, such as sponsors' preferences. This issue is crucial for determining whether our empirical setting is appropriate to test our model.

As an illustration, we use the most recent year of our data to investigate a runner's potential prize winnings, taking the behavior of all other runners as given. We then construct the counterfactual outcome by counting the number of races in which the runner

[37] Note that, since effort and ability are hard to separate, finishing times may be related to prize money. An advantage of using origin is, therefore, that this definition of high-ability is independent of prize money considerations.

could have obtained a higher prize than in the one he actually chose to compete in. We take as given his current time (effort), as well as the times of all other runners, thus neglecting potential effort adjustments.

We find that a surprisingly high fraction of runners choose a race that maximizes their prize winnings ex post. In particular, around 40 percent of the prize winners could not have earned a higher prize in any other marathon. A further 20 percent had only one alternative race in which their prize would have been higher. This suggests that (expected) prize winnings are an important determinant of runners' behavior, relegating other factors as major drivers of contest choice.

2.4.2 *Coordinated race choices*

In some instances runners are managed by athlete representatives. This may lead to the race choices of runners, who are managed by a common representative, to become coordinated. Our theory (Proposition 3) shows that such coordination would have a negative effect on sorting. Hence coordination may confound our result that sorting depends negatively on the number of high-ability contestants but *only* if coordination was easier to achieve in larger groups, which seems unlikely to be the case. In fact, coordination is commonly seen as a small-group phenomenon due to the relative ease to agree on a common decision. If anything, we, therefore, underestimate the actual reduction in sorting implied by an increase in the number of high-ability contestants.

Moreover, with respect to marathon running, we expect the effect of coordination on contest choices to be small. Using affiliation data by Road Race Management Inc. (2015), we find that the number of runners who share a common manager is relatively low with respect to the overall number of runners. Based on the information available for the 1081 (male) East-African runners that year, we find that more than half of the runners have no manager. For those who are represented by a manager (46%), the Herfindhal concentration index calculated for the distribution of runners across managers is only 0.04. This number increases only slightly to 0.10 when we restrict attention to the East-African runners included in the IAAF Top 100 List. In particular, those runners are managed by sixteen different representatives with at most nine runners sharing a common manager. Hence, while athlete representatives may have some influence on race choices, the low level of runners' concentration suggests that their effect is rather small.

2.4.3 *Exogenous variation in prize budgets*

We may be concerned that race organizers adjust their prizes to keep their race attractive to high-ability contestants. If entry falls, race organizers may increase prize money. As a consequence, the coefficient on B_{jt} in equation (8) would be biased downwards. We deal with this problem by instrumenting the value of a race's average prize with the exchange rate of the country where the race takes place, relative to a currency basket.[38] We expect that a move in the exchange rate is associated with an exogenous change in the value of the race's average prize. This change should not be associated directly with race entry. In order to construct a currency basket, we use the annual Special Drawing Rights basket provided by the International Monetary Fund.[39] Table 9 shows that when we instrument for the prize budget, the coefficient is positive and significant, as previously seen. However, compared with OLS, the coefficient is larger, even after controlling for race and year fixed-effects, suggesting that the OLS is, indeed, downward-biased. The first stage of the instrument is reported in Table 11.

2.4.4 *Identification of high-type contests*

In our analysis of sorting in Section 2.3, we identify the Major races as the high-type contests–i.e., as those with high prizes and high concentration. We verify our identification by repeating the participation analysis in Section 2.2 through making a distinction between entry into Major and Minor races. We define the variable Major, which takes the value 1 if the race is a Major race and 0 otherwise, and we use it as an alternative to the winner's share to measure the prize structure's concentration. We find that our main results from Section 2.2 hold. Being a Major race increases entry, but as opposition increases, Major races become less attractive to enter. This provides additional support for our identification of Major races as high-type contests. The results are presented in Table 10.

3 Conclusion

While the incentive effects of rewarding relative performance have been extensively studied in the theoretical and empirical literature, little is known about contest selection. In this

[38]This is preferable over instrumenting with the value of an East-African runner's national currency since changes in the latter affect the attractiveness of all marathons equally.

[39]This basket contains U.S. Dollars, Euros, Japanese Yen, and Pounds Sterling. Weights assigned to each currency are adjusted annually to take account of changes in the share of each currency in world exports and international reserves.

paper, we have presented and tested a simple model that studies both contest and effort choices. Contestants take into account their own ability, the (expected) strength of their competitors, and the reward schemes offered by the different types of contests. We show that contrary to common belief, the contests with the highest and most-concentrated prizes do not always attract the largest number of high-ability contestants. Contest selection depends, in a systematic way, on the overall distribution of talent, and sorting is reversed when the proportion of high-ability individuals increases beyond a certain threshold. We show that the selection and incentive effects of a contest's prize structure can be either aligned or opposed depending on the competitiveness of the environment, highlighting the importance to study both effects.

Data limitations often prevent the empirical study of contest theory. Key model parameters, such as individual ability and performance, are often unobservable. Moreover, in many tournament settings, a wide array of factors confound the variables of interest. In a labor-market setting, for example, it is often difficult to separate worker from firm types. Our real-effort tournament setting overcomes such identification problems and allows us to shed light on important aspects of contest design. Detailed data on marathons and professional road runners, spanning three decades, have provided us with an opportunity to empirically test theoretical predictions on contest selection.

Our empirical findings confirm our theoretical results and provide evidence for the contestants' trade-off between entering a contest with few high prizes or a contest with many low-prizes. Empirically, we have determined the "prize" that contestants are willing to pay to avoid talented opponents and that organizers must offer to guarantee their contest's attractiveness. Using exogenous variation in the level of competition, our results provide evidence for a strong negative relation between the level of sorting and the overall frequency of highly-talented contestants.

This paper sheds light on an aspect of contest design that has been largely overlooked. By focusing on the effect of contest design on participation, we have been able to establish results, both theoretically and empirically, that complement those in the existing literature. Since the basic trade-off between prizes and opposition, which determines contest selection in our framework, is present in other settings, including labor tournaments, procurement contests, and R&D competition, we expect our results to have important implications for contest design in a broad variety of contexts.

