Selecting the Best: The Persistent Effects of Luck*

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Abstract

Meritocratic principles seem to be abandoned when an initial stroke of luck significantly affects the final allocation of economic resources or decision-making authority. This paper proposes a stylized model of organizational learning where agents' performance at each stage is the sum of time-invariant, unobservable ability, privately-chosen effort, and transitory noise. Our main result shows that, to identify the most able agent, selection *must be* biased in favor of agents who perform well initially, *even when* noise swamps ability and effort differentials in the determination of performance. Making early career luck persistent (e.g. through professional fast-tracks or high- potential programs) is thus rationalized as a necessary consequence of organizational learning. The persistence of luck is amplified by the ordinal nature of performance evaluation and by informed agents' strategic behaviour. Rationally biased selection processes also propagate advantages stemming from the luck of possessing the right identity (e.g. race or gender), especially when non-discrimination laws restrict biases to be independent of identity.

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JEL *classification*: D21, D82, D83, M51.

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1 Introduction

Sometimes an individual's success is explained, or even discredited, as resulting from an initial stroke of good luck. Frank (2016) documents a multitude of careers of over-achievers, ranging from the arts to business, that were kick-started by fortunate circumstances or events. Such narratives raise the question: To what extent do economic institutions or organizational practices *amplify* the role of luck by making its effects long-lasting?

A common argument, across different social sciences, is that resources, training, favoritism or, more generally, "*biases*" granted to early strong performers increase the likelihood that an initial stroke of luck translates into a final economic advantage. For example, academic tracking in schools (Gamoran and Mare, 1989) and professional fast-tracks in firms or public agencies (Rosenbaum, 1979; Forbes, 1987; Baker et al., 1994) magnify the importance of early performance for final success.[1] As a consequence of such policies, chance events such as graduating during a recession or being the oldest child in class can have long-lasting effects on both labor market outcomes (Oreopoulos et al., 2012) and educational achievements (Bedard and Dhuey, 2006).[2] Sociologists refer to such phenomena using the term *cumulative advantage* (Merton, 1968, DiPrete and Eirich, 2006) and argue that performance differentials, such as those between the publication records of scientists at elite universities and at other institutions, can be largely explained by accumulated resource advantages rather than inherent differences in talent (Zhang et al., 2022).[3]

If initial success can be attributed at least in part to *merit*, commonly defined as a combination of ability and effort (Sen, 2000), the use of such biases can be rationalized as improving *selective efficiency*, i.e. the allocation of resources to the most productive individuals. However, in the limit where noise swamps merit in the determination of outcomes, using such biases merely makes luck persistent, by inducing final outcomes to depend on early performance that is almost entirely random. In this paper we argue that, while seemingly at odds with meritocratic principles, making luck persistent is a necessary consequence of an organization's pursuit of the goal of "selection of the best"

[1] Singapore's Public Service Leadership Programme selects its candidates upon graduation from college before supporting their careers within the public administration through designated job assignments and leadership workshops (https://www.psd.gov.sg/leadership/public-service-leadership-careers).

[2] There is evidence for these so-called *relative age effects* for both physical achievements, such as becoming a player in the National Hockey League (Deaner et al., 2013) and intellectual achievements, such as becoming CEO of a S&P 500 company (Du et al., 2012).

[3] A related phenomenon in finance, known as *rich-get-richer dynamics*, explains performance persistence of hedge funds (Cong and Xiao, 2022) and venture capitalists (Nanda et al., 2020) by the special investment opportunities originating from a successful initial investment.

in very noisy environments.[4]

By explaining how institutions shape the role of luck for individual success, our theory helps to illuminate the mechanisms behind economic inequality. This is important because inequality appears to be tolerated when based on merit but not when based on luck (Konow, 2000; Fong, 2001; Cappelen et al., 2007; Cappelen et al., 2013). Stronger beliefs in the relevance of luck increase a country's social spending (Alesina and Angeletos, 2005) and its citizens' willingness to implement redistributive policies (Almås et al., 2020). They also affect what recent critics of meritocracy have denoted as the *social divide* between the "deserving" and the "undeserving" (Sandel, 2020). To the extent that political polarization is driven by group-identification (Duclos et al., 2004), beliefs about the role of luck for success may influence political outcomes. This is especially relevant when beliefs determine the choice between a low-redistribution "American" equilibrium emphasizing the role of merit and a high-redistribution "European" equilibrium acknowledging the role of luck (Benabou and Tirole, 2006; Alesina et al., 2018).[5]

In Section 2 we present a stylized model of a two-agent, two-stage selection process in which individual performance, at each stage, is the sum of an agent's time-invariant, unobservable ability, privately-chosen effort, and a transitory shock. Agents are ex ante identical to the organization but may share private information about relative abilities. The organization observes only the ordinal ranking of performances at each stage and attempts to select the more able agent.[6] Agents choose efforts to maximize the probability of being selected, minus their effort costs.[7] The organization's optimal selection rule augments the second-stage performance of the agent who performed best in the first stage with an additive bias and selects the agent who performs best in the second stage. Our main focus of interest is the persistence of early success, i.e. the probability with which, in equilibrium, the agent with the better initial performance is selected in the final stage.

We start our analysis in Section 3 by considering the case where agents are as unin-

[4]The term "meritocracy" originates from Young's (1958) apocalyptic vision of a future society in which "merit" serves as the central determinant of an individual's power and wealth. In spite of a dispute over what constitutes merit, modern democracies claim to adopt merit as a basis for their allocation of resources and decision-making power (Piketty, 2014).

[5]Experimental studies on redistribution find that U.S. subjects implement Gini-coefficients 0.2 points lower when incomes are based on luck than when incomes are based on merit, which would be sufficient to bring down U.S. inequality to European levels (Lefgren et al., 2016).

[6]Ordinal performance measurement arises naturally when performance is hard to quantify. Lazear (2000) documents that for managers, piece rates are employed ten times less frequently than for operatives, and attributes this difference to the absence of a cardinal measure of managerial performance.

[7]Lazear and Rosen (1981) argue that competing to become selected, e.g. for promotion, can provide workers inside firms with efficient incentives to exert effort and may thus substitute incentive schemes that rely on cardinal performance measurement when performance is hard to quantify.

formed about their relative abilities as the organization. We first show that effort choices will be identical across agents in *both* stages, in spite of the asymmetries induced by learning from the first-stage performance and the application of the second-stage bias. In the absence of private information, effort choices thus have no effect on selective efficiency, implemented bias, or persistence. Our main result shows that in the limit as noise swamps ability differences as a determinant of performance, equilibrium bias converges to a strictly positive value. In other words, even when ability differences have only negligible impact on performance, optimal bias makes first-stage winners considerably more likely to be selected than first-stage losers: *Luck is made persistent.* This shows that the persistence of luck illustrated by our motivating examples need not reflect the use of too much or the wrong kind of bias, but rather emerges as a necessary consequence of an organization's attempt to "select the best" in environments where individual performance is noisy.

To provide further insight into the relation between selective efficiency and the persistence of luck, we consider an alternative setting where performance information is *cardinal* rather than ordinal, so bias can condition on the first-stage *margin of victory.* We show that for noise distributions that are normal, or thinner-tailed, organizations will make luck more persistent when individual performance can be ranked but not quantified. Furthermore, this greater persistence of luck under ordinal than cardinal evaluation is equivalent to greater *front-loading* of the dynamic selection process in the former case, in that first-stage noise is given a relatively more important role than second-stage noise. As ordinal performance measurement is more prevalent towards the top of an organization's hierarchy, our theory thus highlights the importance of initial luck for selection into positions where selection is most consequential.

In Section 4 we consider the case where, in comparison to the organization, agents have superior, possibly imperfect, information about their relative abilities. Because, in our setting, effort acts as a substitute for ability in increasing performance, strategic behavior might be expected to decrease the informativeness of the agents' first-stage ranking, thereby reducing or even eliminating the need to make luck persistent through the application of bias. We show that, contrary to this intuition, informed agents' strategic behavior amplifies the persistence of luck, because the agent more likely to be better exerts a strictly larger first-stage effort than his rival, thereby reinforcing the agents' ability differential on average. This result resonates well with the prominent role of biased selection—in the form of fast-tracking and high-potential programs—for careers such as management consulting where collaboration in small, close-knit teams allows workers to obtain an informational advantage over their superiors regarding their co-workers' abilities.

Finally, in Section 5 we extend our model to allow for a type of luck that originates outside of the organization. Following Akerlof and Kranton (2005), we assume that some agent (randomly selected) possesses an "identity" (e.g. ethnicity, gender, socio-economic background) that gives him an additive but transitory exogenous advantage over his rival. Investigating the mechanisms that propagate such forms of "societal luck" by making its effects long-lasting ranks high on the agenda of the literature on cumulative advantage (DiPrete and Eirich, 2006) and cumulative discrimination (Blank, 2005). We highlight that an important factor influencing the persistence of societal luck is whether or not organizational selection can condition on whether early success was achieved with or without an exogenous advantage, i.e. whether bias can depend on agents' identity. We show that, if bias must be identity-independent, then societal luck is *always* made persistent, whereas allowing biases to depend on identity not only increases the organization's selective efficiency but also reduces the persistence of societal luck. We further prove that agents' strategic response to the manner in which biases are set *amplifies* these beneficial effects of identity-dependent biases. These results suggest that non-discrimination policies that constrain an organization's selection process may backfire by propagating the disadvantages from unequal opportunities, especially when agents have the opportunity to adjust their behavior in response to such policies.

All proofs are in the Appendix.

Related literature Our paper contributes to the literature on organizational learning. Driven by emerging evidence about the functioning of internal labor markets (Baker et al., 1994), the seminal studies by Farber and Gibbons (1996), Gibbons and Waldman (1999, 2006), and Altonji and Pierret (2001) have identified firms' learning about workers' productivity as a key factor explaining wage and promotion dynamics. A robust empirical finding is that early wage increases and early promotions increase the probability of later promotions. Whether this correlation is caused by workers' inherent productivity differentials or by a "fast-track effect" is controversial, with U.S. evidence in favor of the former (Belzil and Bognanno, 2008) and Japanese evidence pointing towards the latter (Ariga et al., 1999). While in the seminal models serial correlation of promotion rates arises from workers' time-invariant ability differences, our model shows that even when ability differences become negligible, serial correlation can be explained by the non-vanishing optimality of fast-tracking (bias). The special relevance of early performance for careers is underlined by Lange's (2007) finding that "employers learn fast".[8] Pastorino (2024)

[8]Using Armed Forces Qualification Test scores as measures of unobserved ability, Lange (2007) finds that it takes only 3 years for employers' expectation error about workers' productivity to decline by one half. Similarly, Lluis (2005) finds evidence that employer learning affects mobility between upper and

supports this view by documenting firms' tendency to assign newly employed managers to tasks that are particularly informative about their abilities. According to our theory, such a task assignment augments the persistence of luck because a greater bias is required to raise the informativeness of the less informative later tasks. Her structural estimates provide strong evidence that learning, besides human capital accumulation, has a sizeable impact on career outcomes.

Our analysis builds on the organizational learning model of Meyer (1991) but incorporates the possibility that agents exert effort to influence their performance. Including effort is important in light of an ongoing controversy over what constitutes merit (Sen, 2000) and given our focus on the relevance of luck—as opposed to merit—for individual success. It allows us to decompose merit into a non-strategic part ("ability") and a strategic part ("effort") and to study how these two components interact to shape the role of luck for economic outcomes. Notably, the broader notion of merit that makes agents "responsible" for their performance induces organizational selection to assign an even greater role to luck, but only if agents are informed about their relative abilities.

Finally, our results about the effects of biased selection on the persistence of advantages individuals derive from their identity contributes to a literature investigating whether selection (e.g. college admission, hiring) should be group-contingent. Sethi and Somanathan (2023) provide an argument for why a disproportionately large representation of members from disadvantaged groups can be necessary to achieve selective efficiency. In our model, allowing organizations to bias selection more strongly in favor of disadvantaged agents is similarly beneficial for selective efficiency but part of the effect comes in form of agents' strategic responses.

Our finding that organizations can make inequality emerge even when individuals have starting points that are approximately equal is shared by Bardhi et al. (2023). They show that small disadvantages which workers experience early in their careers, e.g. due to discrimination, can have long-lasting effects in professions that track on-the-job-failures, such as pilots or surgeons. The underlying learning mechanism differs from ours because it is based on a scarcity of tasks that requires the organization to chose from which agent to learn at any moment in time, rather than to learn from a sequence of relative performance evaluations.

executive levels of German firms but only for workers below 35 years of age. For more experienced workers learning is found to continue to matter when workers differ in how their productivity evolves over time (Kahn and Lange, 2014).

2 Model

We consider an organization consisting of a risk-neutral principal and two agents $i \in \{A, B\}$ with heterogeneous abilities a_i. The difference in abilities or *heterogeneity* is given by $h > 0$, i.e. $\Delta a \equiv a_A - a_B \in \{-h, h\}$. The principal observes the agents' relative performance during two stages, $t \in \{1, 2\}$. After the second stage, the principal needs to select one of the agents for a higher-level task whose payoff to the principal is increasing in the selected agent's ability. The principal's goal is thus simple: to select the more able agent.

Agent i's performance at stage t, $x_{i,t} \in \Re$, is the sum of three elements: the agent's time-invariant ability a_i, multiplied by a stage-specific weight $\lambda_t > 0$; the agent's private choice of effort $e_{i,t} \geq 0$; and a time-varying random component $\epsilon_{i,t} \in \Re$.[9] That is,

$$x_{i,t} \equiv \lambda_t a_i + e_{i,t} + \epsilon_{i,t}. \tag{1}$$

Variation in λ_t across stages accounts for potential differences in the impact of ability on performance. This is especially relevant when agents' task changes over time.

Information and choices The principal and the agents share a common prior, $q^0 \equiv \mathbb{P}(\Delta a = h) \geq \frac{1}{2}$, but for the principal, agents A and B are indistinguishable. If $q^0 = \frac{1}{2}$, the agents are as uninformed as the principal, while if $q^0 > \frac{1}{2}$, the agents have superior information about their relative abilities, with both agents believing that agent A is more likely to be better.[10]

The principal can observe only the ranking of the two agents' performances after the first stage. In the second stage, the principal may costlessly and publicly assign a bias $\beta \in \Re$ to the winner of the first stage. If $\beta > 0$, the bias increments the winner's second-stage performance, and we say that the bias "favors" the first-stage winner, whereas if $\beta < 0$, it reduces his second-stage performance. Having won the first stage, agent i is then identified as the winner of the second stage if $x_{i,2} + \beta > x_{j,2}$.

The principal's chooses the size of the bias β and the selection rule to maximize *selective efficiency*, $S(\beta; h)$, defined as the probability that the more able agent is selected. The agents exert efforts $e_{i,t}$ in each stage to maximize the probability of being selected minus the effort costs. The value of being selected is the same for both agents and is normalized to 1. The cost-of-effort functions $C_t(e_{i,t}), t = 1, 2$, are strictly increasing and

[9] By logarithmic transformation, our results remain qualitatively unchanged when performance equals the product rather than the sum of ability, effort, and noise.