Tables and Figures

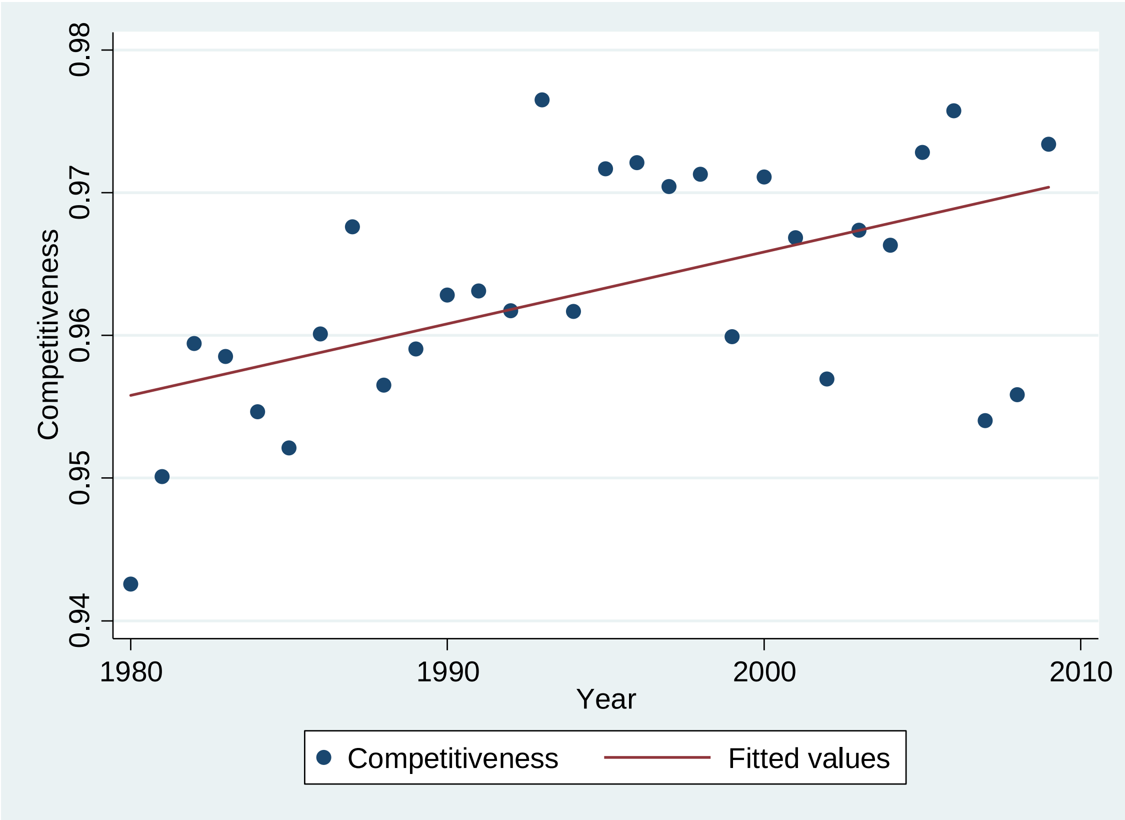

Figure 1: *Competitiveness of Marathon Running.*

Notes. Competitiveness is defined as the ratio of the fastest (male) winning time of a year over the average finishing times of the top 20 (male) finishers in all races. Finishing times are adjusted for racecourse differences.

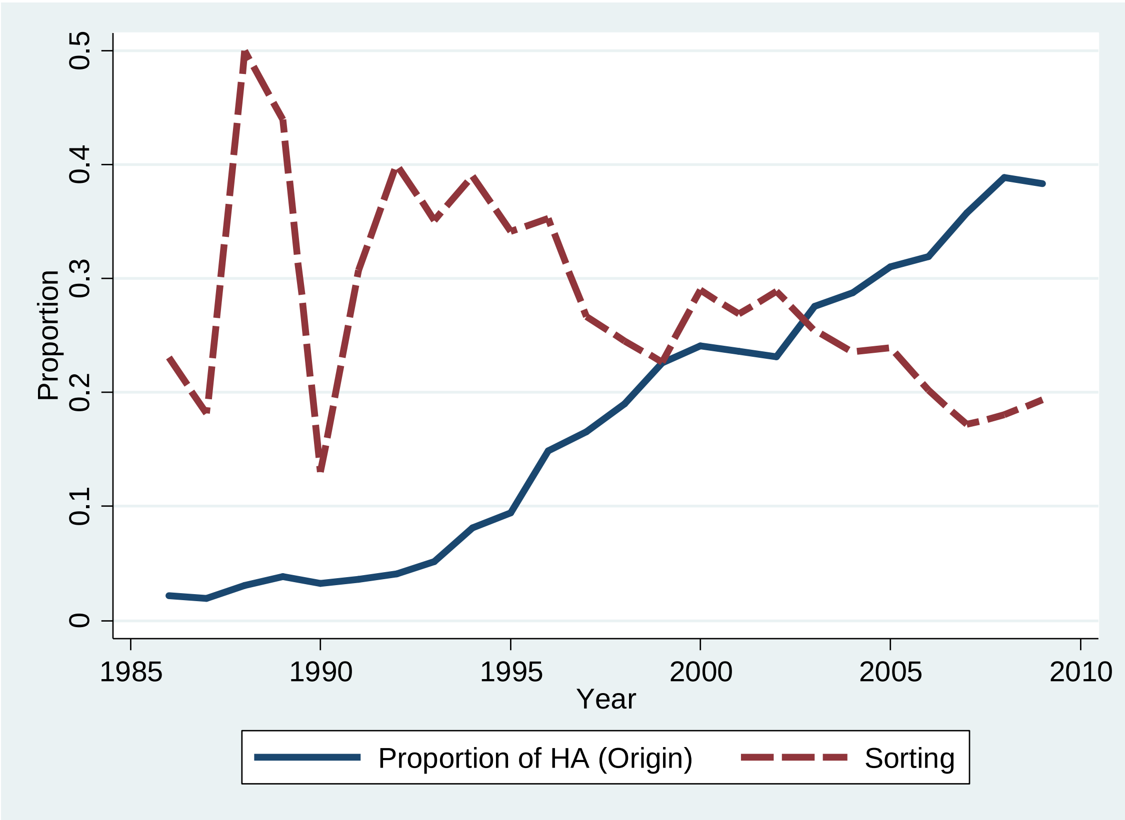

Figure 2: *Sorting of High-Ability Runners.*

Notes. Time-series plot of the overall proportion of high-ability runners ("Proportion of HA (Origin)") and the fraction who chose to participate in a Major rather than a Minor marathon ("Sorting").

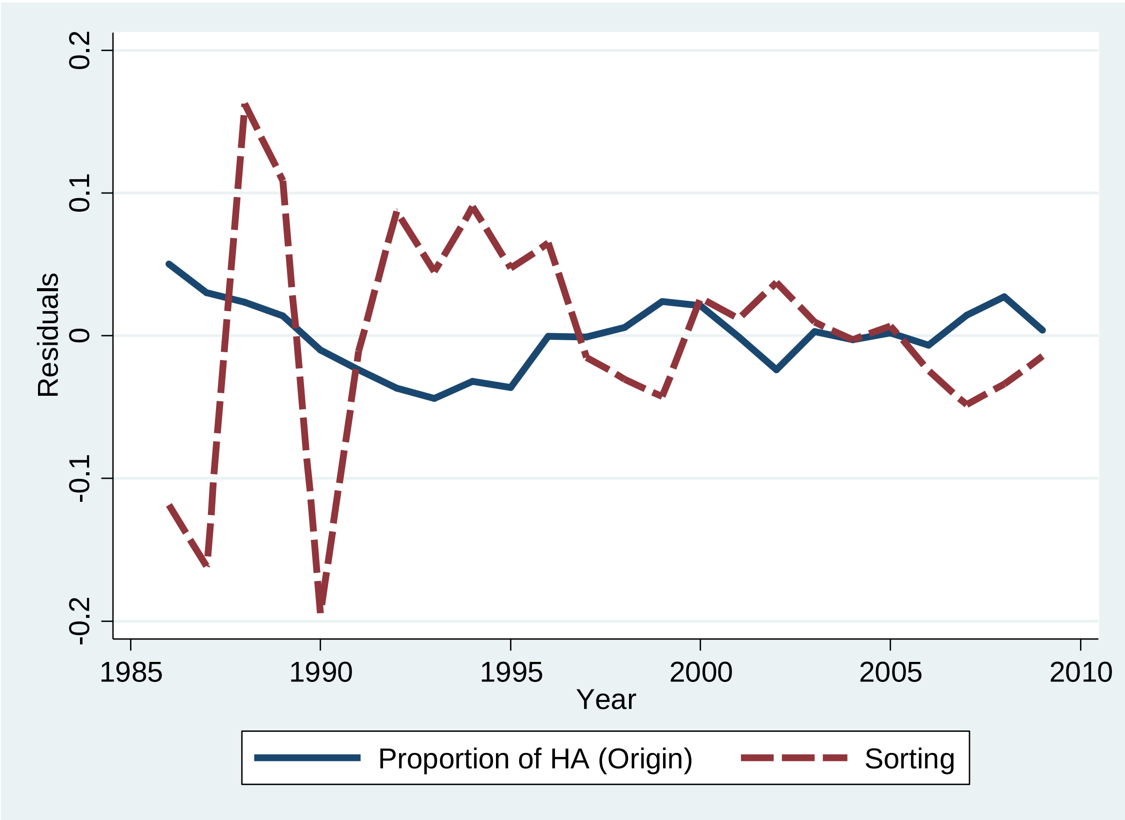

Figure 3: *Sorting of High-Ability Runners (De-trended).*

Notes. De-trended time-series plot of the overall proportion of high-ability runners ("Proportion of HA (Origin)") and the fraction who chose to participate in a Major rather than a Minor marathon ("Sorting"). The variables are de-trended by linear trend.

Table 1: *Descriptive Statistics (Races)*

	Major Races			All other Races		
Variables	Obs	Mean	Std. Dev.	Obs	Mean	Std. Dev.
Average Prize ($)	238	17,277	9,372	1381	3,240	4,331
1st/Total	238	0.34	0.12	1381	0.27	0.27
High Concentration	238	0.57	0.5	1381	0.35	0.48
No. of Participants	236	22,332	10,143	859	6,838	6,462
Winning Time (hh:min)	238	02:17	00:09	1381	02:25	00:13
High Ability (Origin)	238	0.18	0.18	1381	0.14	0.22
High Ability (1%)	238	0.03	0.06	1381	0.00	0.02
High Ability (5%)	238	0.29	0.26	1381	0.08	0.17
High Ability (10%)	238	0.66	0.29	1381	0.36	0.36

Notes. Means and standard deviations for Major and Minor marathons, respectively. Major races are the Berlin, Boston, Chicago, London, and New York marathons. The sample period is 1986 to 2009. "Average Prize" is the sum of all prizes awarded in a race (US dollars at 2000 prices) divided by the number of prize winners. "1st/Total" is the winner's prize divided by the sum of all prizes in a race. "High Concentration" takes value 1 if the Herfindahl index, calculated for the top three prizes, is above its mean value. "No. of Participants" is the total number of participants, including amateurs, in a race. These data were collected separately from various sources, including ARRS and race websites. "Winning Time" is adjusted using ARRS conversion factors to ensure that times are comparable across races. "High Ability (Origin)" refers to the fraction of runners from East Africa among the first 20 finishers of a race. Similarly, "High Ability (1%) (5%), (10%)" refers to the fraction of runners among the first 20 finishers of a race, finishing within 1%, 5%, and 10% of the best time of the year, respectively.

Table 2: *Descriptive Statistics (Runners)*

	Male Runners								
	East-African			Top 100 Non-East-African			All others		
Variables	Obs	Mean	Std. Dev.	Obs	Mean	Std. Dev.	Obs	Mean	Std. Dev.
Age	2892	28.78	4.54	2684	30.05	4.14	4619	30.96	5.16
Prize ($)	2892	7,676	17,780	2684	8,284	16,048	4619	833	2,075
No. Races	2892	1.42	0.6	2684	1.44	0.61	4619	1.17	0.45
Fraction entering Major Race	2892	0.23	0.42	2684	0.38	0.49	4619	0.14	0.34
Finish Time	2892	2:14	0:05	2684	2:12	0:02	4619	2:20	0:05

	Female Runners								
	East-African			Top 100 Non-East-African			All others		
Variable	Obs	Mean	Std. Dev.	Obs	Mean	Std. Dev.	Obs	Mean	Std. Dev.
Age	646	27.69	4.44	2621	30.82	5.35	4840	32.26	6.31
Prize ($)	646	12,420	25,536	2621	10,339	18,319	4840	815	1,885
No. Races	646	1.45	0.59	2621	1.54	0.72	4840	1.19	0.46
Fraction entering Major Race	646	0.32	0.47	2621	0.43	0.49	4840	0.19	0.39
Finish Time	646	2:33	0:08	2621	2:32	0:04	4840	2:46	0:07

Notes. Means and standard deviations (by gender category) for East-African runners, Top 100 Non-East-African runners, and all other runners, respectively. The sample period is 1986 to 2009. "No. of Races" is the number of races run in a given year. "Prize" is the prize money in US dollars at 2000 prices that a runner wins (on average) per race. "Finishing Times" have been adjusted using ARRS conversion factors to ensure that race courses are comparable.

Table 3: *Probability of Entering a Race (OLS).*

	OLS	OLS	OLS	OLS
	[1]	[2]	[3]	[4]
Variables	enter	enter	enter	enter
Expected Opposition$_{t-1}$	-0.0271***	-0.0250***	-0.0162***	-0.0109***
	[0.002]	[0.003]	[0.004]	[0.004]
Average Prize ('00000)	0.3381***	0.3404***	0.3197***	0.0211*
	[0.017]	[0.017]	[0.017]	[0.012]
1st/Total	-0.0259***	-0.0271***	-0.0108***	0.0086***
	[0.002]	[0.002]	[0.002]	[0.003]
Female		-0.0013	0.0033*	0.0032
		[0.001]	[0.002]	[0.002]
Age		-0.0000**	-0.0000**	-0.0000**
		[0.000]	[0.000]	[0.000]
At Home		0.1164***	0.1200***	0.1223***
		[0.006]	[0.006]	[0.002]
Nationality: US		0.0070***	0.0063***	0.0065***
		[0.001]	[0.001]	[0.002]
Rank$_{t-1}$		-0.0001***	-0.0001***	-0.0001***
		[0.000]	[0.000]	[0.000]
Constant	0.0312***	0.0281***	0.0261***	0.0215***
	[0.001]	[0.001]	[0.004]	[0.005]
Time Fixed Effects	No	No	Yes	Yes
Race Fixed Effects	No	No	No	Yes
Observations	144,880	144,120	144,120	144,120
R-Squared	0.015	0.036	0.041	0.059

Notes. *,**,*** denotes significance at the 10%, 5%, and 1% level, respectively. The standard errors are clustered at the runner-year level. The sample is restricted to the runners who were among the Top 100 Non-East-African runners in the previous year. The sample period is 1986 to 2009. "Expected Opposition (t-1)" is the fraction of East-African runners among the top 20 finishers of the race in the previous year. "Average Prize" is the sum of all prizes awarded in the race (US dollars at 2000 prices) divided by the number of prize winners. "1st/Total" is the winner's prize divided by the sum of all prizes in the race. "At home'" takes the value 1 if the runner is racing in his or her home country. "Nationality" takes the value 1 if the runner is from the US and 0 otherwise. "Rank (t-1)" is the ranking of the runner in the previous year (between 1 and 100). The time fixed-effects include a complete set of month and year dummies, as well as year and gender interactions.

Table 4: *Probability of Entering a Race (Robustness).*

	OLS	OLS
	[1]	[2]
Variables	enter	enter
Expected Opposition$_{t-1}$	-0.0114***	-0.0147***
	[0.004]	[0.004]
Average Prize ('00000)	0.0199	0.0235**
	[0.019]	[0.012]
1st/Total	0.0092***	0.0083***
	[0.002]	[0.003]
Female	0.0027	0.0011
	[0.002]	[0.001]
Age	-0.0000**	-0.0000**
	[0.000]	[0.000]
At Home	0.1204***	0.1217***
	[0.005]	[0.002]
Nationality US	0.0058***	0.0063***
	[0.001]	[0.002]
Rank$_{t-1}$	-0.0000***	-0.0001***
	[0.000]	[0.000]
Constant	-0.0087***	0.0345***
	[0.003]	[0.004]
Time Fixed Effects	Yes	Yes
Race Fixed Effects	Yes	Yes
Observations	168,461	144,120
R-Squared	0.054	0.058

Notes. *,**,*** denotes significance at the 10%, 5%, and 1% level, respectively. The standard errors are clustered at the runner-year level. In Column 1 the sample is extended to include the race choices of those runners who were among the Top 100 Non-East-African runners in any of the previous *three* years. In Column 2 the definition of "Expected Opposition (t-1)" is narrowed to include only those East-African participants of the previous year's race whose performance was within the Top 100 finishing times of that year. All other variables are as described previously in Table 3.