[10] Virtually all the employer learning models reviewed in the Introduction assume that workers are ignorant about their own ability, and hence correspond to the case $q^0 = \frac{1}{2}$.

convex. Thus, effort costs are identical across agents but may differ across stages.

Noise The distribution of the difference in the individual noise terms, $\Delta\epsilon_t \equiv \epsilon_{A,t} - \epsilon_{B,t}$, is a key primitive in our model because outcomes depend only on performance *differentials*. We assume that $\Delta\epsilon_t$ are identically and independently distributed across stages and denote the corresponding support by $[-z, z]$ (where z may be infinite), the cumulative distribution function by G, and its density by g. We make the following distributional assumptions:

Assumption 1 *(i) g is symmetric about 0; (ii) g is strictly log-concave; (iii) g is differentiable on $(-z, z)$; (iv) $\lim_{y \to z} L(y) = \infty$, where*

$$L(y) \equiv -\frac{g'(y)}{g(y)}. \tag{2}$$

The symmetry of g captures the idea that the only source of heterogeneity across agents is their difference in abilities; it is a weaker assumption than individual shocks, $\epsilon_{i,t}$, being i.i.d. across agents. Log-concavity of g is equivalent to the monotone likelihood ratio property in our setting; it guarantees that, in either stage, a larger performance differential $\Delta x_t \equiv x_{A,t} - x_{B,t}$ implies a higher likelihood that A's ability exceeds B's. It also implies that L is increasing. Strict log-concavity makes all the implications strict. Together with the remaining two assumptions it ensures that the principal's problem is well-behaved.

Timing In the beginning of the first stage, the agents observe who is likely to be better. Then they exert efforts. The noise is realized and both the principal and the agents observe who has higher first-stage performance. In the beginning of the second stage, the principal chooses the level of bias. Then the agents exert efforts. The noise is realized and both the principal and the agents observe who has a higher second-stage performance. The principal then selects one of the agents. Note that the principal chooses the bias *after* the first stage rather than committing to it in the beginning.[11]

Equilibrium The solution concept is perfect Bayesian equilibrium (PBE). In a PBE, (i) the effort choice by each agent at each stage is optimal for him given his conjectures about the effort choices of the other agent and the bias set by the principal; (ii) the bias and the selection rule are optimal for the principal given her conjectures about the agents' efforts; and (iii) the conjectures of both agents and the principal are correct.

It is easy to confirm that when the principal chooses the bias optimally, the optimal selection rule is to select the winner of the second stage.

[11] All of our results in Section 3 continue to hold when the principal can commit to the bias before agents choose their efforts in the first stage.

Persistence Our main focus of interest is the persistence of outcomes induced by the interaction between the principal's pursuit of selective efficiency and the agents' desire to be selected. We define *persistence* as the probability, in equilibrium, that the winner of the first stage is selected after the second stage.

It is important to note that the key parameter of our model, $h > 0$, which captures the degree of agents' heterogeneity in abilities, also has a broader interpretation as the *ratio* of agents' heterogeneity to the scale of noise.[12] To shed light on the role of initial luck for final outcomes, much of the analysis in Sections 3 and 4 will focus on the setting in which h is very small: Here the scale of noise is large *relative* to the agents' heterogeneity and, as we show, differences in agents' efforts vanish. Note that even in this environment, the selection decision may still be important to the principal, because the selected agent's performance in the higher-level task may be very sensitive to ability.

When, in this limiting environment, persistence turns out to be strictly larger than one-half, we will say that *luck is made persistent*, because the first-stage winner has a greater chance of ultimately being selected than the first-stage loser, in spite of the fact that the first-stage outcome is almost entirely determined by random factors.

3 Uninformed agents

In this section, we consider the case where agents are as uninformed as the principal about their relative abilities. We show that, in this case, the agents' ability to influence their performance through the exertion of effort has no impact on selective efficiency, and hence no impact on the principal's choice of bias or on the persistence of early success. This allows us to develop the basic intuition for the connection between these variables, before turning our attention in the next section to the effects of informed agents' strategic behavior. The main results of this section are that an organization's optimal use of bias makes luck have a persistent effect on final selection (Section 3.1) and that, under mild conditions on the distribution of noise, luck is made more persistent when agents' performances can be ranked but not quantified (Section 3.2).

[12]To see this, introduce a scaling transformation $\Delta\epsilon_t \to \sigma\Delta\epsilon_t$, with $\sigma > 1$, which makes the difference in the noise terms more dispersed: The cdf becomes $G(\frac{\Delta\epsilon_t}{\sigma})$, the pdf $\frac{1}{\sigma}g(\frac{\Delta\epsilon_t}{\sigma})$, and the support $[-\sigma z, \sigma z]$. If the underlying heterogeneity in abilities is H, then $G(\frac{\lambda_1 H}{\sigma})$ is the probability that, when the first-stage effort differential is zero, the more able agent wins the first stage. It depends on H and σ only through the *heterogeneity-to-noise ratio* $h \equiv \frac{H}{\sigma}$.

3.1 Persistence of luck

The following lemma is critical, as it shows that, in equilibrium, the efforts of uninformed agents cancel each other in the determination of relative performance.

Lemma 1 (Identical efforts) *Let $q^0 = \frac{1}{2}$. Then for any anticipated choice of bias $\beta \geq 0$ by the principal, there exists a unique pure-strategy equilibrium in efforts. In this equilibrium, agents choose identical efforts, both in the first stage and in the second stage.*

In the second stage, despite the asymmetries due to learning and the use of bias, the marginal benefit of effort is the same for the two agents. This is because the value of winning, the marginal impact of effort on performance, and the pivotal realizations of $\Delta\epsilon_2$ (whether the stage-one winner was in fact the more or the less able agent) are all identical for A and B, cf. Lazear and Rosen (1981). In the first stage, given the symmetry of the agents' situation, there exists a pair of identical efforts that are best responses to each other. We show by contradiction that unequal efforts could not be best responses, whatever value of $\beta \geq 0$ the agents expect the principal to choose.[13] Specifically, if agent A were to exert more effort than agent B in stage one, then a stage-one win by B would be a stronger signal of ability than a win by A. Hence, the biased stage-two contest would be more unbalanced following a win by B and would therefore induce lower stage-two effort. But lower stage-two efforts after a win by B would generate stronger stage-one incentives for B than for A, which contradicts the initial assumption.

Given Lemma 1, the probability $S(\beta; h)$ with which the more able agent is selected ("selective efficiency"), when the principal selects the winner of the biased second stage, is given by

$$S(\beta; h) = G(\lambda_1 h)G(\lambda_2 h + \beta) + [1 - G(\lambda_1 h)]G(\lambda_2 h - \beta). \tag{3}$$

The first term in the sum is the probability that the more able agent wins the first stage and then wins the second stage with bias β in his favor. The second term is the probability that the more able agent loses the first stage but then wins the second stage despite being disadvantaged by the bias.

Differentiating with respect to β and rearranging yields the following first-order condition for the principal's choice of bias:

$$\frac{G(\lambda_1 h)}{1 - G(\lambda_1 h)} = \frac{g(\lambda_2 h - \beta)}{g(\lambda_2 h + \beta)}. \tag{4}$$

The ratio on the left-hand side is the relative likelihood that a first-stage win is achieved

[13] Even if the principal expected a non-zero stage-1 effort differential, it would never be optimal to choose $\beta < 0$, i.e. to favor the stage-1 loser; for more detail, see Section 5.

by the more able agent compared to the less able one. The higher this likelihood ratio, the stronger a signal about relative ability is a victory in the unbiased first stage. The term on the right-hand side is also a likelihood ratio: It is the relative likelihood that a stage-2 draw when agent j is disadvantaged by bias β, i.e. $x_{j,2} - \beta = x_{i,2}$, is achieved when j is the more able agent compared to when j is the less able one. Equation (4) shows that optimal bias strikes a balance between the informativeness of the ordinal stage-1 ranking – an unbiased win – and the informativeness of the marginal stage-2 outcome – a draw achieved despite being handicapped by bias. In equilibrium, bias is such that, if the principal were to observe a draw in stage two, she would be indifferent about which agent to select.

Note that, for $\beta = 0$, the right-hand side of (4) is equal to one and hence strictly smaller than the left-hand side. This is because, for $\beta = 0$, a second-stage draw is uninformative about the agents' abilities. Moreover, given the strict log-concavity of g, as the size of the bias disadvantaging the stage-1 loser increases, a stage-2 draw becomes a strictly stronger signal about that agent's relative ability. It thus follows from Assumption 1 that the first-order condition (4) has a unique solution, $\beta^*(h) > 0$, which maximizes selective efficiency. Moreover, since the left-hand (right-hand) side of (4) is increasing in λ_1 (λ_2), which measures the sensitivity of stage-1 (stage-2) performance to ability, $\beta^*(h)$ is increasing in λ_1 and decreasing in λ_2.

While these arguments establish that a positive bias will emerge in equilibrium for any level $h > 0$ of heterogeneity in abilities, it is not clear what happens in the limit as $h \to 0$. Does bias converge to zero? The following proposition characterizes the limiting value of the equilibrium bias as the scale of the noise swamps the heterogeneity in abilities.

Proposition 1 (Equilibrium bias) *Let $q^0 = \frac{1}{2}$. The principal's equilibrium choice of bias, $\beta^*(h)$, is strictly positive, even in the limit as noise swamps agents' ability differences. More specifically, $\beta_0^* \equiv \lim_{h \to 0} \beta^*(h) > 0$ is given by the unique solution of the equation*

$$2\lambda_1 g(0) = \lambda_2 L(\beta_0^*). \tag{5}$$

At first sight, the fact that bias remains strictly positive, even in the limit, may seem counter intuitive, because when h tends to zero, a first-stage win becomes completely uninformative about relative abilities. However, this reasoning neglects the fact that, as h tends to zero, a second-stage draw also becomes uninformative, for any level of bias. Formally, as h tends to zero, both sides of equation (4) approach one. Proposition 1 thus characterizes equilibrium bias in this limit by equating the *rates* at which the informativeness of each stage tends to zero as h gets small. Since L is a strictly increasing function, $L(0) = 0$, and the LHS of (5) is positive, the limiting value of bias must be positive. More intuitively, observe that, when bias is zero, achieving a second-stage draw is uninformative about relative abilities *for any* ratio h of heterogeneity to noise, whereas the informativeness of a first-stage win rises with h. Thus, a strictly positive bias emerges in the limit because, unless first-stage losers are disadvantaged relative to first-stage winners even when ability differences are negligible, the informativeness of a second-stage draw cannot keep up with the informativeness of a first-stage win when ability differences start to matter.

An alternative interpretation of the limiting value of optimal bias is illustrated in Figure 1. In the limit as $h \to 0$, bias is chosen to maximize not the *level* of selective efficiency—since selective efficiency becomes independent of bias in the limit—but the *rate* at which selective efficiency increases with the agents' heterogeneity. In the limit, optimal bias thus maximizes the potential gains to selective efficiency from a marginal increase in agents' heterogeneity; were bias set to zero, these gains would not be fully realized.

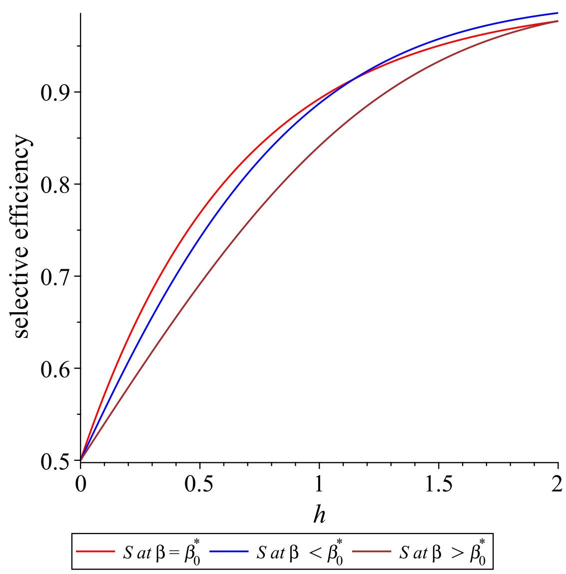

Figure 1: **Selective efficiency.** The figure depicts selective efficiency S as a function of agents' heterogeneity h for different values of bias. $\beta_0^* > 0$ maximizes the slope of S at $h = 0$.

Though the logic behind the equilibrium level of bias is clear in the limit, the dependence of $\beta^*(h)$ on the heterogeneity-to-noise ratio for $h > 0$ can be complex. This is because an increment in h increases both sides of equation (4): It raises *both* the informativeness of a first-stage win *and*—by log-concavity of g—the informativeness of a second-stage draw, for any given level of bias. The complex dependence of $\beta^*(h)$ on h is illustrated in Figure 2. The left panel plots the density functions for the family of expo-

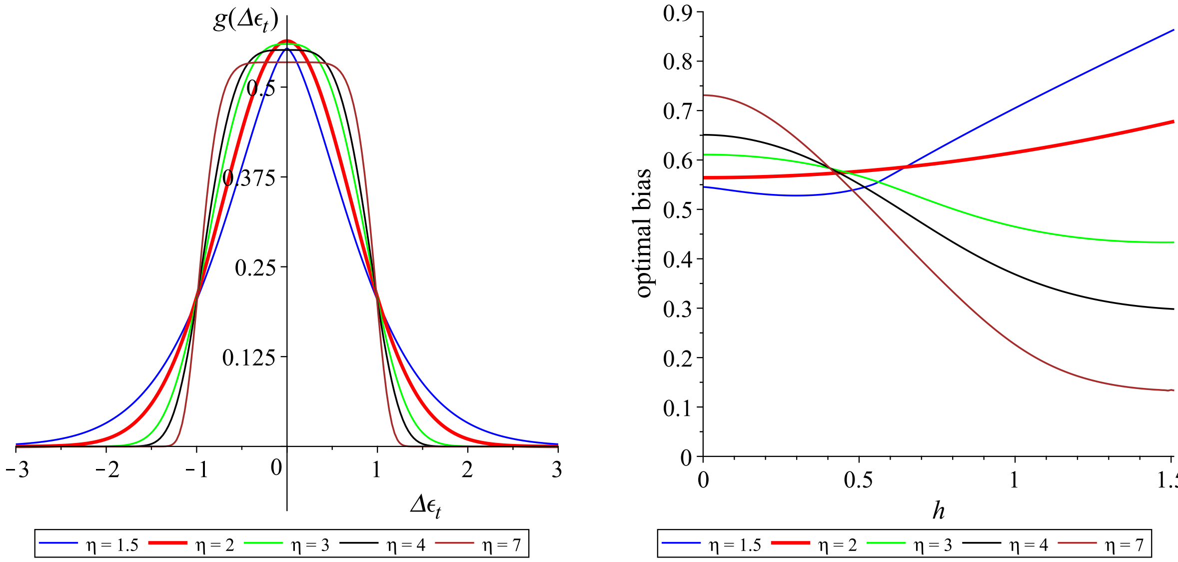

Figure 2: **Example distributions of noise and equilibrium bias.** The left panel depicts the density functions when noise follows an exponential power distributions with mean zero and shape parameter $\eta \in \{1.5, 2, 3, 4, 7\}$. The right panel plots the corresponding equilibrium bias, as h varies, assuming that the impact of ability on performance is time-invariant, i.e. $\lambda_1 = \lambda_2 = 1$.

nential power distributions with mean zero and shape parameter $\eta > 1$.[14] The right panel in Figure 2 plots the equilibrium bias $\beta^*(h)$ as a function of h, for $\lambda_1 = \lambda_2 = 1$. Despite the myriad possibilities illustrated, we see that, as shown by Proposition 1, equilibrium bias remains positive even as h gets small for all members of the family.