Table 5: *Probability of Entering a Race (Instrument for Expected Opposition).*

	OLS	OLS	OLS	OLS	IV	IV	IV	IV
	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]
Variables	enter	enter	enter	enter	enter	enter	enter	enter
Exp.Opposition$_{t-1}$	0.0003	0.0061	0.0084*	-0.0031	-0.0391			
	[0.003]	[0.004]	[0.004]	[0.005]	[0.024]			
Av. Prize ('00000)	0.3462***	0.3264***	0.3273***	0.0272**	0.0364**	0.0387***	0.0146	0.0374*
	[0.017]	[0.017]	[0.017]	[0.012]	[0.016]	[0.013]	[0.017]	[0.020]
1st/Total	-0.0158***	-0.0007	-0.0008	0.0117***	0.0144***	0.0166***	0.0232***	0.0126***
	[0.002]	[0.002]	[0.002]	[0.003]	[0.002]	[0.003]	[0.006]	[0.004]
Exp.Opp.$_{t-1}$*1st/Total	-0.0927***	-0.0853***	-0.0849***	-0.0264**	-0.0582***	-0.0551***		
	[0.008]	[0.008]	[0.008]	[0.010]	[0.010]	[0.015]		
Female		-0.0009	0.0013	0.0027	-0.0067*	0.0036*	-0.0598***	-0.0494**
		[0.001]	[0.002]	[0.002]	[0.004]	[0.002]	[0.021]	[0.023]
Age		0.0000	-0.0000*	0.0000	0.0000	0.0000	0	0
		[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]
At Home		0.1192***	0.1196***	0.1222***	0.1225***	0.1222***	0.1446***	0.1012***
		[0.006]	[0.006]	[0.002]	[0.004]	[0.002]	[0.003]	[0.003]
Nationality: US		0.0062***	0.0063***	0.0064***	0.0063***	0.0063***	0.0070**	0.0053
		[0.001]	[0.001]	[0.002]	[0.002]	[0.002]	[0.003]	[0.003]
Rank$_{t-1}$		-0.0001***	-0.0001***	-0.0001***	-0.0001***	-0.0001***	-0.0001**	-0.0001***
		[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]
Trend					0.0007*			
					[0.000]			
Constant	0.0284***	0.0221***	0.0246***	0.0211***	-0.0043	0.0211***	0.0042	0.0379***
	[0.001]	[0.004]	[0.004]	[0.005]	[0.005]	[0.005]	[0.005]	[0.005]
Time Fixed Effects	No	No	Yes	Yes	No	Yes	Yes	Yes
Race Fixed Effects	No	No	No	Yes	Yes	Yes	Yes	Yes
Observations	144,880	144,120	144,120	144,120	144,120	144,120	75,369	68,751
R-squared	0.016	0.042	0.042	0.059	0.058	0.059	0.065	0.056
P-Val. F-test exc. ins.					0.0000	0.0000	0.0000	0.0000

Notes. *,**,*** denotes significance at the 10%, 5%, and 1% level, respectively. The standard errors are clustered at the runner-year level and bootstrapped with 200 replications in the IV regressions. Expected opposition is instrumented with the commodity price index and (log) rainfall in Kenya and Ethiopia in the previous year. Separate regressions for men and women are shown in Column 7 and 8 respectively. See Table 3 for other definitions.

Table 6: *Sorting of High-Ability Runners (Origin).*

	Top 10 Races				All 35 Races			
	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]
Variables	Sorting	Sorting	Sorting	Sorting	Sorting	Sorting	Sorting	Sorting
Proportion of HA (Origin)	-0.7742***	-0.3551**	-1.0272**	-1.2758**	-0.6222***	-0.5091***	-1.4293***	-1.5077***
	[0.187]	[0.171]	[0.501]	[0.494]	[0.146]	[0.151]	[0.409]	[0.390]
Proportion of Prize		1.1128***	1.1749***	1.2193***		0.4795***	0.4640***	0.4907***
		[0.190]	[0.195]	[0.189]		[0.124]	[0.119]	[0.126]
Female	-0.0894*	-0.0734*	-0.2516	-0.2575*	-0.0224	-0.039	-0.0208	-0.0354
	[0.050]	[0.042]	[0.153]	[0.148]	[0.040]	[0.040]	[0.168]	[0.174]
Trend			0.0125	0.02			0.0301*	0.0320**
			[0.017]	[0.016]			[0.015]	[0.015]
Trend*Female			0.0014	-0.0008			-0.0102	-0.0105
			[0.008]	[0.008]			[0.008]	[0.008]
Olympic Year				0.0967**				0.0548*
				[0.039]				[0.030]
Constant	0.8727***	-0.2134	-0.1688	-0.2579	0.4689***	0.1342	-0.0132	-0.0408
	[0.097]	[0.202]	[0.280]	[0.273]	[0.081]	[0.122]	[0.183]	[0.185]
Observations	79	79	79	79	79	79	79	79
R-squared	0.19	0.448	0.471	0.513	0.274	0.381	0.419	0.445

Notes. *,**,*** denotes significance at the 10%, 5%, and 1% level, respectively. High-ability runners are defined as those who originate from Kenya or Ethiopia. Top 10 Races include the Major races (Berlin, Boston, Chicago, London, and New York), as well as Hamburg, Honolulu, Frankfurt, Paris, Rome. The dependent variable, "Sorting", is the proportion of high-ability runners who enter a Major rather than a Minor race. "Proportion of HA" is the overall proportion of high-ability runners in the population of runners. Both variables are calculated separately for each race season (spring, autumn). "Proportion of Prize" is the proportion of the overall prize money awarded in the Major races. "Trend" is a linear trend for the sample period 1986 to 2009. "Olympic Year" takes value 1 in years 1988, 1992, 1996, 2000, 2004, and 2008 and 0 in all other years.

Table 7: *Sorting of High-Ability Runners (Flexible Time Trends).*

Variables	[1] Sorting	[2] Sorting	[3] Sorting
Proportion of HA (Origin)	-1.5077***	-1.2337***	
	[0.390]	[0.335]	
Proportion of HA*8691			-1.2782
			[1.485]
Proportion of HA*9297			-1.2646**
			[0.522]
Proportion of HA*9803			-1.2311***
			[0.349]
Proportion of HA*0409			-1.0226***
			[0.322]
Trend	0.0320**	0.1272*	0.0416
	[0.015]	[0.069]	[0.112]
Trend Sq.		-0.0058	-0.0014
		[0.004]	[0.006]
Trend Cub.		0.0001	0.0001
		[0.000]	[0.000]
Proportion of Prize	0.4907***	0.5285***	0.5247***
	[0.126]	[0.127]	[0.121]
Female	-0.0354	-0.1574	-0.1958
	[0.174]	[0.187]	[0.239]
Trend*Female	-0.0105	-0.0025	0.0005
	[0.008]	[0.007]	[0.009]
Olympic Year	0.0548*	0.0530*	0.0391
	[0.030]	[0.030]	[0.027]
Constant	-0.0408	-0.5	0.046
	[0.185]	[0.425]	[0.638]
Observations	79	79	79
R-squared	0.445	0.49	0.546

Notes. *,**,*** denotes significance at the 10%, 5%, and 1% level, respectively. The table shows sorting of high ability runners (origin) across all 35 races."Trend" is a linear trend for the sample period 1986-2009. "Trend Sq." and "Trend Cub." are the trend squared and trend cubic, respectively. In Column 3, the proportion of high ability runners is interacted with dummy variables indicating time periods 1986-1991, 1992-1997, 1998-2003, 2004-2009. The regression considers sorting across all 35 races and definitions of all remaining variables are as in Table 6.