Our results about the optimal use of bias for selection have implications for our understanding of the relevance of luck for the determination of economic outcomes. According to meritocratic principles, the allocation of resources and decision-making power should be attributable to *merit*—a combination of ability and effort—rather than luck. In light of this principle, it is important to ask how institutions and organizational practices shape the dynamic relationship between performance and outcomes. A straightforward but important implication of the introduction of bias is that it raises the correlation between initial success and final selection. To see this, define the *persistence* of the selection process as the probability with which the first-stage winner is selected after the second stage,

[14] These density functions are given by $g(\Delta\epsilon_t; \eta) = \frac{\eta}{2\Gamma(\frac{1}{\eta})} \exp(-|\Delta\epsilon_t|^\eta)$, and for all $\eta > 1$, they satisfy Assumption 1. For $\eta = 2$, $g(\Delta\epsilon_t; \eta)$ is a normal distribution with variance $\frac{1}{2}$; as $\eta \to \infty$, $g(\Delta\epsilon_t; \eta)$ approaches a uniform distribution with support $[-1, 1]$; and as $\eta \to 1$, $g(\Delta\epsilon_t; \eta)$ approaches a Laplace distribution with scale parameter 1. At $\eta = 1$, Assumption 1 is violated because the Laplace density is not differentiable at 0 and is not strictly log-concave.

in equilibrium. Given Lemma 1, persistence is independent of efforts and is given by:

$$P(\beta^*(h); h) = G(\lambda_1 h)G(\lambda_2 h + \beta^*(h)) + [1 - G(\lambda_1 h)][1 - G(\lambda_2 h - \beta^*(h)]. \tag{6}$$

Of course, even in the absence of bias, initial success and final selection are positively correlated, and hence $P(0; h) > \frac{1}{2}$, because the outcomes of both stages are affected by the time-invariant ability differential $h > 0$. However, in the limit as $h \to 0$, this correlation would vanish, and hence persistence would approach $\frac{1}{2}$, *unless* it were induced through the use of bias. That is, defining $P_0^* \equiv \lim_{h\to 0} P(\beta^*(h); h)$, we have from (6) that

$$P_0^* = G(\beta_0^*) \qquad \text{and} \qquad P_0^* > \frac{1}{2} \iff \beta_0^* > 0. \tag{7}$$

Hence, a direct implication of Proposition 1 is that *luck is made persistent*: $P_0^* > \frac{1}{2}$. Also note that (5), coupled with the strict monotonicity of L, implies that β_0^*, and hence P_0^*, is increasing in the ratio λ_1/λ_2, which measures the relative sensitivity to ability of first-stage compared to second-stage performance. This is true even though in the limit, ability has only a negligible impact on performance. Recent work by Pastorino (2024) shows that firms tend to allocate to newly-hired workers those tasks that are relatively more informative about their abilities; our results show that this pattern of task allocation enhances the persistence of luck.

In addition, because persistence in (6) is increasing both in the bias and in heterogeneity, the fact that, as shown by Figure 2, equilibrium bias $\beta^*(h)$ can be decreasing suggests that, overall, persistence could also be decreasing in h. This possibility is confirmed by Figure 3, where for $\eta = 7$ and relatively small h, equilibrium persistence falls as the difference in agents' abilities rises.

Contrasted with meritocratic principles, the observations in the preceding two paragraphs are noteworthy, and we summarize them formally:

Corollary 1 (Persistence of luck) *Let $q^0 = \frac{1}{2}$. When bias is set to maximize selective efficiency,*

(i) Luck is made persistent, i.e. $P_0^* > \frac{1}{2}$, and even more so when early performance is relatively more sensitive to ability, i.e. P_0^* is strictly increasing in $\frac{\lambda_1}{\lambda_2}$.

(ii) Initial performance may have a greater impact on final selection in situations where performance differences are less attributable to ability differentials, i.e. there exist noise distributions $g(\cdot)$ and ranges of h for which $P(\beta^*(h); h)$ is decreasing in h.

Corollary 1 shows that two apparent violations of meritocratic principles can be rationalized by the very fact that organizations aim to allocate resources to the most talented

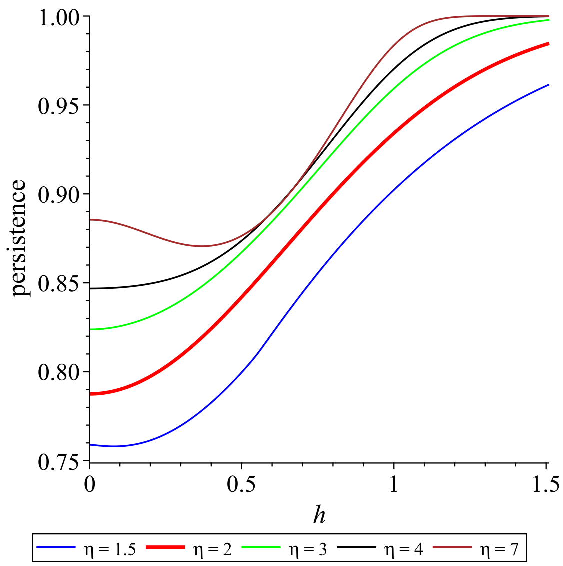

Figure 3: **Equilibrium persistence.** The figure plots the likelihood $P(\beta^*(h); h)$ that winning the initial stage results in becoming ultimately selected, in dependence of the ratio h of agents' heterogeneity to noise. It is assumed that ability has time-invariant impact on performance, $\lambda_1 = \lambda_2 = 1$, and that noise follows an exponential power distributions with mean 0 and shape parameter $\eta \in \{1.5, 2, 3, 4, 7\}$.

individuals. The first part shows that making luck persistent, that is, biasing selection in favor of early strong performers even when initial success is almost entirely due to luck, is a necessary consequence of maximizing selective efficiency. The second part shows that the use of bias for selection may make final success *less* correlated with initial performance in settings where performance differentials are *more* attributable to ability differences. Our analysis implies that neither of these features should necessarily be considered an abandonment of meritocratic principles.

3.2 Cardinal performance evaluation

In the remainder of this section we analyze how the use of bias, and its consequences for the persistence of luck, vary with the way in which performance differentials are measured, contrasting the case of *ordinal* information, studied so far, to that of *cardinal* information. Lazear (2018) argues that ordinal performance evaluation is prevalent towards the top of an organization's hierarchy, given the difficulty of quantifying the performance of increasingly complex tasks. This means that in situations where selection matters most, for both the organization and the agents themselves, ordinal performance measurement may be the most relevant case. However, a comparison with the case where the principal can quantify the agents' performance differentials helps to highlight the specific contribution of rank-order information to the persistence of luck. It may also help to assess whether luck can be expected to play a more important role for selection into positions with higher ranks.

When all parties can observe the stage-1 performance differential, $\Delta x_1 = x_{A,1} - x_{B,1}$, and can condition their stage-2 actions on it, there exists an equilibrium in which, in both stages, agent A exerts the same effort as agent B.[15] Hence, in this equilibrium, similarly to Section 3.1, efforts do not matter for the selective efficiency or persistence. The probability of a *margin of victory* $|\Delta x_1|$ being generated by the stronger agent is $g(|\Delta x_1| - \lambda_1 h)$, whereas for the weaker agent the corresponding probability is $g(|\Delta x_1| + \lambda_1 h)$. Given observed $|\Delta x_1|$, the principal then chooses the bias to maximize

$$S^{card}(\beta, |\Delta x_1|; h) = g(|\Delta x_1| - \lambda_1 h)G(\lambda_2 h + \beta) + g(|\Delta x_1| + \lambda_1 h)G(\lambda_2 h - \beta). \tag{8}$$

Intuitively, a larger margin of victory $|\Delta x_1|$ is a stronger signal about the winner's ability and thus induces the principal to choose a larger bias $\beta^{card}(|\Delta x_1|, h)$.

Optimal bias under cardinal information is particularly transparent when performance in the two stages is equally sensitive to ability, that is, when $\lambda_1 = \lambda_2$. It is then optimal for the principal to select the agent with the higher *aggregate performance*, $x_{i,1} + x_{i,2}$. This selection rule can be implemented by biasing the second stage in favor of the first-stage winner by exactly $|\Delta x_1|$, the first-stage margin of victory. Hence, in this case, $\beta^{card}(|\Delta x_1|, h) = |\Delta x_1|$, for all $|\Delta x_1|$ and h.

In general, the equilibrium bias when performance evaluation is ordinal, $\beta^*(h)$ given by (4), can be thought of as a form of an average of the equilibrium biases $\beta^{card}(|\Delta x_1|, h)$ under cardinal evaluation, as $|\Delta x_1|$ varies. Proposition 2 makes this intuition precise for the limiting case where noise swamps ability. We define $\beta_0^{card}(|\Delta x_1|) \equiv \lim_{h\to 0} \beta^{card}(|\Delta x_1|, h)$.

Proposition 2 (Cardinal bias) *Let $q^0 = \frac{1}{2}$. When the principal can condition bias on cardinal performance information $|\Delta x_1|$, the following holds as $h \to 0$:*

(i) $\beta_0^{card}(|\Delta x_1|) > 0$ whenever $|\Delta x_1| > 0$, and $\beta_0^{card}(|\Delta x_1|)$ solves

$$L(\beta_0^{card}(|\Delta x_1|)) = \frac{\lambda_1}{\lambda_2} L(|\Delta x_1|). \tag{9}$$

(ii) Cardinal bias and ordinal bias are related according to

$$\mathbb{E}[L(\beta_0^{card}(|\Delta x_1|))] = L(\beta_0^*), \tag{10}$$

[15] In close analogy to Lemma 1, agents exerting identical efforts constitutes the *unique* pure-strategy equilibrium if agents anticipate that the principal chooses bias optimally, based on cardinal performance information and given her conjecture about agents' efforts.

and when the difference in the agents' noise terms is normally distributed, so $L(\cdot)$ is linear, $\mathbb{E}[\beta_0^{card}(|\Delta x_1|)] = \beta_0^*$.

A direct implication of Proposition 2(i), recalling (6) and (7), is that with cardinal performance evaluation, luck is made persistent *on average*, i.e.

$$P_0^{card} \equiv \lim_{h\to 0} \mathbb{E}[P(\beta^{card}(|\Delta x_1|, h), h)] = \mathbb{E}[G(\beta_0^{card}(|\Delta x_1|)] > \frac{1}{2}. \tag{11}$$

For the special case of $\lambda_1 = \lambda_2$, since the principal selects the agent with the higher aggregate performance $x_{i,1} + x_{i,2}$, we have, for any noise distribution g,

$$P_0^{card} = \lim_{h\to 0} \mathbb{P}(\Delta x_1 + \Delta x_2 \geq 0 | \Delta x_1 \geq 0) = \mathbb{P}(\Delta\epsilon_1 + \Delta\epsilon_2 \geq 0 | \Delta\epsilon_1 \geq 0) = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}, \tag{12}$$

since $\Delta\epsilon_1$ and $\Delta\epsilon_2$ are i.i.d.

To highlight the specific contribution of ordinal evaluation to the persistence of luck, we compare P_0^{card} with P_0^* in (7), the persistence under ordinal evaluation. Figure 3, which is plotted for $\lambda_1 = \lambda_2 = 1$, shows that P_0^* is larger than $\frac{3}{4}$ for all values of the shape parameter depicted. For $\eta = 2$ ($\Delta\epsilon_t$ normally distributed), this is not surprising, given that $\mathbb{E}[\beta_0^{card}(|\Delta x_1|)] = \beta_0^*$ and given that persistence in (11) is the expectation of a concave function of bias.[16] In fact, combining this insight with (10), a sufficient condition for P_0^* to exceed P_0^{card} is that the function $L(.)$ is convex, since convexity of L implies that the limiting ordinal bias is at least as large as the expected limiting cardinal bias.[17] Distributions for which $L(\cdot)$ is convex are those with densities $\tilde{g}$ that are *thinner-tailed* than the normal distribution, more precisely, those that are more log-concave than the normal in the sense that $\ln \tilde{g}$ is a concave transform of $\ln g$, for g normal.

The following corollary shows that this insight extends to the case of arbitrary λ_1, λ_2.

Corollary 2 (Persistence of Luck: Cardinal versus ordinal evaluation) *Let $q^0 = \frac{1}{2}$ and suppose that the function $L(\cdot)$ is convex.*

(i) The persistence of luck will be greater when performance evaluation is ordinal than when it is cardinal, i.e.

$$P_0^* = G(\beta_0^*) > \mathbb{E}[G(\beta_0^{card}(|\Delta x_1|)] = P_0^{card}. \tag{13}$$

[16] Concavity of G on the positive domain follows from the log-concavity and symmetry about 0 of g.

[17] Convexity of $L(\cdot)$ is not necessary for this conclusion. For the exponential power family of distributions in fn. 14, L is convex if and only if $\eta \geq 2$, but for $\lambda_1 = \lambda_2$, the persistence of luck is larger under ordinal than under cardinal evaluation for all $\eta > \sim 1.38$.

(ii) The inequality in (13) is equivalent to the organization assigning greater relative weight to stage-1 performance than to stage-2 performance when performance evaluation is ordinal than when it is cardinal, as $h \to 0$, i.e. for all λ_1, λ_2,

$$\frac{\mathbb{P}(select\ A|\Delta\epsilon_1 > 0, \Delta\epsilon_2 < 0, ord.)}{\mathbb{P}(select\ A|\Delta\epsilon_1 < 0, \Delta\epsilon_2 > 0, ord.)} > \frac{\mathbb{P}(select\ A|\Delta\epsilon_1 > 0, \Delta\epsilon_2 < 0, card.)}{\mathbb{P}(select\ A|\Delta\epsilon_1 < 0, \Delta\epsilon_2 > 0, card.)}. \quad (14)$$

Corollary 2 shows that if, as argued by Lazear (2000), organizations are constrained to use ordinal performance measurement at high ranks because of the difficulty of quantifying performance in complex tasks, luck may have especially persistent effects on selection at the top of the hierarchy.

Corollary 2 also shows that greater persistence of luck under ordinal than cardinal evaluation is equivalent to greater "front-loading" of the dynamic selection process when performance evaluation is constrained to be ordinal. This inflation of the importance of early luck under ordinal evaluation is especially transparent when the two stages are *intrinsically* equally informative about abilities, i.e. $\lambda_1 = \lambda_2$: Whereas under cardinal evaluation luck is weighted equally across stages (the RHS of (14) equals one), under ordinal evaluation early luck has a greater impact on selection than later luck (the LHS of (14) is greater than one).

4 Informed agents

Our baseline model shares with the literature on organizational learning (e.g. Gibbons and Waldman, 2006; Lange, 2007; Pastorino, 2024) the assumption that agents are as uninformed about their relative abilities as the principal. But what if agents have an informational advantage relative to the principal right from the start? For example, workers might know each other from college or might have shared experiences with previous employers, allowing them to better judge their relative abilities. We capture this by assuming that $q^0 \equiv \mathbb{P}(\Delta a = h) > \frac{1}{2}$, where $\Delta a = a_A - a_B$. While with uninformed agents, Lemma 1 showed that effort choices had no impact on the principal's learning, because in equilibrium efforts cancelled each other in the determination of relative performance, with informed agents, their efforts may no longer be identical. In this section, we examine how the strategic behavior of informed agents impacts organizational learning and persistence.

Because effort and ability are *substitutes* for agents' performance, the agent thought less likely to be the more able might use effort to try to compensate for his ability disadvantage, thereby decreasing the informativeness of early performance about relative abilities, reducing the optimal bias, and weakening our result about the persistence of luck. In fact, we show that, on the contrary, informed agents' strategic behavior *reinforces* the impact of agents' ability difference, resulting in luck being made even more persistent than when agents are ignorant of relative abilities. The following lemma represents the crucial step in our argument:

Lemma 2 (Informed agents' effort differential) *Let $q^0 > \frac{1}{2}$. Then for any anticipated choice of bias $\beta > 0$, agents choose identical efforts in the second stage, but in the first stage, the agent thought more likely to have higher ability exerts a strictly larger effort than his rival.*

The explanation for why the agents choose identical efforts in stage 2 is the same as for Lemma 1. To understand the sign of the stage-1 effort differential

$$\Delta e_1^*(\beta, h, q^0) \equiv e_{A,1}^*(\beta, h, q^0) - e_{B,1}^*(\beta, h, q^0) > 0, \tag{15}$$

note first that because exactly one agent will be selected after stage 2, the "rewards" of winning the first stage arising from the increased probability of being selected are precisely the same for the two agents. However, in contrast to the case where agents are uninformed, the *level* of effort that agents exert in stage 2, and hence their effort cost, now depends on which agent turns out to be the stage-1 winner. To see this most clearly, suppose for simplicity that $q^0 = 1$, so that agent A is known with certainty to be more able. Recall that the principal is aware of the agents' superior knowledge but cannot distinguish agent A from agent B, so must assign the same level of bias whoever wins stage 1. If agent A wins the first stage, then the bias will reinforce the agents' ability difference, and the pivotal realization of noise $\Delta\epsilon_2$ driving second-stage efforts will be determined by $C_2'(e_{A,2}^*) = g(h+\beta) = C_2'(e_{B,2}^*)$. If, instead, agent A loses the first stage, then bias will mitigate the agents' ability difference, so $C_2'(e_{A,2}^*) = g(h-\beta) = C_2'(e_{B,2}^*)$ will determine the pivotal realization of $\Delta\epsilon_2$ driving second-stage efforts. Because $g(h+\beta) < g(h-\beta)$ by log-concavity of g, agent A faces lower second-stage effort costs after winning the first stage than after losing, so A has a "cost-saving incentive" to win the first stage. For agent B, the argument is reversed because bias mitigates agents' heterogeneity when B wins but reinforces it when B loses, so agent B has a "cost-saving disincentive" for stage-1 effort.

Lemma 2 shows that, in equilibrium, informed agents' stage-1 effort differential on average reinforces the ability difference, thus raising the informativeness of the first-stage outcome. The following result extends Proposition 1 to the case of informed agents under the additional assumption that effort costs are quadratic.[18]

[18]The assumption of quadratic costs simplifies the proof that the equilibrium in the limit as $h \to 0$ is

Proposition 3 (Bias with informed agents) *Let $q^0 > \frac{1}{2}$ and suppose that $C_t(e_{i,t}) = \frac{c}{2}e_{i,t}^2$ for all i, t. In the limit as noise swamps ability differences, equilibrium bias $\beta_0^*(q^0) \equiv \lim_{h\to 0}\beta^*(h, q^0)$ is unique, strictly positive, and strictly increasing in q^0, and it solves:*

$$2g(0)\left[\lambda_1 + (2q^0 - 1)\frac{\partial \Delta e_1(\beta_0^*(q^0), 0, q^0)}{\partial h}\right] = \lambda_2 L(\beta_0^*(q^0)). \tag{16}$$

Proposition 3 shows that our insights about the optimal use of bias for selection are robust to the introduction of private information on the part of the agents about their relative abilities. In particular, equilibrium bias continues to remain positive in the limit as noise swamps ability differences. Even though in this limit, the stage-1 effort differential $\Delta e_1(\beta, h, q^0)$ between the "better" agent A and the "worse" agent B vanishes, $\lim_{h\to 0}\frac{\partial \Delta e_1(\beta,h,q^0)}{\partial h} > 0$, which implies that the informativeness of the stage-1 outcome increases as h rises from 0; hence, the left-hand side of (16), just like the left-hand side of (5), is strictly positive, ensuring that equilibrium bias $\beta_0^*(q^0)$ is strictly positive.

The proposition also reveals that the limiting equilibrium bias is strictly increasing in the precision q^0 of the agents' private information and that there are two distinct forces generating this result. First, (16) shows that the larger is q^0, the greater is the impact of any given $\lim_{h\to 0}\frac{\partial \Delta e_1}{\partial h}$, because the effort differential is more likely to be aligned with the ability difference. Second, the larger is q^0, the larger is $\lim_{h\to 0}\frac{\partial \Delta e_1}{\partial h}$ itself, because in period-1 both A's "cost-saving incentive" for effort and B's "cost-saving disincentive" are stronger the more certain the agents are about which agent is better.

This comparative statics result in Proposition 3 provides further insights about the relevance of luck for selection:

Corollary 3 (Persistence amplified) *When agents have private information about their relative abilities, luck is made even more persistent than when agents are uninformed, i.e. $P_0^*(q^0) = G(\beta_0^*(q^0)) > P_0^*$ for all $q^0 > \frac{1}{2}$. The persistence of luck is increasing in the agents' informational advantage relative to the principal, i.e. $P_0^*(q^0)$ is increasing.*

Corollary 3 emphasizes that making luck persistent can be understood as an organizational response to an informational friction. Organizations employ bias for selection even in extremely noisy environments not only because they know *little* about agents' relative abilities but also because they know *less* than agents themselves. Moreover, because with uninformed agents the persistence of luck takes the same value as in the hypothetical situation where agents cannot influence their performance through effort, Corollary 3 relates our theory to an ongoing discussion of what constitutes "merit" (Sen, 2000). Inherited

unique but is not necessary for this result.

talents, acquired abilities, and costly noble acts are all potential sources of merit, endowing its possessor with a justification for receiving decision-making power or economic prosperity. Our theory allows us to distinguish between the case where performance—or merit—is given by the (noisy) sum of an agent's ability *and* effort, and the case where only ability matters. Section 3 showed that whether or not effort is included in the definition of merit is irrelevant for the outcome of organizational selection when agents are uninformed about their relative abilities. However, Proposition 3 and its corollary suggest that with informed agents, organizational selection becomes more biased when merit depends not only on ability but also on efforts. Perhaps surprisingly, when viewed from this angle, our theory thus predicts a greater relevance of luck for selection in situations where agents carry a greater "responsibility" for their performance.

5 Societal luck

Our theory highlights the relevance of early career luck for an individual's long-term success and explains how it arises from an organizational talent selection problem. In our analysis so far, "luck" derives from the inherent noisiness of individual performance. We have abstracted from other factors that could impact relative performance comparisons such as the luck of possessing the "right identity" in the form of gender, race, ethnic origin, or socioeconomic background. There exists evidence showing that individuals with certain identities derive advantages during the early stages of their career that can have long-lasting effects on social and economic outcomes.[19] Conceptually, these forms of "societal luck" are different from what we have considered so far in that actions might condition on them. More specifically, agents' incentives to exert effort might vary with their identity, and organizations might want to reward good performance with biases that depend on whether success was achieved with or without an initial advantage.

In this section, we analyze the impact of societal luck on our organizational selection problem, by assuming that one agent $i \in \{A, B\}$ obtains an exogenous additive advantage of size $\alpha > 0$ that augments the agent's initial performance $x_{i,1}$ but is uncorrelated with ability. We emphasize the transitory nature of the exogenous advantage, which distinguishes it from the agents' ability. In line with our earlier analysis, we examine under what conditions, and to what extent, organizational learning about abilities induces the societal luck of receiving a transitory advantage to have a persistent effect on ultimate

[19] Ciocca Eller (2023) provides evidence that differences in educational achievement of students attending colleges of *equal* selectivity can be traced to heterogeneous socioeconomic backgrounds. Bukodi et al. (2024) document the impact of "parental class" on the attainment of ultra-elite scientific status in the UK.

success, captured here as the probability of being selected.

For simplicity, our remaining analysis focuses on the case in which $q^0 = \frac{1}{2}$.[20] Without loss of generality, we let agent A receive the advantage α, and we assume that this information is common knowledge. Under "identity-dependent (ID) biases", the principal can condition the bias β_i granted to the winner of the first stage on the winner's identity $i \in \{A, B\}$, i.e. on whether or not the winning agent received an advantage. In contrast, under "identity-independent (II) bias", the principal is required to set $\beta_A = \beta_B$. Identity-independent bias might be a consequence of legislation aimed at preventing discriminatory practices.[21] Alternatively, there may be behavioral reasons why advantages or disadvantages are not taken into account, even when they are known to exist.[22]

Given the exogenous advantage α and a bias regime (ID or II), agents choose efforts optimally in response to the bias(es) they anticipate the principal will choose. For the by-now familiar reasons, second-stage efforts are identical across agents, so we can focus on the agents' first-stage effort differential $\Delta e_1 = e_{A,1} - e_{B,1}$. If $\Delta e_1 > 0$ ($\Delta e_1 < 0$), agents' strategic behavior augments (mitigates) the exogenous advantage. We define $\tilde{\alpha} = \alpha + \Delta e_1$ to be the *net advantage* of the advantaged agent. Under ID biases, agents' optimization results in $\Delta e_1 = \Delta e_1^*(\beta_A, \beta_B)$ and hence $\tilde{\alpha} = \alpha + \Delta e_1^*(\beta_A, \beta_B)$. The analogous notation under II bias is $\Delta e_1 = \Delta e_1^*(\beta)$ and $\tilde{\alpha} = \alpha + \Delta e_1^*(\beta)$.

Given α, the principal chooses bias(es) optimally in response to the unobservable yet contemplated stage-1 effort differential and the corresponding net advantage $\tilde{\alpha}$. Denote the principal's optimal choice of biases in the ID-regime by $\beta_A^*(\tilde{\alpha})$ and $\beta_B^*(\tilde{\alpha})$, and let $\beta^*(\tilde{\alpha})$ be her optimal II-bias. An equilibrium with ID biases is a combination of biases and net advantage $(\beta_A^{ID}, \beta_B^{ID}, \tilde{\alpha}^{ID})$ that are mutual best responses, that is, $\beta_A^{ID} = \beta_A^*(\tilde{\alpha}^{ID})$, $\beta_B^{ID} = \beta_B^*(\tilde{\alpha}^{ID})$, and $\tilde{\alpha}^{ID} = \alpha + \Delta e_1^*(\beta_A^{ID}, \beta_B^{ID})$. Similarly, an equilibrium with II bias is a combination $(\beta^{II}, \tilde{\alpha}^{II})$ satisfying $\beta^{II} = \beta^*(\tilde{\alpha}^{II})$ and $\tilde{\alpha}^{II} = \alpha + \Delta e_1^*(\beta^{II})$. We use Δe_1^{ID} and Δe_1^{II} to denote $\Delta e_1^*(\beta_A^{ID}, \beta_B^{ID})$ and $\Delta e_1^*(\beta^{II})$, respectively.

[20] Our result in Corollary 4 on the persistence of societal luck generalizes to arbitrary q^0, for small values of the exogenous advantage.

[21] Title VII of the 1964 Civil Rights Act declares as "an unlawful employment practice [...] to discriminate against any individual because of his race, color, religion, sex, or national origin in admission to, or employment in, any program established to provide apprenticeship or other training."

[22] Exley and Nielsen (2024) document that evaluators correctly expect women to be less confident than men in the assessment of their own abilities but fail themselves to account for this gender gap in their evaluations. We will show that the optimal II bias becomes insensitive to the size of the exogenous advantage α, for α small (i.e. $\lim_{\alpha \to 0} \frac{\partial \beta^*}{\partial \alpha} = 0$); hence for small α our analysis approximates the case where an advantage exists but is neglected by the principal.

For any given net advantage $\tilde{\alpha}$, selective efficiency can be written as:

$$\begin{aligned} S(\beta_A, \beta_B, \tilde{\alpha}) &= \frac{1}{2}[G(\lambda_1 h + \tilde{\alpha})G(\lambda_2 h + \beta_A) + G(-\lambda_1 h - \tilde{\alpha})G(\lambda_2 h - \beta_B)] \\ &+ \frac{1}{2}[G(\lambda_1 h - \tilde{\alpha})G(\lambda_2 h + \beta_B) + G(-\lambda_1 h + \tilde{\alpha})G(\lambda_2 h - \beta_A)]. \end{aligned} \tag{17}$$

The terms in the first (respectively, second) square brackets are the probability that the better agent is selected conditional on being advantaged (respectively, disadvantaged). The principal's optimal identity-dependent biases β_A^* and β_B^* solve the first-order conditions

$$\frac{G(\lambda_1 h + \tilde{\alpha})}{G(-\lambda_1 h + \tilde{\alpha})} = \frac{g(\lambda_2 h - \beta_A^*)}{g(\lambda_2 h + \beta_A^*)} \qquad \text{and} \qquad \frac{G(\lambda_1 h - \tilde{\alpha})}{G(-\lambda_1 h - \tilde{\alpha})} = \frac{g(\lambda_2 h - \beta_B^*)}{g(\lambda_2 h + \beta_B^*)}. \tag{18}$$

In the presence of societal luck, optimal ID biases equate the informativeness of a (hypothetical) second-stage draw to the informativeness of a first-stage win, taking into account that this informativeness depends on whether the stage-1 winner was advantaged or disadvantaged, and by how much, net of the induced effort differential. The log-concavity of g implies that for any positive net advantage $\tilde{\alpha} > 0$, $\beta_B^*(\tilde{\alpha}) > \beta_A^*(\tilde{\alpha}) > 0$. This is because a first-stage win against a net disadvantage is a stronger positive signal about the winner's ability than a first stage-win with the net advantage in the winner's favor.