Table 8: *Sorting of High-Ability Runners (Performance).*

	Top 10 Races			All Races		
	[1]	[2]	[3]	[4]	[5]	[6]
Variables	Sorting	Sorting	Sorting	Sorting	Sorting	Sorting
Proportion of HA (1%)	-1.9589***			-4.6357**		
	[0.707]			[2.148]		
Proportion of HA (5%)		-0.2751*			-0.7163***	
		[0.159]			[0.214]	
Proportion of HA (10%)			-0.1194			-0.3075***
			[0.146]			[0.110]
Proportion of Prize	0.3263*	1.0318***	1.1413***	1.2664***	0.7091***	0.4475***
	[0.176]	[0.126]	[0.119]	[0.286]	[0.140]	[0.082]
Female	-0.1602	-0.0097	-0.0432	0.1364	-0.0995	-0.1608*
	[0.128]	[0.105]	[0.122]	[0.221]	[0.112]	[0.083]
Trend	-0.0193**	-0.0139**	-0.0036	0.0171	-0.0170**	-0.0166***
	[0.008]	[0.007]	[0.008]	[0.014]	[0.007]	[0.005]
Trend*Female	0.0102*	0.0027	-0.0007	-0.0086	0.0060	0.0063**
	[0.006]	[0.005]	[0.005]	[0.010]	[0.005]	[0.003]
Olympic Year	0.0233	-0.0231	0.0071	-0.0634	-0.0008	0.0137
	[0.035]	[0.025]	[0.023]	[0.061]	[0.028]	[0.016]
Constant	1.0270***	0.1000	-0.1355	-0.3148	0.4351*	0.5788**
	[0.259]	[0.242]	[0.336]	[0.380]	[0.240]	[0.220]
Observations	79	79	79	79	79	79
R-squared	0.314	0.719	0.692	0.364	0.603	0.622

Notes. *,**,*** denotes significance at the 10%, 5%, and 1% level, respectively. High-ability runners are defined as those with an (adjusted) finishing time within 1% (5%, 10%) of the race seasons's fastest time in their gender category. The dependent variable "Sorting" is the proportion of high-ability runners who enter a Major rather than a Minor race. "Proportion of HA 1% (5%, 10%)", is the overall proportion of high-ability runners in the population of runners. Both variables are calculated separately for each race season (spring, autumn). For definition of other variables, see Table 6.

Table 9: *Probability of Entering a Race (Instrument for Prizes).*

	IV	IV
	[1]	[2]
Variables	enter	enter
Expected Opposition$_{t-1}$	-0.0124***	0.011
	[0.004]	[0.009]
Average Prize ('00000)	0.3499**	0.3257**
	[0.168]	[0.160]
1st/Total	-0.0172	-0.0049
	[0.013]	[0.009]
Exp.Opp.$_{t-1}$*1st/Total		-0.0787***
		[0.030]
Female	0.0060**	0.004
	[0.003]	[0.002]
Age	0	0
	[0.000]	[0.000]
At Home	0.1220***	0.1219***
	[0.002]	[0.002]
Nationality: US	-0.0001***	-0.0001***
	[0.000]	[0.000]
Rank$_{t-1}$	0.0064***	0.0064***
	[0.002]	[0.002]
Constant	0.0247***	0.0230***
	[0.005]	[0.005]
Time Fixed Effects	Yes	Yes
Race Fixed Effects	Yes	Yes
Observations	144,120	144,120
R-squared	0.053	0.055
P-Value of F-test of exc. ins.	0.0000	0.0000

Notes. *,**,*** denotes significance at the 10%, 5%, and 1% level, respectively. The standard errors are clustered at the runner-year level. Average Prize is instrumented with the exchange rate of the country of the race relative to the Special Drawing Rights currency basket provided by the IMF. For definition of variables, see Table 3.

Table 10: *Probability of Entering a Race (Major Race as Indicator for High Concentration).*

	OLS	OLS	OLS	OLS	OLS	OLS	IV
	[1]	[2]	[3]	[4]	[5]	[6]	[7]
Variables	enter	enter	enter	enter	enter	enter	enter
Expected Opposition$_{t-1}$	-0.0156***	-0.0137***	-0.0204***	-0.0048**	-0.0048*	-0.0126***	
	[0.002]	[0.003]	[0.004]	[0.002]	[0.003]	[0.004]	
Average Prize ('00000)	0.1092***	0.1109***	0.1069***	0.1542***	0.1562***	0.1527***	0.1872***
	[0.016]	[0.016]	[0.017]	[0.016]	[0.016]	[0.017]	[0.011]
Major Race	0.0633***	0.0630***	0.0639***	0.0819***	0.0812***	0.0787***	0.0851***
	[0.004]	[0.004]	[0.004]	[0.004]	[0.004]	[0.005]	[0.002]
Exp. Opp.$_{t-1}$*Major Race				-0.1401***	-0.1376***	-0.1238***	-0.1863***
				[0.014]	[0.014]	[0.015]	[0.010]
Female		-0.0013	-0.0001		-0.0031***	-0.0026	0.0024
		[0.001]	[0.002]		[0.001]	[0.002]	[0.002]
Age		0	0		0	0	0
		[0.000]	[0.000]		[0.000]	[0.000]	[0.000]
At Home		0.1155***	0.1193***		0.1152***	0.1190***	0.1187***
		[0.006]	[0.006]		[0.006]	[0.006]	[0.002]
Nationality: US		0.0065***	0.0063***		0.0065***	0.0063***	0.0060**
		[0.001]	[0.001]		[0.001]	[0.001]	[0.002]
Rank$_{t-1}$		-0.0001***	-0.0001***		-0.0001***	-0.0001***	-0.0001***
		[0.000]	[0.000]		[0.000]	[0.000]	[0.000]
Constant	0.0252***	0.0219***	0.0232***	0.0221***	0.0200***	0.0200***	0.0229***
	[0.001]	[0.001]	[0.004]	[0.001]	[0.001]	[0.004]	[0.005]
Time Fixed Effects	No	No	Yes	No	No	Yes	Yes
Observations	144,880	144,120	144,120	144,880	144,120	144,120	144,120
R-squared	0.022	0.043	0.047	0.024	0.045	0.049	0.049
P-Value of F-test of exc. ins.							0.0000

Notes. *,**,*** denotes significance at the 10%, 5%, and 1% level, respectively. The standard errors are clustered at the runner-year level. "Major Race" takes value 1 if the race is a Berlin, Boston, Chicago, London, or New York marathon. Expected opposition is instrumented with the commodity price index in Kenya and Ethiopia in the previous year, as well as the (log) rainfall in Kenya and Ethiopia in the previous year. For definition of variables, see Table 3.

Table 11: *First Stage Regressions.*

Variables	Exp. Opposition$_{t-1}$ (for Table 5)	Exp. Opposition$_{t-1}$ (for Table 10)	Average Prize ('00000) (for Table 9)
Commodity Price Index in Kenya$_{t-1}$	0.0021***	0.0003***	
	[0.000]	[0.000]	
Log Rainfall in Kenya$_{t-1}$	0.1833***	0.0193***	
	[0.005]	[0.005]	
Commodity Price Index in Ethiopia$_{t-1}$	-0.001***	-0.002***	
	[0.000]	[0.000]	
Log Rainfall in Ethiopia$_{t-1}$	0.0115***	0.0201***	
	[0.003]	[0.003]	
Exchange Rate			0.0001***
			[0.000]
Constant	-0.6411***	-0.1305***	-0.0369**
	[0.0184]	[0.0165]	[0.0039]
Controls	Yes	Yes	Yes
Time Trends	Yes	Yes	Yes
Race Fixed Effects	Yes	Yes	Yes
Observations	144,120	144,120	144,120
R-Squared	0.49	0.61	0.743

Notes. *,**,*** denotes significance at the 10%, 5%, and 1% level, respectively. Standard errors are clustered at the runner-year level. "Commodity Price Index Kenya (Ethiopia) in t-1" is constructed using the international commodity price data from International Monetary Fund. "Log Rainfall in Kenya (Ethiopia) in t-1" is annual rainfall data from the NASA Global Precipitation Climatology Project. "Exchange Rate" is the exchange rate of the country of the race relative to the Special Drawing Rights currency basket provided by the International Monetary Fund.

Appendix

Proof of Lemma 1

Consider a contest with M_j prizes of size b_j that has attracted N_H high-ability participants and $N+1-N_H$ low-ability participants. Index the $N+1$ participants of the contest in a way such that players $n \in \{1, \ldots, N_H\}$ are of type H and players $n \in \{N_H+1, \ldots, N+1\}$ are of type L. Our model satisfies the definition of a (separable) all-pay contest in Siegel (2009) with a player n's valuation for winning given by $v_n = b_j - c_n e$ where $c_n = c_H$ for $n \in \{1, \ldots, N_H\}$ and $c_n = c_L$ for $n \in \{N_H + 1, \ldots, N + 1\}$.

In order to satisfy Siegel's conditions for a *generic* contest, we now perturb the model by assuming that player n's (perturbed) valuation of winning is given by $\tilde{v}^n = v_n - n\epsilon$ with $\epsilon \in (0, \frac{b_j}{N+1})$. This can be motivated by the existence of (small) differences in the players' benefits from obtaining one of the contest's prizes. Theorem 1 of Siegel (2009) then implies that, in any equilibrium, the expected payoff of player n is given by

$$\max\{0, b_j - n\epsilon - \frac{c_n}{c_{M_j+1}}[b_j - (M_j + 1)\epsilon]\}. \tag{10}$$

Note that expected payoffs are zero for all players $n \in \{M_j + 1, \ldots, N + 1\}$. Also note that for $N_H \geq M_j + 1$, all players $n \in \{1, \ldots, M_j + 1\}$ have marginal cost $c_n = c_H$, which implies that the expected payoff of player $n \in \{1, \ldots, M_j\}$ is given by $(M_j + 1 - n)\epsilon$. Finally, for $N_H < M_j + 1$, it holds that $c_{M_j+1} = c_L$. In this case, the expected payoff of player $n \in \{1, \ldots, N_H\}$ is given by $b_j - n\epsilon - \frac{c_H}{c_L}[b_j - (M_j + 1)\epsilon]$ whereas the expected payoff of player $n \in \{N_H + 1, \ldots, M_j\}$ is $(M_j + 1 - n)\epsilon$. Taking the limit $\epsilon \to 0$ leads to the payoffs described in Lemma 1. ■

Proof of Proposition 1

To abbreviate notation in this and in most of the subsequent proofs, we suppress the number of opponents N as an argument in the (cumulative) distribution functions defined in (2) by letting $f(k; p) \equiv f(k; N, p)$ and $F(k; p) \equiv F(K; N, p)$.

It is immediate that $E[U_H]$ is increasing in b_j and M_j, but decreasing in p_j. To prove the last claim of Proposition 1, increase the concentration of the contest's prize structure by letting $\tilde{M}_j < M_j$ and $\tilde{b}_j > b_j$, and consider

$$\begin{aligned} \frac{E[U_H] - E[\tilde{U}_H]}{1-c} &= b_j F(M_j - 1; p_j) - \tilde{b}_j F(\tilde{M}_j - 1; p_j) \qquad (11) \\ &= b_j[F(M_j - 1; p_j) - F(\tilde{M}_j - 1; p_j)] - (\tilde{b}_j - b_j)F(\tilde{M}_j - 1; p_j). \end{aligned}$$

The first term represents the advantage of the less-concentrated prize structure. When the number of high-ability opponents turns out to be between $\tilde{M}_j$ and $M_j - 1$, the less-concentrated prize structure guarantees a positive payoff, b_j, whereas payoffs are zero for the more-concentrated prize structure. The second term represents the advantage of the more-concentrated prize structure. When the number of high-ability opponents is smaller or equal to $\tilde{M}_j - 1$, payoffs are positive for both prize structures, but the more-concentrated prize structure offers an extra payoff $\tilde{b}_j - b_j > 0$. Now $E[U_H] - E[\tilde{U}_H] \geq 0$ is equivalent to

$$\frac{b_j}{\tilde{b}_j - b_j} \geq \left[\frac{F(M_j - 1; p_j)}{F(\tilde{M}_j - 1; p_j)} - 1\right]^{-1}. \tag{12}$$

We show below that the likelihood ratio $\frac{F(M_j-1;p_j)}{F(\tilde{M}_j-1;p_j)}$ is strictly increasing in p_j, tends to infinity for $p_j \to 1$, and converges to 1 for $p_j \to 0$. Hence, there exists a $\bar{p}_j \in (0,1)$ such that $E[U_H] - E[\tilde{U}_H] \geq 0$ if and only if $p_j > \bar{p}_j$. The more-concentrated prize structure $(\tilde{M}_j, \tilde{b}_j)$ guarantees a higher payoff if and only if the likelihood p_j with which opponents have high ability is smaller than $\bar{p}_j$. The threshold $\bar{p}_j$ is decreasing in $M_j - \tilde{M}_j$ and increasing in $\frac{\tilde{b}_j}{b_j}$. To complete the proof, consider

$$\begin{aligned}
\frac{\partial F(K;p)}{\partial p} &= \sum_{k=0}^{K} \binom{N}{k} [kp^{k-1}(1-p)^{N-k} - (N-k)p^k(1-p)^{N-k-1}] \qquad (13) \\
&= \frac{1}{p(1-p)} \sum_{k=0}^{K} f(k;p)(k - Np) \\
&= \frac{F(K;p)}{p(1-p)} \{E_p[k|k \leq K] - E_p[k]\} < 0.
\end{aligned}$$

Here $E_p[k] = Np$ denotes the expected number of successes under the binomial distribution $f(k;p)$ and $E_p[k|k \leq K]$ is the expected number of successes conditional on this number being smaller or equal to K. Using (13) we obtain for $K > \tilde{K}$:

$$\frac{\partial}{\partial p}\left[\frac{F(K;p)}{F(\tilde{K};p)}\right] = \frac{F(K;p)}{F(\tilde{K};p)} \frac{E_p[k|k \leq K] - E_p[k|k \leq \tilde{K}]}{p(1-p)} > 0. \tag{14}$$

For $p \to 0$ it holds that $F(K;p) \to 1$ for all K implying that $\frac{F(K;p)}{F(\tilde{K};p)} \to 1$. Finally, using l'Hopital's theorem we obtain

$$\lim_{p\to 1} \frac{F(K;p)}{F(\tilde{K};p)} = \lim_{p\to 1} \frac{\frac{\partial F(K;p)}{\partial p}}{\frac{\partial F(\tilde{K};p)}{\partial p}} = \lim_{p\to 1} \frac{(N-K)\binom{N}{K}}{(N-\tilde{K})\binom{N}{\tilde{K}}} (\frac{p}{1-p})^{K-\tilde{K}} = \infty \tag{15}$$

where we have used the representation of F in terms of the regularized incomplete beta function

$$F(K;p) = (N-K)\binom{N}{K}\int_0^{1-p} x^{N-K-1}(1-x)^K dx \tag{16}$$

to get

$$\frac{\partial F(K;p)}{\partial p} = -(N-K)\binom{N}{K}(1-p)^{N-K-1}p^K. \tag{17}$$

■

Proof of Proposition 2

The high-ability players' preferences over contests are given by (4) with $p_h = 2yq_H$ and $p_l = 2y(1-q_H)$. It follows from (13) that

$$\frac{d\Delta}{dq_H} = 2y\left[b_h\frac{dF(M_h-1;p_h)}{dp_h} + b_l\frac{dF(M_l-1;p_l)}{dp_l}\right] < 0. \tag{18}$$