The principal's optimal identity-independent bias β^* solves $\frac{\partial S}{\partial \beta} = 0$ under the constraint that $\beta_A = \beta_B = \beta$:

$$\frac{G(\lambda_1 h + \tilde{\alpha}) + G(\lambda_1 h - \tilde{\alpha})}{G(-\lambda_1 h + \tilde{\alpha}) + G(-\lambda_1 h - \tilde{\alpha})} = \frac{g(\lambda_2 h - \beta^*)}{g(\lambda_2 h + \beta^*)}. \tag{19}$$

The optimal II bias cannot be adjusted according to the "true" informativeness of the first stage but only in accordance with its "average" informativeness: Given $\tilde{\alpha}$, the principal must set bias without conditioning on whether or not the first-stage winner was helped or hindered by $\tilde{\alpha}$. Consequently, for all $\tilde{\alpha} > 0$, the log-concavity of g implies that

$$\beta_B^*(\tilde{\alpha}) > \beta^*(\tilde{\alpha}) > \beta_A^*(\tilde{\alpha}) > 0. \tag{20}$$

Moreover, $\beta^*(\tilde{\alpha})$ is strictly decreasing in $\tilde{\alpha}$, and $\lim_{\tilde{\alpha} \to 0} \frac{\partial \beta^*}{\partial \tilde{\alpha}} = 0$. These last two properties hold because constraining the magnitude of the bias to be independent of whether the stage-1 winner was advantaged or disadvantaged converts $\tilde{\alpha}$, from the principal's point of view, into an unobservable binary, symmetric shock affecting the stage-1 outcome. The larger this shock, the less informative is a stage-1 win, so the smaller is the optimal II bias; and as the magnitude of this shock tends to 0, it has only a second-order effect on stage-1 informativeness, and hence only a second-order effect on optimal II bias.

We can now characterize and compare equilibrium stage-1 effort differentials:

Proposition 4 (Effort differentials with societal luck) *Let $q^0 = \frac{1}{2}$ and suppose agent A's first-stage performance is augmented by $\alpha > 0$.*

(i) Both in the regime with identity-dependent biases and in that with identity-independent bias, in equilibrium agent A exerts a lower stage-1 effort than agent B but maintains a strict net advantage:

$$-\alpha < \Delta e_1^{II} < 0 \qquad \text{and} \qquad -\alpha < \Delta e_1^{ID} < 0. \tag{21}$$

(ii) If effort costs are $C_t(e_{i,t}) = \frac{c}{2}e_{i,t}^2$, with $c > 0$ sufficiently large for equilibrium in both bias regimes to be unique, and if ability difference h is sufficiently small, the net advantage of agent A in the equilibrium under identity-dependent biases is strictly less than A's net advantage in the equilibrium under identity-independent bias:

$$\tilde{\alpha}^{ID} = \alpha + \Delta e_1^{ID} < \alpha + \Delta e_1^{II} = \tilde{\alpha}^{II}. \tag{22}$$

Similarly to Section 4, where the stage-1 competition was asymmetric due to $q^0 > \frac{1}{2}$, the difference in A's and B's stage-1 efforts arises exclusively from the impact of the stage-1 outcome on stage-2 effort costs. Because a net advantage $\alpha + \Delta e_1 > 0$ for agent A in stage 1 makes A more likely to win that stage, both the agents and the principal are less confident in the first-stage winner's ability when it is A compared to when it is B. Under II bias, the agents will therefore expect the biased second-stage competition to be more balanced and consequently more costly after a stage-1 win by A than after a stage-1 win by B. This difference in stage-2 effort costs gives the advantaged agent, A, a weaker incentive than his rival to exert stage-1 effort, resulting in $\Delta e_1 < 0$. Identity-dependent biases augment this "future effort-cost effect", reducing the induced Δe_1 further below zero, because the principal will optimally choose $\beta_A < \beta_B$ for any anticipated $\alpha + \Delta e_1 > 0$. As long as the ability difference h is not too large, a reduction in β_A makes stage-2 competition even more balanced following a win by A, and an increase in β_B makes stage-2 competition even less balanced following a win by B.[23]

Note that, due to the inability to commit to the bias(es), the principal might not do better in the ID case. Yet, Proposition 4(ii) implies that selective efficiency is always

[23]The assumption of quadratic costs allows us to compare the agents' stage-1 effort differential across the two regimes by focusing on the size of the difference in their marginal benefits of effort.

higher under ID biases than under II bias. Agents' strategic behavior augments the direct benefit for selective efficiency (for any given $\tilde{\alpha}$) of the principal's ability to condition biases on the identity of the first-stage winner.[24]

Our main interest, however, is in the comparison of the *persistence of societal luck*, P_α, across these two equilibria. The probability P_α that the initially advantaged agent is ultimately selected depends on the principal's choice of biases (β_A, β_B) and on the endogenous net advantage $\tilde{\alpha} = \alpha + \Delta e_1$ as follows:

$$P_\alpha(\beta_A, \beta_B, \tilde{\alpha}) = \frac{1}{2} \sum_{\Delta a} [G(\lambda_1 \Delta a + \tilde{\alpha}) G(\lambda_2 \Delta a + \beta_A) + G(-\lambda_1 \Delta a - \tilde{\alpha}) G(\lambda_2 \Delta a - \beta_B)]. \quad (23)$$

For each possible value of the ability difference $\Delta a \in \{-h, h\}$, the first term in the sum in square brackets in (23) is the probability that the advantaged agent wins both stages, while the second term is the probability that this agent loses stage 1 but wins stage 2.

Under identity-independent bias, the effect of a randomly assigned advantage α is *always* persistent, that is, $P_\alpha(\beta^{II}, \beta^{II}, \alpha + \Delta e_1^{II}) > \frac{1}{2}$. This is because, for $\beta_A = \beta_B = \beta$, $P_\alpha(\beta, \beta, \tilde{\alpha})$ can be expressed as

$$P_\alpha(\beta, \beta, \tilde{\alpha}) = \frac{1}{2} \left\{ 1 + [G(\lambda_1 h + \tilde{\alpha}) - G(\lambda_1 h - \tilde{\alpha})][G(\lambda_2 h + \beta) - G(\lambda_2 h - \beta)] \right\}, \quad (24)$$

and in equilibrium, both the advantaged agent's net advantage, $\tilde{\alpha}^{II} = \alpha + \Delta e_1^{II}$, and the principal's choice of bias, β^{II}, are strictly positive, as shown by Proposition 4 and (20). Intuitively, the advantaged agent is selected with higher probability than his rival, because he is more likely to win the first stage (despite his lower effort), and the second stage is biased by the *same* amount, no matter the identity of the first-stage winner. When the II bias β is chosen optimally given $\tilde{\alpha}$, P_α in (24) is strictly increasing in $\tilde{\alpha}$ at $\tilde{\alpha} = 0$ since at $\tilde{\alpha} = 0$, $\frac{\partial \beta^*}{\partial \tilde{\alpha}} = 0$, as explained above. Moreover, P_α is typically hump-shaped in $\tilde{\alpha}$.

In striking contrast, allowing biases to be identity-dependent may completely eliminate the persistence of societal luck. For example, we can show that, when the difference in agents' noise terms has a logistic distribution, then $P_\alpha(\beta_A^{ID}, \beta_B^{ID}, \alpha + \Delta e_1^{ID}) = \frac{1}{2}$ for *all* values of exogenous advantage $\alpha > 0$. The following corollary to Proposition 4 provides a general comparison of the persistence of societal luck between the equilibria under identity-independent and identity-dependent biases. It also compares the expected utility differential ΔU between the advantaged and the disadvantaged agent across these

[24] This result follows because, under II bias, maximized selective efficiency is decreasing in net advantage, by the envelope theorem.

equilibria:

$$\Delta U(\beta_A, \beta_B, \alpha + \Delta e_1) \equiv [2P_\alpha(\beta_A, \beta_B, \alpha + \Delta e_1) - 1] - [C(e_{A,1}) - C(e_{B,1})]. \tag{25}$$

Corollary 4 (Persistence of societal luck) *Under the assumptions of Proposition 4 (ii) and for small headstarts α, allowing bias to be identity-dependent*

(i) reduces the persistence of societal luck:

$$P_\alpha(\beta_A^{ID}, \beta_B^{ID}, \alpha + \Delta e_1^{ID}) < P_\alpha(\beta^{II}, \beta^{II}, \alpha + \Delta e_1^{II}) \tag{26}$$

(ii) reduces the expected utility differential between the advantaged and the disadvantaged agent:

$$\Delta U(\beta_A^{ID}, \beta_B^{ID}, \alpha + \Delta e_1^{ID}) < \Delta U(\beta_A^{II}, \beta_B^{II}, \alpha + \Delta e_1^{II}). \tag{27}$$

We stress that there are two distinct drivers of the reduction in the persistence of societal luck when biases are allowed to be identity-dependent. For any *given* net advantage $\tilde{\alpha}$, persistence is reduced because $\beta_A^*(\tilde{\alpha}) < \beta^*(\tilde{\alpha}) < \beta_B^*(\tilde{\alpha})$, and P_α in (23) is increasing in β_A and decreasing in β_B. This reduction in persistence reflects the fact that ID biases, compared to II bias, effectively penalize in stage 2 the agent who benefited from the advantage in stage 1, independently of the stage-1 outcome.

The second driver behind the reduced persistence under identity-dependent biases is the strategic response of the agents themselves. Because identity-dependent biases induce the advantaged agent to reduce stage-1 effort relative to his rival *even more* than when bias is identity-independent, as shown by Proposition 4, this generates a further reduction in the persistence of societal luck, as long as α is sufficiently small for P_α in (24) to still be increasing in $\tilde{\alpha}$.[25]

The second part of Corollary 4 shows that, for small values of exogenous advantage, allowing for identity-dependent biases lowers the persistence of societal luck by so much that the disadvantaged agent benefits in relative terms, despite his equilibrium effort costs rising relative to those of his rival.

Roemer (2000) advocates that an *equal-opportunity principle* should be applied at the entry level of careers, e.g. for admissions to medical school, while a *non-discrimination*

[25]The mitigating effect of agents' strategic behavior on persistence emerges here with ordinal performance comparisons. In contrast, if cardinal information were available and identity-dependent biases were allowed, the principal could completely filter out the contribution of α, the unique equilibrium stage-1 effort differential would be 0, and the advantaged agent would be no more likely to be selected than his rival.

principle should govern the selection for final positions, such as the licensing of surgeons. Proposition 4 shows that the application of a non-discrimination principle in the selection for positions can backfire, by propagating disadvantages stemming from a failure to establish equal opportunity. In particular, Corollary 4 suggests that the use of identity-dependent biases for selection can help to reduce, if not eliminate, the persistent effects of disadvantages individuals experience during the early stages of their careers. This is accomplished by rewarding early strong performance with larger biases, e.g. in the form of larger grants or swifter fast-tracks, when early success was achieved in spite of disadvantages; the benefits of this policy are further amplified by agents' strategic responses.

6 Conclusion

When the careers of professional hockey players or CEOs are kick-started by the proximity of their birthday to a cut-off or when hedge funds or venture capitalists persistently outperform the market following a fortunate initial investment, luck seems to play an unjustified role in the selection of the most gifted. Such findings and related anecdotes play into the hands of recent critics of a meritocratic worldview (e.g. Piketty, 2014; Sandel, 2020), which, in spite of forming the basis of modern democratic societies, is claimed to be a myth, used as a justification for their exorbitant degrees of economic and social inequality. The main contribution of this paper is to show that making initial luck have a persistent effect on selection is consistent with—if not a necessary feature of—a society aiming to allocate resources and decision-making power to the most able individuals.

Our theory thus illuminates a basic mechanism behind inequality by rationalizing the persistence of luck as an equilibrium outcome of the strategic interaction between an organization aiming to maximize selective efficiency and a group of heterogeneous agents capable of influencing their likelihood of becoming selected through costly efforts. We have characterized the scenarios where the role of initial luck can be expected to be most amplified. This happens when agents are informed about their relative abilities and the organization is restricted to use ordinal rather than cardinal performance information. Both conditions seem more likely to be met towards the top of an organizations hierarchy, which means that we have identified luck as a determinant of selection where, arguably, selection matters most, both, for the organization's payoff and for the induced inequality amongst agents.

We have also analyzed the contribution of organizational learning to the persistence of a different type of luck, "societal luck", reflecting initial advantages that some individuals derive from their identities, e.g. their race or gender. We have shown that non-discrimination laws that restrict organizational learning may backfire by making advantages or disadvantages stemming from unequal opportunities have longer-lasting effects, especially when individuals strategically react to such policies.

Appendix

Proof of Lemmas 1 and 2

Use superscripts w and l, respectively, to distinguish the cases where agent A won and lost the first stage. Define $\Delta e_1 = e_{A,1} - e_{B,1}$, $\Delta e_2^w = e_{A,2}^w - e_{B,2}^w$, and $\Delta e_2^l = e_{A,2}^l - e_{B,2}^l$. Let $q^w(\Delta e_1, q^0)$ and $q^l(\Delta e_1, q^0)$ denote the posterior probabilities that the *winner* of the first stage is the more able agent, given q^0 and Δe_1. When there is no risk of confusion, we suppress the arguments of the posteriors.

We first show that agents exert identical effort in the second stage and that this holds *independently* of q^0 and Δe_1. In case w, A's and B's first-order conditions determining second-stage efforts are:

$$C_2'(e_{A,2}^w) = q^w g\left(h + \beta + \Delta e_2^w\right) + (1 - q^w) g\left(-h + \beta + \Delta e_2^w\right) \tag{28}$$

$$C_2'(e_{B,2}^w) = q^w g\left(-h - \beta - \Delta e_2^w\right) + (1 - q^w) g\left(h - \beta - \Delta e_2^w\right). \tag{29}$$

By the symmetry of g, the marginal returns to effort are identical, so $e_{A,2}^w = e_{B,2}^w$. An analogous argument for case l shows that $e_{A,2}^l = e_{B,2}^l$.