The higher the fraction of high-ability players who choose high-type contests, the less willing are high-ability players to enter such contests. The fact that $b_h > b_l$ implies that

$$\Delta(q_H = 0) = b_h - b_l F(M_l - 1; 2y) > 0. \tag{19}$$

Hence, there cannot exist an equilibrium in which $q_H^* = 0$. Moreover,

$$\Delta(q_H = 1) = b_h F(M_h - 1; 2y) - b_l. \tag{20}$$

Note that $\Delta(q_H = 1)$ is strictly decreasing in y with $\Delta(q_H = 1) \to -b_l < 0$ for $y \to \frac{1}{2}$ and $\Delta(q_H = 1) \to b_h - b_l > 0$ for $y \to 0$. Hence, there exists a unique $\bar{y} \in (0, \frac{1}{2})$ such that $\Delta(q_H = 1) \geq 0$ if and only if $y \leq \bar{y}$. Therefore, an equilibrium in which $q_H^* = 1$ exists if and only if $y \leq \bar{y}$. Moreover, the equation $\Delta(q_H^*) = 0$ has a solution $q_H^* \in (0,1)$ if and only if $y > \bar{y}$. This solution and, hence, the equilibrium are unique. To determine how q_H^* depends on y for $y > \bar{y}$, use (13) to get

$$\begin{aligned} y\frac{d\Delta}{dy} &= b_h p_h \frac{dF(M_h-1;p_h)}{dp_h} - b_l p_l \frac{dF(M_l-1;p_l)}{dp_l} && (21)\\ &= \frac{b_h F(M_h-1;p_h)}{1-p_h}\{E_{p_h}[k|k \leq M_h - 1] - E_{p_h}[k]\} && (22)\\ &- \frac{b_l F(M_l-1,p_l)}{1-p_l}\{E_{p_l}[k|k \leq M_l - 1] - E_{p_l}[k]\}. \end{aligned}$$

For q_H such that $\Delta = 0$, we can substitute $b_h = b_l \frac{F(M_l - 1; p_l)}{F(M_h - 1; p_h)}$ to get

$$\begin{aligned}\frac{y\frac{d\Delta}{dy}}{b_l F(M_l - 1; p_l)} &= \frac{E_{p_h}[k|k \leq M_h - 1] - E_{p_h}[k]}{1 - p_h} - \frac{E_{p_l}[k|k \leq M_l - 1] - E_{p_l}[k]}{1 - p_l} \\ &< \frac{E_{p_h}[k|k \leq M_l - 1] - E_{p_h}[k]}{1 - p_h} - \frac{E_{p_l}[k|k \leq M_l - 1] - E_{p_l}[k]}{1 - p_l}\end{aligned} \tag{23}$$

where the inequality follows from $M_h < M_l$. Note that

$$\frac{\partial}{\partial p} \frac{E_p[k|k \leq K] - E_p[k]}{1 - p} = \frac{(1-p)\frac{\partial E_p[k|k \leq K]}{\partial p} + E_p[k|k \leq K] - N}{(1-p)^2} \leq 0 \tag{24}$$

because $E_p[k|k \leq K] \leq E_p[k] = pN$ and $\frac{\partial E_p[k|k \leq K]}{\partial p} \leq \frac{\partial E_p[k]}{\partial p} = N$ (see below).

In summary, since $p_h \geq p_l \Leftrightarrow q_H \geq \frac{1}{2}$, we have thus shown that at any equilibrium such that $q_H^* \in [\frac{1}{2}, 1)$ it holds that $\frac{d\Delta}{dy}|_{q_H = q_H^*} < 0$. Together with $\frac{d\Delta}{dq_H} < 0$, this implies that q_H^* is strictly decreasing in y as long as $q_H^* \in [\frac{1}{2}, 1)$. This also means that once q_H^* has crossed $\frac{1}{2}$ from above, it will stay below $\frac{1}{2}$ for all higher values of y. In other words, there exists a $\bar{\bar{y}} \in (\bar{y}, \frac{1}{2}]$ such that $q_H^* \leq \frac{1}{2}$ for all $y \geq \bar{\bar{y}}$.

It remains to show that $\frac{\partial E_p[k|k \leq K]}{\partial p} \leq \frac{\partial E_p[k]}{\partial p}$. Following Jones (1990), let $\tilde{k}$ be a so called weighted random variable with distribution function $\tilde{f}(\tilde{k}) \equiv \frac{\tilde{k}}{E_p[k]} f(\tilde{k}; p, N)$. Let $\tilde{F}$ denote the corresponding cumulative distribution function. Supressing p as an argument we can write

$$E[k|k \leq K] = \sum_{k=0}^{K} \frac{kf(k; N)}{F(K; N)} = \frac{E[k]}{F(K; N)} \sum_{k=0}^{K} \frac{kf(k; N)}{E[k]} = \frac{E[k]}{F(K; N)} \tilde{F}(K) \tag{25}$$

and the result follows if $\frac{\tilde{F}(K)}{F(K;N)}$ is decreasing in p. Note that $\tilde{f}(0) = 0$ and that for $\tilde{k} > 0$:

$$\tilde{f}(\tilde{k}) = \frac{\tilde{k}}{Np} \binom{N}{\tilde{k}} p^{\tilde{k}} (1-p)^{N-\tilde{k}} = \binom{N-1}{\tilde{k}-1} p^{\tilde{k}-1} (1-p)^{N-1-(\tilde{k}-1)}. \tag{26}$$

Hence $\tilde{F}(K) = F(K-1; N-1)$ and

$$\frac{\partial}{\partial p} \left[\frac{\tilde{F}(K)}{F(K; N)} \right] = \frac{F(K-1; N-1)}{F(K; N)} \frac{E_{N-1}[k|k \leq K-1] - E_N[k|k \leq K] + p}{p(1-p)} \tag{27}$$

where E_{N-1} and E_N denote expectations for binomial distributions $f(k; p, N-1)$ and $f(k; p, N)$ respectively. To see that this term is negative, write $k = \sum_{n=1}^{N} x_n$ with x_n, $n = 1, \ldots, N$, denoting N independent Bernoulli trials with success probability p and note that

$$\begin{aligned} E_N[k|k \leq K] &= E[\sum_{n=1}^{N} x_n | \sum_{n=1}^{N} x_n \leq K] > E[\sum_{n=1}^{N} x_n | \sum_{n=1}^{N-1} x_n \leq K-1 \wedge x_N \leq 1] \quad (28) \\ &= E[\sum_{n=1}^{N-1} x_n | \sum_{n=1}^{N-1} x_n \leq K-1] + E[x_N | x_N \leq 1] \\ &= E_{N-1}[k|k \leq K-1] + p. \end{aligned}$$

Here the inequality holds since $\sum_{n=1}^{N} x_n \leq K$ is the union of two disjoint events: $\sum_{n=1}^{N-1} x_n = K$ and $x_N = 0$ or $\sum_{n=1}^{N-1} x_n \leq K-1$ and $x_N \leq 1$ with the former dominating the latter in terms of the expected value of $\sum_{n=1}^{N} x_n$. ■

Proof of Proposition 3

Consider the concavity of the coordinator's objective function:

$$\frac{\partial \Delta_C}{\partial q_H} = 2\frac{\partial \Delta}{\partial q_H} + 2y\left[b_h p_h \frac{\partial^2 F(M_h-1;p_h)}{\partial p^2} + b_l p_l \frac{\partial^2 F(M_l-1;p_l)}{\partial p^2}\right]. \quad (29)$$

Taking the derivative of (17) we get

$$\frac{\partial^2 F(K;p)}{\partial p^2} = (N-K)[(N-1)p-K]\binom{N}{K} p^{K-1}(1-p)^{N-K-2}. \quad (30)$$

Substituting (17) and (30) into (29) and using $(N-M+1)\binom{N}{M-1} = M\binom{N}{M}$ we get

$$\begin{aligned} \frac{\frac{\partial \Delta_C}{\partial q_H}}{2y} &= b_h M_h[Np_h - M_h - (1-p_h)]\binom{N}{M_h} p_h^{M_h-1}(1-p_h)^{N-M_h-1} \quad (31) \\ &+ b_l M_l[Np_l - M_l - (1-p_l)]\binom{N}{M_l} p_l^{M_l-1}(1-p_l)^{N-M_l-1}. \end{aligned}$$