Now consider the agents' incentives for stage-1 effort. We can write the overall utility of agent A as follows:

$$\begin{aligned} -C_1(e_{A,1}) &+ q^0\{G\left(\lambda_1 h + \Delta e_1\right)\left[G\left(\lambda_2 h + \beta + \Delta e_2^w\right) - C_2\left(e_{A,2}^w\right)\right] \\ &+ \left[1 - G\left(\lambda_1 h + \Delta e_1\right)\right]\left[G\left(\lambda_2 h - \beta + \Delta e_2^l\right) - C_2\left(e_{A,2}^l\right)\right]\} \\ &+ (1 - q^0)\{G\left(-\lambda_1 h + \Delta e_1\right)\left[G\left(-\lambda_2 h + \beta + \Delta e_2^w\right) - C_2\left(e_{A,2}^w\right)\right] \\ &+ \left[1 - G\left(-\lambda_1 h + \Delta e_1\right)\right]\left[G\left(-\lambda_2 h - \beta + \Delta e_2^l\right) - C_2\left(e_{A,2}^l\right)\right]\}. \end{aligned} \tag{30}$$

A change in $e_{A,1}$ does not affect $e_{B,2}^w$, $e_{B,2}^l$, or β, because it is unobservable, and the local effect via the induced changes in $e_{A,2}^w$ and $e_{A,2}^l$ is zero by the envelope theorem. Using $\Delta e_2^w = \Delta e_2^l = 0$ and the symmetry of g around 0, the first-order condition for $e_{A,1}$ can be written as

$$\begin{aligned} C_1'(e_{A,1}) = &\left[q^0 g\left(\lambda_1 h + \Delta e_1\right) + (1 - q^0) g\left(-\lambda_1 h + \Delta e_1\right)\right] \\ &\cdot \left\{\left[G\left(\lambda_2 h + \beta\right) - G\left(\lambda_2 h - \beta\right)\right] - \left[C_2\left(e_2^w\right) - C_2\left(e_2^l\right)\right]\right\} \end{aligned} \tag{31}$$

Analogously, for agent B the first-order condition for $e_{B,1}$ can be written as

$$\begin{aligned} C'(e_{B,1}) &= \left[(1-q^0)g\left(\lambda_1 h - \Delta e_1\right) + q^0 g\left(-\lambda_1 h - \Delta e_1\right)\right] \\ &\quad \cdot \left\{\left[G\left(\lambda_2 h + \beta\right) - G\left(\lambda_2 h - \beta\right)\right] + \left[C_2\left(e_2^w\right) - C_2\left(e_2^l\right)\right]\right\} \end{aligned} \tag{32}$$

Again using the symmetry of g, and noting that the component of the marginal benefit of stage-1 effort stemming from the enhanced probability of selection is identical for the two agents, when we subtract (32) from (31), we get

$$\frac{C'(e_{A,1}) - C'(e_{B,1})}{C(e_2^l) - C(e_2^w)} = 2\left[q^0 g\left(\lambda_1 h + \Delta e_1\right) + (1-q^0)g\left(-\lambda_1 h + \Delta e_1\right)\right] \tag{33}$$

Given that costs are strictly increasing and strictly convex, we conclude that in equilibrium, $\Delta e_1 = e_{A,1} - e_{B,1}$ and $e_2^l - e_2^w$ must have the same sign.

To determine the sign of $e_2^l - e_2^w$, compare agent A's first-order conditions for stage-2 effort, after a stage-1 win by A vs. after a stage-1 loss by A, respectively:

$$C'(e_2^w) = q^w(\Delta e_1, q^0)g(\lambda_2 h + \beta) + (1 - q^w(\Delta e_1, q^0))g(-\lambda_2 h + \beta), \tag{34}$$

$$C'(e_2^l) = q^l(\Delta e_1, q^0)g(-\lambda_2 h - \beta) + (1 - q^l(\Delta e_1, q^0))g(\lambda_2 h - \beta). \tag{35}$$

Subtracting the second FOC from the first, and using the symmetry of g, gives

$$C'(e_2^w) - C'(e_2^l) = [q^w(\Delta e_1, q^0) - q^l(\Delta e_1, q^0)][g(\lambda_2 h + \beta) - g(-\lambda_2 h + \beta)]. \tag{36}$$

The strict log-concavity and symmetry of g imply that for any $\beta > 0$, $g(\lambda_2 h + \beta) - g(-\lambda_2 h + \beta) < 0$, while for $\beta = 0$, $g(\lambda_2 h + \beta) - g(-\lambda_2 h + \beta) = 0$. Hence, since costs are strictly convex,

$$e_2^l - e_2^w \gtreqless 0 \iff q^w(\Delta e_1, q^0) - q^l(\Delta e_1, q^0) \gtreqless 0. \tag{37}$$

The posterior beliefs $q^w(\Delta e_1, q^0)$ and $q^l(\Delta e_1, q^0)$ are given by

$$q^w(\Delta e_1, q^0) = \frac{q^0 G(\lambda_1 h + \Delta e_1)}{q^0 G(\lambda_1 h + \Delta e_1) + (1-q^0)G(-\lambda_1 h + \Delta e_1)}, \tag{38}$$

$$q^l(\Delta e_1, q^0) = \frac{(1-q^0)G(\lambda_1 h - \Delta e_1)}{(1-q^0)G(\lambda_1 h - \Delta e_1) + q^0 G(-\lambda_1 h - \Delta e_1)} \tag{39}$$

Observe that q^w and q^l are, respectively, strictly decreasing and strictly increasing in Δe_1. For $q^0 = \frac{1}{2}$, they are equal at $\Delta e_1 = 0$, while for $q^0 > \frac{1}{2}$, they are equal at some $\Delta e_1 > 0$.

We are now in a position to complete the proof of Lemma 1. Let $q^0 = \frac{1}{2}$. Assume first that agents anticipate a bias $\beta = 0$. Then from (36), $e_2^l = e_2^w$. Hence, as shown by the right-hand sides of (31) and (32), at any value of Δe_1, the marginal benefit of stage-1 effort is 0 for both agents, so the unique equilibrium stage-1 efforts are $e_{A,1} = e_{B,1} = 0$. Now let agents anticipate a bias $\beta > 0$. Suppose, for contradiction, that $\Delta e_1 > 0$. Then $q^w(\Delta e_1, q^0) - q^l(\Delta e_1, q^0) < 0$, so by (37), $e_2^l < e_2^w$. In turn, this implies, using (33), that $\Delta e_1 < 0$, which is a contradiction. Analogously, assuming that $\Delta e_1 < 0$ would also lead to a contradiction. Hence, equilibrium requires equal first-stage efforts: $e_{A,1} = e_{B,1}$. These are unique since with $\Delta e_1 = 0$, the right-hand sides of (31) and (32) are independent of the common level of e_1.

To complete the proof of Lemma 2, we need to show that for any $q^0 > \frac{1}{2}$ and any $\beta > 0$, equilibrium entails $e_{A,1} - e_{B,1} > 0$. Suppose, for contradiction, that $\Delta e_1 \leq 0$. Then from (38) and (39), $q^w(\Delta e_1, q^0) - q^l(\Delta e_1, q^0) > 0$, because the agents' prior is that A is more able *and* a stage-1 win by A despite an effort disadvantage is per se a stronger signal of ability than a stage-1 win by B with an effort advantage. By (37), it follows that $e_2^l > e_2^w$. In turn, this implies, using (33), that $\Delta e_1 > 0$, which is a contradiction. ■

Proof of Proposition 1

Equilibrium bias maximizes selective efficiency, $S(\beta; h)$, which for $q_0 = \frac{1}{2}$ by Lemma 1 is given by (3). We use sub-indices to denote partial derivatives. For any $h > 0$, Assumption 1 ensures that the first-order condition $S_\beta(\beta; h) = 0$ uniquely determines the optimal bias $\beta^*(h)$:

$$S_\beta(\beta^*(h); h) = G(\lambda_1 h)\, g(\lambda_2 h + \beta^*(h)) - [1 - G(\lambda_1 h)]\, g(\lambda_2 h - \beta^*(h)) = 0.$$

To see that $\beta^*(h) > 0$ for all $h > 0$ note that $G(\lambda_1 h) > 1 - G(\lambda_1 h)$. However, $\lim_{h\to 0} S_\beta(\beta, h) = 0\ \forall \beta$. Characterizing $\beta_0^* \equiv \lim_{h\to 0} \beta^*(h)$ thus requires totally differentiating $S_\beta(\beta^*(h); h)$ with respect to h, setting it equal to 0, and letting $h \to 0$. Total differentiation yields

$$\frac{d}{dh} S_\beta(\beta^*(h); h) = S_{\beta h}(\beta^*(h); h) + S_{\beta\beta}(\beta^*(h); h) \frac{\partial \beta^*(h)}{\partial h}, \tag{40}$$

where $\lim_{h\to 0} S_{\beta\beta}(\beta; h) = 0\ \forall \beta$ (since $\lim_{h\to 0} S_\beta(\beta; h) = 0\ \forall \beta$). Hence, (40) and Assumption 1(i) imply that β_0^* solves

$$\lim_{h\to 0} S_{\beta h}(\beta^*(h); h) = S_{\beta h}(\beta_0^*, 0) = 2\lambda_1 g(0) g(\beta_0^*) + \lambda_2 g'(\beta_0^*) = 0, \tag{41}$$

which gives (5). Since Assumptions 1(i) and 1(iii) guarantee that $L(0) = 0$ and that L is strictly increasing, it follows that $\beta_0^* > 0$. ■

Proof of Proposition 2

To abbreviate notation we let $k = |\Delta x_1| \geq 0$ denote the observed first-stage margin of victory.

Part (i) Having observed the margin of victory, k, the principal chooses β to maximize the objective in (8), and the first-order condition is

$$S_\beta^{card}(\beta, k; h) = g(k - \lambda_1 h)g(\lambda_2 h + \beta) - g(k + \lambda_1 h)g(\lambda_2 h - \beta) = 0. \tag{42}$$

By Assumption 1, (42) uniquely determines the optimal cardinal bias $\beta^{card}(k, h)$ as a strictly increasing function of k, equal to zero for $k = 0$. Since $\lim_{h\to 0} S_\beta^{card}(\beta, k; h) = 0\ \forall \beta, k$, characterizing $\beta_0^{card}(k) \equiv \lim_{h\to 0} \beta^{card}(k, h)$ requires totally differentiating the value $S_\beta^{card}(\beta^{card}(k, h), k; h)$ with respect to h, setting it equal to zero, and letting $h \to 0$. Doing so shows that $\beta_0^{card}(k)$ solves $\lim_{h\to 0} S_{\beta h}^{card}(\beta, k; h) = 0$, which yields

$$L(\beta_0^{card}(k)) = \frac{\lambda_1}{\lambda_2} L(k),$$

which is equation (9). By Assumption 1, $L(0) = 0$ and $L(k) > 0\ \forall k > 0$. Hence, $\beta_0^{card}(k) > 0\ \forall k > 0$.

Part (ii) Given (5) and (9), we need only show that $\mathbb{E}[L(k)] = 2g(0)$. As $h \to 0$, the density of k converges to $2g(k)$ on support $[0, z]$. Hence

$$\mathbb{E}[L(k)] = \int_0^z L(k)2g(k)dk = -2\int_0^z g'(k)dk = 2g(0),$$

using $g(z) = 0$, which is implied by Assumption 1(iii). ■

Proof of Corollary 2

Part (i) When $L(\cdot)$ is convex, (10) implies that $\beta_0^* \geq E[\beta_0^{card}(k)]$ and hence

$$G(\beta_0^*) \geq G(E[\beta_0^{card}(k)]), \tag{43}$$

since $G(\cdot)$ is strictly increasing. Strict log-concavity and symmetry of $g(\cdot)$ imply that $G(\cdot)$ is strictly concave on the positive domain, so

$$G(E[\beta_0^{card}(k)]) > E[G(\beta_0^{card}(k))]. \tag{44}$$

Inequalities (43) and(44) together imply (13).

Part (ii) Whichever type of information, ordinal or cardinal, is used, and given the ex ante symmetry of the selection process with respect to agents A and B, the limiting value of persistence as $h \to 0$ can be expressed as

$$\begin{aligned} & 2\mathbb{P}(\text{select A}, \Delta\epsilon_1 > 0) \\ = \; & 2\left[\mathbb{P}(\text{select A}, \Delta\epsilon_1 > 0, \Delta\epsilon_2 > 0) + \mathbb{P}(\text{select A}, \Delta\epsilon_1 > 0, \Delta\epsilon_2 < 0)\right] \\ = \; & \frac{1}{2}\left[\mathbb{P}(\text{select A} \,|\, \Delta\epsilon_1 > 0, \Delta\epsilon_2 > 0) + \mathbb{P}(\text{select A} \,|\, \Delta\epsilon_1 > 0, \Delta\epsilon_2 < 0)\right], \end{aligned} \tag{45}$$

where we have used the fact that $\mathbb{P}(\Delta\epsilon_1 > 0, \Delta\epsilon_2 > 0) = \mathbb{P}(\Delta\epsilon_1 > 0, \Delta\epsilon_2 < 0) = \frac{1}{4}$. Since

$$\mathbb{P}(\text{select A} \,|\, \Delta\epsilon_1 > 0, \Delta\epsilon_2 > 0, ord.) = \mathbb{P}(\text{select A} \,|\, \Delta\epsilon_1 > 0, \Delta\epsilon_2 > 0, card.) = 1, \tag{46}$$

it follows that $P_0^* > P_0^{card}$ if and only if

$$\mathbb{P}(\text{select A} \,|\, \Delta\epsilon_1 > 0, \Delta\epsilon_2 < 0, ord.) > \mathbb{P}(\text{select A} \,|\, \Delta\epsilon_1 > 0, \Delta\epsilon_2 < 0, card.). \tag{47}$$

Whether ordinal or cardinal information is used, the ex ante symmetry of the selection process with respect to A and B means that the ex ante probability of selecting A is $\frac{1}{2}$. Using the first equality in (46), and the trivial fact that

$$\mathbb{P}(\text{select A} \,|\, \Delta\epsilon_1 < 0, \Delta\epsilon_2 < 0, ord.) = \mathbb{P}(\text{select A} \,|\, \Delta\epsilon_1 < 0, \Delta\epsilon_2 < 0, card.) = 0, \tag{48}$$

it thus has to hold that

$$\begin{aligned} & \mathbb{P}(\text{select A} \,|\, \Delta\epsilon_1 > 0, \Delta\epsilon_2 < 0, ord.) + \mathbb{P}(\text{select A} \,|\, \Delta\epsilon_1 < 0, \Delta\epsilon_2 > 0, ord.) \\ & = \mathbb{P}(\text{select A} \,|\, \Delta\epsilon_1 > 0, \Delta\epsilon_2 < 0, card.) + \mathbb{P}(\text{select A} \,|\, \Delta\epsilon_1 < 0, \Delta\epsilon_2 > 0, card.) \end{aligned} \tag{49}$$

Using (49), it is then straightforward to confirm that (47) holds if and only if (14) is satisfied. ■

Proof of Proposition 3

In the limit as $h \to 0$, (36) implies that $e_2^w - e_2^l \to 0$, since agents' posterior beliefs about their relative ability become irrelevant to their stage-2 effort incentives. The stage-1 effort differential $\Delta e_1 = e_{A,1} - e_{B,1}$ therefore approaches 0 as $h \to 0$, since the right-hand sides of (31) and (32) become equal.