If in both types of contests, the expected number of high-talent opponents is smaller than the number of prizes, i.e. if $Np_h < M_h$ and $Np_l < M_l$ then $\frac{\partial \Delta_C}{\partial q_H} < 0$. Since p_h and p_l are both smaller than $2y$ and $M_h < M_l$, a sufficient condition for the above to holds is that $2yN < M_h$ or $y < \frac{M_h}{2N}$. This condition is sufficient for the manager's objective function to be concave and for a unique maximizer q_H^C to exist. In order to see how q_H^C compares to q_H^*, consider $\Delta_C(q_H^*)$. Given concavity of the manager's objective, it holds that $q_H^C < q_H^* \Leftrightarrow \Delta_C(q_H^*) < 0$. We have

$$\begin{aligned} \Delta_C(q_H^*) &= 2yq_H^* b_h \frac{\partial F(M_h-1, 2yq_H^*)}{\partial p} - 2y(1-q_H^*)b_l \frac{\partial F(M_l-1, 2y(1-q_H^*))}{\partial p} \\ &= y\frac{\partial \Delta}{\partial y}|_{q_H=q_H^*}. \quad (32) \end{aligned}$$

In the proof of Proposition 2 it was shown that this term is negative for all $q_H^* \geq \frac{1}{2}$. ■

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References

[1] Amegashie, A., J., Wu, X. (2004) "Self-Selection in Competing All-Pay Auctions." Unpublished Manuscript.

[2] Azmat, G., Möller, M. (2009) "Competition Amongst Contests." *RAND Journal of Economics* **40**, 743-768.

[3] Bagger, J., Lentz, R. (2012) "An Empirical Model of Wage Dispersion with Sorting." Unpublished Manuscript.

[4] Barut, Y., Kovenock, D. (1998) "The Symmetric Multiple Prize All-Pay Auction with Complete Information" *European Journal of Political Economy* **14**, 627-644.

[5] Becker, B., E., Huselid, M., A. (1992) "The Incentive Effects of Tournament Compensation Systems" *Administrative Science Quarterly* **37**, 336-350.

[6] Bognanno, M., L. (2001) "Corporate Tournaments" *Journal of Labor Economics* **19**(2), 290-315.

[7] Brown, J. (2011) "Quitters Never Win: The (Adverse) Incentive Effects of Competing with Superstars" *Journal of Political Economy* **119**, 982-1013.

[8] Brückner, M., Ciccone, A. (2011) "Rain and the Democratic Window of Opportunity." *Econometrica* **79**(3), 923-947.

[9] Bulow, J., Levin, J. (2006). "Matching and Price Competition." *American Economic Review*, **96**(3), 652-668.

[10] Clark, D., J., Riis, C. (1998) "Competition over more than one Prize." *American Economic Review* **88**, 276-289.

[11] Cohen, C., Sela, A. (2008) "Allocation of Prizes in Asymmetric All-Pay Auctions." *European Journal of Political Economy* **24**, 123-132.

[12] Damiano, E., Hao, L., and Suen, W. (2010) "First in Village or Second in Rome?" *International Economic Review* **51**(1), 263-288.

[13] Damiano, E., Hao, L., and Suen, W. (2012) "Competing for Talents." *Journal of Economic Theory* **147**(6), 2190-2219.

[14] Deaton, A., (1999) "Commodity Prices and Growth in Africa." *Journal of Economic Perspectives* **13**(3), 23-40.

[15] Dohmen, T., Falk, A. (2011) "Performance Pay and Multidimensional Sorting: Productivity, Preferences, and Gender." *American Economic Review* **101**, 556-590.

[16] Eeckhout, J., Kircher, P. (2011) "Identifying Sorting–In Theory" *Review of Economic Studies* **78** (3), 872-906.

[17] Ehrenberg, R., G, Bognanno, M., L. (1990) "Do Tounaments have Incentive Effects?" *Journal of Political Economy* **98** (6), 1307-1324.

[18] Eriksson, T. (1999) "Executive Compensation and Tournament Theory: Empirical Tests on Danish Data." *Journal of Labor Economics* **17**(2), 262-280.

[19] Eriksson, T., Teyssier, S., Villeval, M. (2009) "Self-Selection and the Efficiency of Tournaments." *Economic Inquiry* **47**(3), 530-548.

[20] Fibich, G., Gavious, A., Sela, A. (2006) "All-Pay Auctions with Risk Averse Players." *International Journal of Game Theory* **34**, 583-599.

[21] Finn, A. (2012) *Running with the Kenyans: Discovering the Secrets of the Fastest People on Earth.*, New York: Random House.

[22] Jones, M., C. (1990) "The Relationship Between Moments of Truncated and Original Distributions Plus Some Other Simple Structural Properties of Weighted Distributions." *Metrica* **37**, 233-243.

[23] Konrad, K. A. (2009) *Strategy and Dynamics in Contests*, Oxford: Oxford University Press.

[24] Konrad, K. A., Kovenock, D. (2012) "The Lifeboat Problem." *European Economic Review* **56**, 552-559.

[25] Krishna, V., Morgan, J. (1998) "The Winner-Take-All Principle in Small Tournaments." *Advances in Applied Microeconomics* **7**, 61-74.

[26] Lazear, E., P. (2000) “Performance Pay and Productivity.” *American Economic Review* **90**(5), 1346-1361.

[27] Lazear, E., P., Rosen, S. (1981) “Rank-Order Tournaments as Optimal Labor Contracts.” *Journal of Political Economy* **89**, 841-864.

[28] Leuven, E., Oosterbeek, Van der Klaauw, B. (2010) “Splitting Tournaments.” *IZA Discussion Paper* No. 5186.

[29] Leuven, E., Oosterbeek, H., Sonnemans, J., Van der Klaauw, B. (2011) “Incentives versus Sorting in Tournaments: Evidence from a Field Experiment.” *Journal of Labor Economics* **29**(3), 637-658.

[30] Lise, J., Meghir, C., Robin, J. M. (2016) “Mismatch, Sorting and Wages.” *Review of Economic Dynamics* **19**(1), 63-87.

[31] Lopes de Melo, R. (2013) “Firm Wage Differentials and Labor Market Sorting: Reconciling Theory and Evidence.” Unpublished Manuscript.

[32] Lynch, J., G., Zax, J., S. (2000) “The Rewards to Running: Prize Structure and Performance in Professional Road Racing” *Journal of Sports Economics* **1**, 323-340.

[33] Miguel, E., S. Satyanath, and E. Sergenti (2004) “Economic Shocks and Civil Conflict: An Instrumental Variables Approach.” *Journal of Political Economy*, **112**, 725-753.

[34] Moldovanu, B., Sela, A. (2001) “The Optimal Allocation of Prizes in Contests.” *American Economic Review* **91**(3), 542-558.

[35] Moldovanu, B., Sela, A. (2006) Contest Architecture. *Journal of Economic Theory* **126**(1), 70-97.

[36] Morgan, J., Sisak, D., and F. Várdy (2015) “The Ponds Dilemma” Unpublished Manuscript.

[37] Olszewsky, W., Siegel, R. (2016) “Large Contests.” *Econometrica*, **64**(2), 835-854.

[38] Shimer, R., Smith, L. (2000) “ Assortative Matching and Search.” *Econometrica* **68**(2), 343-369.

[39] Siegel, R. (2009) “All-Pay Contests.” *Econometrica* **77**(1), 71-92.

[40] Urquiola, M. (2005) "Does School Choice Lead to Sorting? Evidence from Tiebout Variation." *American Economic Review* **95**(4), 1310-1326.

[41] Xiao, J. (2016) "Asymmetric All-Pay Contests with Heterogeneous Prizes." *Journal of Economic Theory*, forthcoming.

[42] Yun, J. (1997) "On the Efficiency of the Rank-Order Contract under Moral Hazard and Adverse Selection." *Journal of Labor Economics* **15**(3), 466-494.