Using this result, we now characterize the principal's optimal choice of bias, for any anticipated stage-1 effort differential Δe_1. The principal chooses β to maximize selective efficiency $S(\beta, h, q^0)$, where

$$\begin{aligned} S(\beta; h, q^0) &= [q^0 G(\lambda_1 h + \Delta e_1) + (1-q^0) G(\lambda_1 h - \Delta e_1)] G(\lambda_2 h + \beta) \\ &+ [q^0 G(-\lambda_1 h - \Delta e_1) + (1-q^0) G(-\lambda_1 h + \Delta e_1)] G(\lambda_2 h - \beta). \end{aligned} \tag{50}$$

The first-order condition for β is

$$\begin{aligned} S_\beta(\beta; h, q^0) &= [q^0 G(\lambda_1 h + \Delta e_1) + (1-q^0) G(\lambda_1 h - \Delta e_1)] g(\lambda_2 h + \beta) \\ &- [q^0 (1 - G(\lambda_1 h + \Delta e_1)) + (1-q^0)(1 - G(\lambda_1 h - \Delta e_1))] g(\lambda_2 h - \beta) = 0. \end{aligned} \tag{51}$$

Since $\lim_{h\to 0} \Delta e_1 = 0$, $\lim_{h\to 0} S_\beta(\beta; h, q^0) = 0$ for all β. As in the proof of Proposition 1, characterizing the optimal bias $\beta^*(h)$ in the limit as $h \to 0$ thus requires totally differentiating $S_\beta(\beta^*(h), h, q^0)$ with respect to h, setting it equal to 0, and letting $h \to 0$. Since $\lim_{h\to 0} S_\beta(\beta; h, q^0) = 0$ for all β, $\lim_{h\to 0} S_{\beta\beta}(\beta; h, q^0)$ for all β. Hence the limiting optimal bias as $h \to 0$, β_0^*, solves the first-order condition

$$\begin{aligned} 0 = \lim_{h\to 0} S_{\beta h}(\beta^*(h); h, q^0) &= S_{\beta h}(\beta_0^*; 0, q^0) \\ &= 2g(0)g(\beta_0^*)\left[\lambda_1 + (2q^0 - 1)\frac{\partial \Delta e_1}{\partial h}\bigg|_{h \to 0}\right] + \lambda_2 g'(\beta_0^*). \end{aligned} \tag{52}$$

To complete the characterization of equilibrium in the limit as $h \to 0$, we must determine how the limiting derivative with respect to h of the agents' best-response effort differential, $\lim_{h\to 0} \frac{\partial \Delta e_1}{\partial h}$, depends on their anticipations about the principal's choice of β. The derivation of $\lim_{h\to 0} \frac{\partial \Delta e_1}{\partial h}$ is simplified by the following observation, which is based on a symmetry argument: $\lim_{h\to 0} \frac{\partial \beta^*(h)}{\partial h} = 0$.

To show that $\lim_{h\to 0} \frac{\partial \beta^*(h)}{\partial h} = 0$, we begin by observing that since $\lim_{h\to 0} \beta^*(h)$ solves the first-order condition $\lim_{h\to 0} S_{\beta h} = 0$, the sign of $\lim_{h\to 0} \frac{\partial \beta^*(h)}{\partial h} = 0$ will be determined by the sign of $\lim_{h\to 0} S_{\beta hh}(\beta^*(h); h, q^0)$. We will show that $\lim_{h\to 0} S_{hh}(\beta; h, q^0) = 0$ for all β, q^0, from which it follows that $\lim_{h\to 0} S_{\beta hh}(\beta^*(h); h, q^0) = 0$ for all β, q^0 and therefore $\lim_{h\to 0} \frac{\partial \beta^*(h)}{\partial h} = 0$.

To prove that $\lim_{h\to 0} S_{hh}(\beta; h, q^0) = 0$ for all β, q^0, we will show that, for any β, $S(\beta; h, q^0)$, regarded as a function of $h \in \Re$, displays 180° rotational symmetry around the point $(h = 0, S = \frac{1}{2})$, that is, $S(\beta; h, q^0) = 1 - S(\beta; -h, q^0)$. To interpret the mathematical expression $S(\beta; -h, q^0)$, temporarily set $\Delta e_1 = 0$; $S(\beta; -h, q^0)$ then gives the probability of selecting the more able agent when the principal assigns bias β in favor of the stage-1 *loser*. In such a setting, the endogenous stage-1 effort differential would switch sign, that is, $\Delta e_1(-h) = -\Delta e_1(h)$, as can be seen from (33) and (36). Using $\Delta e_1(-h) = -\Delta e_1(h)$, we have

$$\begin{aligned} S(\beta; -h, q^0) &= [q^0 G(\lambda_1 h + \Delta e_1(h)) + (1-q^0) G(\lambda_1 h - \Delta e_1(h))] G(-\lambda_2 h - \beta) \\ &+ [q^0 G(-\lambda_1 h - \Delta e_1(h)) + (1-q^0) G(-\lambda_1 h + \Delta e_1(h))] G(-\lambda_2 h + \beta). \end{aligned} \tag{53}$$

It follows from (53) and (50) that for all h, β, q^0, $S(\beta; h, q^0) = 1 - S(\beta; -h, q^0)$. Differentiating this identity twice with respect to h and letting $h \to 0$ then yields $\lim_{h\to 0} S_{hh}(\beta; h, q^0) = 0$ for all β, q^0.

Having established that $\lim_{h\to 0} \frac{\partial \beta^*(h)}{\partial h} = 0$, we now return to the analysis of how $\lim_{h\to 0} \frac{\partial \Delta e_1}{\partial h}$ depends on agents' anticipations about the principal's choice of β. Differentiating the agents' first-order conditions for stage-1 effort, (31) and (32), with respect to h, letting $h \to 0$, and using $\lim_{h\to 0} \frac{\partial \beta^*(h)}{\partial h} = 0$, yields

$$C_1''(e_0) \left[\frac{\partial e_{A,1}}{\partial h} - \frac{\partial e_{B,1}}{\partial h} \right] = -4\lambda_2 g(0) X'(g(\beta)) g'(\beta)(2q^0 - 1), \tag{54}$$

where e_0 is the agents' common limiting period-1 effort, given β, which solves $C_1'(e_0) = g(0)[2G(\beta) - 1]$, and the function $X(\cdot) \equiv C_2((C_2')^{-1}(\cdot)$. Note that e_0 is independent of q^0 and that strict convexity of $C_2(\cdot)$ ensures that $X(\cdot)$ is strictly increasing. For any anticipated $\beta > 0$ and any $q^0 > \frac{1}{2}$, the right-hand side of (54) is strictly positive, so $\lim_{h\to 0} \frac{\partial \Delta e_1}{\partial h} > 0$.

An *equilibrium* value of β as $h \to 0$, β_0^*, solves the fixed-point equation derived from (52), recognizing the dependence of $\lim_{h\to 0} \frac{\partial \Delta e_1}{\partial h}$ on β_0:

$$2g(0) \left[\lambda_1 + (2q^0 - 1) \frac{\partial \Delta e_1(\beta_0^*; 0, q^0)}{\partial h} \right] = \lambda_2 L(\beta_0^*). \tag{55}$$

Since $\lim_{h\to 0} \frac{\partial \Delta e_1}{\partial h} > 0$ for all $\beta > 0, q^0 > \frac{1}{2}$, the left-hand size of (55) is strictly positive, so any fixed point β_0^* must be strictly positive. To show that the fixed point is unique, use (54) to substitute for $\frac{\partial \Delta e_1(\beta_0; 0, q^0)}{\partial h}$ in (55). This yields, after rearrangement,

$$2\lambda_1 g(0) = \lambda_2 L(\beta_0^*) \left[1 - \frac{8(g(0))^2}{C''(e_0)} X'(g(\beta_0^*)) g(\beta_0^*)(2q^0 - 1)^2 \right]. \tag{56}$$

For $C_t(e_{i,t}) = \frac{c_t}{2} e_{i,t}^2$, $C_1''(e_0)$ is a constant, and $X'(\cdot)$ is linear, so the expression in square brackets on the right-hand side of (56) is strictly increasing in β_0. For quadratic costs, therefore, the right-hand side of (56) is strictly increasing in β_0 whenever the expression in square brackets is positive. Since the left-hand side of (56) is strictly positive, there is a unique equilibrium value β_0^*. Finally, since the right-hand side of (56) is strictly decreasing in q^0 for all $\beta_0 > 0$, the equilibrium β_0^* is increasing in q^0. ■

Proof of Proposition 4

We first derive properties of the principal's optimal bias, given her belief (correct in equilibrium) about the agents' effort differential and the corresponding net advantage $\tilde{\alpha}$. First note that, given net advantage $\tilde{\alpha}$, the principal's optimal biases $\beta_A^*(\tilde{\alpha})$, $\beta_B^*(\tilde{\alpha})$, and $\beta^*(\tilde{\alpha})$ are strictly positive. This is because the left hand sides of the first-order conditions (18) and (19) are strictly larger than one, while the right hand sides are equal to one when bias is zero and strictly increasing in bias by the log-concavity of g. Moreover, $\beta_A^*(\tilde{\alpha}) < \beta^*(\tilde{\alpha}) < \beta_B^*(\tilde{\alpha})$ for $\tilde{\alpha} > 0$ because the principal's "confidence" in the first-stage winner's ability is strictly decreasing in the net advantage the first-stage winner has benefited from, i.e.

$$\frac{G(\lambda_1 h + \tilde{\alpha})}{G(-\lambda_1 h + \tilde{\alpha})} > \frac{G(\lambda_1 h + \tilde{\alpha}) + G(\lambda_1 h - \tilde{\alpha})}{G(-\lambda_1 h + \tilde{\alpha}) + G(-\lambda_1 h - \tilde{\alpha})} > \frac{G(\lambda_1 h - \tilde{\alpha})}{G(-\lambda_1 h - \tilde{\alpha})}. \tag{57}$$

For the same reason, $\beta_A^*(\tilde{\alpha})$ and $\beta^*(\tilde{\alpha})$ are strictly decreasing whereas $\beta_B^*(\tilde{\alpha})$ is strictly increasing. As all three terms in (57) converge to $\frac{G(\lambda_1 h)}{G(-\lambda_1 h)}$ for $\tilde{\alpha} \to 0$, it holds that $\lim_{\alpha\to 0} \beta_A^{ID} = \lim_{\alpha\to 0} \beta_B^{ID} = \lim_{\alpha\to 0} \beta^{II}$. Finally, differentiating the left hand side of (19) with respect to $\tilde{\alpha}$ gives

$$\frac{2[g(\lambda_1 h + \tilde{\alpha}) - g(\lambda_1 h - \tilde{\alpha})]}{[G(-\lambda_1 h + \tilde{\alpha}) + G(-\lambda_1 h - \tilde{\alpha})]^2}, \tag{58}$$

which converges to zero for $\alpha \to 0$, proving that $\lim_{\tilde{\alpha}\to 0} \frac{\partial \beta^*}{\partial \tilde{\alpha}} = 0$. And since the first-order conditions (18) determining optimal identity-dependent biases are identical except for the sign of $\tilde{\alpha}$, it has to hold that $\lim_{\tilde{\alpha}\to 0} \frac{\partial \beta_A^*}{\partial \tilde{\alpha}} = -\lim_{\tilde{\alpha}\to 0} \frac{\partial \beta_B^*}{\partial \tilde{\alpha}}$. Having established the properties of the principal's optimal bias response we can now turn our attention to the Proposition's claims about agents' first-stage effort differential.

Part (i) The proof of this claim treats jointly the cases of identity-dependent and identity-independent bias, for the latter simply impose $\beta_A = \beta_B = \beta$ and all arguments go through for all $\beta > 0$. In the second stage, agent A's effort equals agent B's effort. The proof of this claim is analog to Lemma 1 and will thus be skipped. Let e_2^w and e_2^l denote the agents' (identical) second-stage efforts after the advantaged agent A won or lost the first stage, respectively. Agent A's expected utility in stage one is then given by

$$-C(e_{A,1}) + \frac{1}{2} \sum_{\Delta a \in \{-h,h\}} \{G(\lambda_1 \Delta a + \alpha + \Delta e_1)[G(\lambda_2 \Delta a + \beta_A) - C(e_2^w)] \\ +G(-\lambda_1 \Delta a - \alpha - \Delta e_1)[G(\lambda_2 \Delta a - \beta_B) - C(e_2^l)]\}, \tag{59}$$

and the corresponding first order condition is

$$2C'(e_{A,1}) = \sum_{\Delta a} g(\lambda_1 \Delta a + \alpha + \Delta e_1)[G(\lambda_2 \Delta a + \beta_A) - G(\lambda_2 \Delta a - \beta_B) + C(e_2^l) - C(e_2^w)] \tag{60}$$

Similarly, for agent B we get

$$2C'(e_{B,1}) = \sum_{\Delta a} g(\lambda_1 \Delta a + \alpha + \Delta e_1)[G(-\lambda_2 \Delta a + \beta_B) - G(-\lambda_2 \Delta a - \beta_A) + C(e_2^w) - C(e_2^l)]$$

Comparing the marginal benefits of effort across agents it follows from $G(x) = 1 - G(x)$ that those parts stemming from the enhanced probability of selection are identical and subtraction of the two equations yields:

$$\frac{C'(e_{A,1}) - C'(e_{B,1})}{C(e_2^l) - C(e_2^w)} = \sum_{\Delta a \in \{-h,h\}} g(\lambda_1 \Delta a + \alpha + \Delta e_1). \tag{61}$$

Given that costs are increasing and convex, in equilibrium, $\Delta e_1 = e_{A,1} - e_{B,1}$ and $e_2^l - e_2^w$ have to have the same sign. To determine the latter, consider the advantaged agent A's expected utility in the second stage, separately for the two cases where the advantaged agent won (w) or lost (l) the first stage, respectively:

$$q^w G(\lambda_2 h + \beta_A + \Delta e_2^w) + (1 - q^w) G(-\lambda_2 h + \beta_A + \Delta e_2^w) - C(e_{A,2}^w), \tag{62}$$

$$q^l G(-\lambda_2 h - \beta_B + \Delta e_2^w) + (1 - q^l) G(\lambda_2 h - \beta_B + \Delta e_2^w) - C(e_{A,2}^w). \tag{63}$$

Here we have introduced

$$q^w = \frac{G(\lambda_1 h + \alpha + \Delta e_1)}{G(\lambda_1 h + \alpha + \Delta e_1) + G(-\lambda_1 h + \alpha + \Delta e_1)}, \tag{64}$$

$$q^l = \frac{G(\lambda_1 h - \alpha - \Delta e_1)}{G(\lambda_1 h - \alpha - \Delta e_1) + G(-\lambda_1 h - \alpha - \Delta e_1)} \tag{65}$$

to denote the updated probabilities that the winner of the first-stage constitutes the more able agent. The corresponding first order conditions determining e_2^w and e_2^l are given by:

$$C'(e_2^w) = q^w g(\lambda_2 h + \beta_A) + (1 - q^w) g(-\lambda_2 h + \beta_A), \quad (66)$$

$$C'(e_2^l) = q^l g(-\lambda_2 h - \beta_B) + (1 - q^l) g(\lambda_2 h - \beta_B). \quad (67)$$

Note that q^w and q^l are decreasing, respectively, increasing functions of the advantaged agent's net advantage and that

$$q^l > q^w \Leftrightarrow \alpha + \Delta e_1 > 0. \quad (68)$$

Also note that, as we argued above, with identity-dependent biases, the principal awards a larger bias when she is more certain that the first-stage winner constitutes the more able agent, that is, in equilibrium $\beta_A - \beta_B$ and $q^w - q^l$ have to have the same sign.

We now argue, by contradiction, that $-\alpha < \Delta e_1 < 0$. Suppose, instead, that, $\Delta e_1 \leq -\alpha$. Then $\alpha + \Delta e_1 \leq 0$ implies that $q^l \leq q^w$ and thus $\beta_A \geq \beta_B$. (For identity-independent bias, this condition holds trivially.) We have, for all $\beta \in (0, \beta_A]$:

$$\frac{q^w}{1 - q^w} = \frac{g(\lambda_2 h - \beta_A)}{g(\lambda_2 h + \beta_A)} \geq \frac{g(\lambda_2 h - \beta)}{g(\lambda_2 h + \beta)} > \frac{g'(\lambda_2 h - \beta)}{g'(\lambda_2 h + \beta)} = -\frac{g'(-\lambda_2 h + \beta)}{g'(\lambda_2 h + \beta)} \quad (69)$$

where the first equality is the principal's first-order condition for β_A, the two inequalities follow from $\beta \in (0, \beta_A]$ and the strict log-concavity of g, and the second equality holds because g is symmetric. Hence, for $\beta \in (0, \beta_A]$:

$$q^w g'(\lambda_2 h + \beta) + (1 - q^w) g'(-\lambda_2 h + \beta) < 0 \quad (70)$$

and therefore

$$q^w g(\lambda_2 h + \beta_A) + (1 - q^w) g(-\lambda_2 h + \beta_A) \leq q^w g(\lambda_2 h + \beta_B) + (1 - q^w) g(-\lambda_2 h + \beta_B) \quad (71)$$

with strict inequality if $\beta_B < \beta_A$. Since $\beta_B > 0$ and, under the hypothesis, $q^l \leq q^w$, the right hand side of (71) is less than or equal to $q^l g(\lambda_2 h + \beta_B) + (1 - q^l) g(-\lambda_2 h + \beta_B)$. Hence (66) and (67) imply that $C'(e_2^w) \leq C'(e_2^l)$ and by the convexity of C it follows that $e_2^w \leq e_2^l$. As in equilibrium, Δe_1 has to have the same sign as $e_2^l - e_2^w \geq 0$ we obtain a contradiction to our assumption that $\Delta e_1 \leq -\alpha < 0$.

Similarly, if $\Delta e_1 \geq 0$ then it follows from $\alpha + \Delta e_1 > 0$ that $q^l > q^w$, so $\beta_B > \beta_A$. Now we have, for all $\beta \in (0, \beta_B)$:

$$\frac{q^l}{1-q^l} = \frac{g(\lambda_2 h - \beta_B)}{g(\lambda_2 h + \beta_B)} > \frac{g(\lambda_2 h - \beta)}{g(\lambda_2 h + \beta)} > \frac{g'(\lambda_2 h - \beta)}{g'(\lambda_2 h + \beta)} = -\frac{g'(-\lambda_2 h + \beta)}{g'(\lambda_2 h + \beta)}, \tag{72}$$

and thus

$$q^l g'(\lambda_2 h + \beta) + (1-q^l) g'(-\lambda_2 h + \beta) < 0. \tag{73}$$

Hence, since $\beta_B > \beta_A > 0$,

$$q^l g(\lambda_2 h + \beta_B) + (1-q^l) g(-\lambda_2 h + \beta_B) < q^l g(\lambda_2 h + \beta_A) + (1-q^l) g(-\lambda_2 h + \beta_A) \tag{74}$$

which is strictly smaller than $q^w g(\lambda_2 h + \beta_A) + (1-q^w) g(-\lambda_2 h + \beta_A)$ because $q^l > q^w$. So it follows from (66) and (67) that $C'(e_2^w) > C'(e_2^l)$ and thus $e_2^w > e_2^l$. As in equilibrium, Δe_1 has to have the same sign as $e_2^l - e_2^w < 0$ we obtain a contradiction to our assumption that $\Delta e_1 \geq 0$.

Part (ii) Let $(\beta^{II}, \Delta e_1^{II})$ and $(\beta_A^{ID}, \beta_B^{ID}, \Delta e_1^{ID})$ denote the unique equilibrium with identity-independent and identity-dependent biases, respectively. Assume that h is sufficiently small such that $-\lambda_2 h + \beta_A^{ID} \geq 0$. Choosing h like that is possible because, by analogy to Proposition 1, it holds that $\lim_{h \to 0} \beta_A^{ID} > 0$. We now show that

$$\Delta e_1^{ID} < \Delta e_1^{II}. \tag{75}$$

By contradiction, assume that $\Delta e_1^{ID} \geq \Delta e_1^{II}$. Starting from $(\beta_A^{ID}, \beta_B^{ID}, \Delta e_1^{ID})$ suppose the principal is restricted to use identity-independent bias, resulting in the choice $\hat{\beta} = \beta^*(\alpha + \Delta e_1^{ID})$. Consider the agents' corresponding effort response $\Delta e_1^*(\hat{\beta}, \hat{\beta})$. As costs are quadratic it follows from (61) that the agents' first-stage effort-differential satisfies the implicit equation

$$c\Delta e_1 - [C(e_2^l) - C(e_2^w)] \sum_{\Delta a \in \{-h, h\}} g(\lambda_1 \Delta a + \alpha + \Delta e_1) = 0, \tag{76}$$

with

$$\begin{aligned} C(e_2^l) - C(e_2^w) &= \frac{1}{c}[q^l g(-\lambda_2 h - \beta_B) + (1-q^l) g(\lambda_2 h - \beta_B)]^2 \\ &- \frac{1}{c}[q^w g(\lambda_2 h + \beta_A) + (1-q^w) g(-\lambda_2 h + \beta_A)]^2. \end{aligned} \tag{77}$$

Because $\beta_A^{ID} < \beta^*(\alpha + \Delta e_1^{ID}) < \beta_B^{ID}$ as shown above, the move from $\beta_A = \beta_A^{ID}$ and

$\beta_B = \beta_B^{ID}$ to $\beta_A = \beta_B = \beta^*(\alpha + \Delta e_1^{ID})$ decreases $g(\lambda_2 h + \beta_A)$ and increases $g(-\lambda_2 h - \beta_B)$ and, given $-\lambda_2 h + \beta_A^{ID} \geq 0$ (which implies $\lambda_2 h - \beta_B^{ID} < 0$) it also decreases $g(-\lambda_2 h + \beta_A)$ and increases $g(\lambda_2 h - \beta_B)$. The move from $(\beta_A^{ID}, \beta_B^{ID})$ to $\beta^*(\alpha + \Delta e_1^{ID})$ thus reduces (76) for any fixed Δe_1 by increasing $C(e_2^l) - C(e_2^w)$ which is negative, as shown in the proof of claim (i). Given that (76) is negative for $\Delta e_1 = -\alpha$ and positive for $\Delta e_1 = 0$ and equilibrium is unique (which is guaranteed by the assumption that c is sufficiently large), the move from $(\beta_A^{ID}, \beta_B^{ID})$ to $\beta^*(\alpha + \Delta e_1^{ID})$ thus leads to an increase in Δe_1, i.e. we have shown that $\Delta e_1^*(\beta^*(\alpha + \Delta e_1^{ID}), \beta^*(\alpha + \Delta e_1^{ID})) > \Delta e_1^*(\beta_A^{ID}, \beta_B^{ID}) = \Delta e_1^{ID}$.

To see that this leads to a contradiction, let $\gamma = (\beta^*)^{-1} - \alpha$. Then $\gamma(\beta)$ gives the effort-differential conjecture, Δe_1, that makes β the principal's optimal choice of identity-independent bias. Given uniqueness of the equilibrium $(\beta^{II}, \Delta e_1^{II})$ the curves $\gamma(\beta)$ and $\Delta e_1^*(\beta, \beta)$ intersect exactly once. And because $\Delta e_1^*(\beta, \beta)$ goes to zero for $\beta \to 0$ and for $\beta \to \infty$ and $\gamma(\beta)$ is strictly decreasing, $\Delta e_1^*(\beta, \beta)$ has to cross $\gamma(\beta)$ from below. In particular, for any $\beta < \beta^{II}$ it has to hold that $\gamma(\beta) > \Delta e_1^*(\beta, \beta)$. Note that $\hat{\beta} = \beta^*(\alpha + \Delta e_1^{ID}) < \beta^*(\alpha + \Delta e_1^{II}) = \beta^{II}$ because β^* is decreasing and we have assumed that $\Delta e_1^{ID} > \Delta e_1^{II}$. Hence $\gamma(\hat{\beta}) > \Delta e_1^*(\hat{\beta}, \hat{\beta})$, or formulated equivalently, $\Delta e_1^{ID} = \Delta e_1^*(\beta_A^{ID}, \beta_B^{ID}) > \Delta e_1^*(\beta^*(\alpha + \Delta e_1^{ID}), \beta^*(\alpha + \Delta e_1^{ID}))$, which contradicts our earlier finding. ■

Proof of Corollary 4

This proof assumes that α is sufficiently small such that $P_\alpha(\beta^*(\tilde{\alpha}), \beta^*(\tilde{\alpha}), \tilde{\alpha})$ is increasing in $\tilde{\alpha}$ for all $\tilde{\alpha} < \alpha + \Delta e_1^{II}$. Choosing α like that is possible because, as shown above, $\lim_{\alpha \to 0} \Delta e_1^{II} = 0$ and $\lim_{\tilde{\alpha} \to 0} \frac{\partial \beta^*}{\partial \tilde{\alpha}} = 0$, so that $P_\alpha(\beta^*(\tilde{\alpha}), \beta^*(\tilde{\alpha}), \tilde{\alpha})$ is increasing in $\tilde{\alpha}$ for small α by the envelope theorem.

Part (i) This claim is true because $P_\alpha(\beta_A^{ID}, \beta_B^{ID}, \alpha + \Delta e_1^{ID}) < P_\alpha(\beta^{II}, \beta^{II}, \alpha + \Delta e_1^{II})$ follows from (75). To see this note first that, as P_α is increasing in β_A but decreasing in β_B, it holds that $P_\alpha(\beta_A^{ID}, \beta_B^{ID}, \alpha + \Delta e_1^{ID}) < P_\alpha(\beta^*(\alpha + \Delta e_1^{ID}), \beta^*(\alpha + \Delta e_1^{ID}), \alpha + \Delta e_1^{ID})$, because, as shown above, the principal's optimal biases satisfy $\beta_A^{ID} = \beta_A^*(\alpha + \Delta e_1^{ID}) < \beta^*(\alpha + \Delta e_1^{ID}) < \beta_B^*(\alpha + \Delta e_1^{ID}) = \beta_B^{ID}$. And because, by assumption, $P_\alpha(\beta^*(\tilde{\alpha}), \beta^*(\tilde{\alpha}), \tilde{\alpha})$ is increasing in $\tilde{\alpha}$ for all $\tilde{\alpha} < \alpha + \Delta e_1^{II}$ it follows from (75) that $P_\alpha(\beta^*(\alpha + \Delta e_1^{ID}), \beta^*(\alpha + \Delta e_1^{ID}), \alpha + \Delta e_1^{ID}) < P_\alpha(\beta^*(\alpha + \Delta e_1^{II}), \beta^*(\alpha + \Delta e_1^{II}), \alpha + \Delta e_1^{II}) = P_\alpha(\beta^{II}, \beta^{II}, \alpha + \Delta e_1^{II})$.

Part (ii) To prove the second claim, we determine the effect of a move from identity-independent to identity-dependent bias on the agents' utility differential by considering

$$\lim_{\alpha \to 0} \frac{d}{d\alpha} \Delta U(\beta_A, \beta_B, \alpha + \Delta e_1) = 2 \frac{dP_\alpha}{d\alpha}|_{\alpha=0} + c e_1^* \frac{\partial \Delta e_1}{\partial \alpha}|_{\alpha=0}. \tag{78}$$

Here we used that costs are quadratic and that in the limit agents exert the same first-stage effort $e_1^* = \lim_{\alpha\to 0} e_{A,1}^* = \lim_{\alpha\to 0} e_{B,1}^*$. From the corresponding first order condition we get

$$ce_1^* = g(\lambda_1 h)[G(\lambda_2 h + \beta^*(0)) - G(\lambda_2 h - \beta^*(0))]. \tag{79}$$

Using the fact that, as shown in the proof of Proposition 4, $\lim_{\alpha\to 0} \frac{\partial \beta_A^*}{\partial \alpha} = -\lim_{\alpha\to 0} \frac{\partial \beta_B^*}{\partial \alpha}$ we get

$$\begin{aligned}\frac{dP_\alpha^{ID}}{d\alpha}|_{\alpha=0} &= g(\lambda_1 h)[G(\lambda_2 h + \beta^*(0)) - G(\lambda_2 h - \beta^*(0))]\left(1 + \frac{d\Delta e_1^{ID}}{d\alpha}|_{\alpha=0}\right) \\ &\quad + [G(\lambda_1 h)g(\lambda_2 h + \beta^*(0)) + G(-\lambda_1 h)g(-\lambda_2 h + \beta^*(0))]\frac{d\beta_A^*}{d\alpha}|_{\alpha=0}\end{aligned} \tag{80}$$

whereas $\lim_{\alpha\to 0} \frac{\partial \beta^*}{\partial \alpha} = 0$ implies

$$\frac{dP_\alpha^{II}}{d\alpha}|_{\alpha=0} = g(\lambda_1 h)[G(\lambda_2 h + \beta^*(0)) - G(\lambda_2 h - \beta^*(0))]\left(1 + \frac{d\Delta e_1^{II}}{d\alpha}|_{\alpha=0}\right). \tag{81}$$

For the difference we thus get

$$\begin{aligned}\lim_{\alpha\to 0} \frac{d(\Delta U^{ID} - \Delta U^{II})}{d\alpha} &= g(\lambda_1 h)[G(\lambda_2 h + \beta^*(0)) - G(\lambda_2 h - \beta^*(0))]\frac{d(\Delta e_1^{ID} - \Delta e_1^{II})}{d\alpha}|_{\alpha=0} \\ &+ [G(\lambda_1 h)g(\lambda_2 h + \beta^*(0)) + G(-\lambda_1 h)g(-\lambda_2 h + \beta^*(0))]\frac{d\beta_A^*}{d\alpha}|_{\alpha=0}.\end{aligned}$$

This is strictly negative, because $\frac{d\beta_A^*}{d\alpha}|_{\alpha=0} < 0$ as shown in the proof of Proposition 4 and because our analysis above implies that $\Delta e_1^{ID} - \Delta e_1^{II}$ must be non-increasing for small α. Given that for $\alpha \to 0$, $\Delta U^{ID} = \Delta U^{II} = 0$, for small α it must therefore hold that $\Delta U(\beta_A^{ID}, \beta_B^{ID}, \alpha + \Delta e_1^{ID}) < \Delta U(\beta_A^{II}, \beta_B^{II}, \alpha + \Delta e_1^{II})$. ∎

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