Fighting for Lemons: The Balancing Effect of Private Information on Incentives in Dynamic Contests

Marc Möller* Juan Beccuti†

December 28, 2024

Abstract

In a common value environment with multi-stage competition, losing a stage conveys positive news about a rival's estimation of a contested prize, capable of balancing the discouraging effect of falling behind. We show that, due to players' learning from stage-outcomes, aggregate incentives under private information are often greater than under public information and may even exceed the static competition benchmark. Moreover, laggards can become *more* motivated than leaders, giving rise to long-lasting fights. Our results have implications for the duration of R&D races, the desirability of feedback in labor- and procurement-contests, and the campaign spending and selective efficiency of presidential primaries.

Keywords: Dynamic contests, private information, learning, discouragement effect, information design.
JEL: C72, D72, D82.

*Department of Economics, University of Bern, 3001 Bern, Switzerland. Email: marc.moeller@vwi.unibe.ch. Tel. +41316848078.

†Department of Economics, University of Bern, 3001 Bern, Switzerland. Email: juan.beccuti@vwi.unibe.ch.

1 Introduction

Competition for a scarce resource, such as a patent, political candidacy, promotion, procurement contract, or, generally, a *prize*, is frequently of a dynamic nature, thus featuring the possibility that competitors have taken the lead or have fallen behind. There are concerns that in such situations, incentives are undermined by the so-called *discouragement effect* (Konrad, 2009): As followers feel discouraged by the costs of catching up, leaders can allow themselves to lower their efforts, resulting in a reduction of incentives on aggregate.[1] Evidence supporting these concerns has been reported not only for experimental settings (Mago et al., 2013) and sports (Malueg and Yates, 2010, Iqbal and Krumer, 2019), but also by an influential recent study of online innovation contests (Lemus and Marshall, 2021).

The consequences of the discouragement effect are far reaching and have been noted for a broad variety of settings. In R&D races, an early breakthrough may mute the investment-incentives of rival firms and lead to a slow-down of innovation (Fudenberg et al., 1983; Harris and Vickers, 1987; Judd et al., 2012). In promotion tournaments, workers can become demotivated by the achievements of their co-workers, putting under scrutiny the wide-spread use of interim performance evaluations (Klein and Schmutzler, 2017) and feedback policies (Gershkov and Perry, 2009; Aoyagi, 2010; Ederer, 2010; Goltsman and Mukherjee, 2011). In presidential primaries, overall campaign spending is reduced and early voting in non-representative districts can become decisive for the overall outcome of the election (Klumpp and Polborn, 2006). Finally, in sports competitions, performance differences accumulated during earlier stages may lead to a deterioration of suspense (Chan et al., 2009).

In this article, we argue that, besides their direct, discouraging effect, stage-wins or -losses may have an indirect, *informational* effect, which improves incentives on aggregate by making competition more *balanced*. Our starting point is the observation that, while in many of the aforementioned applications stage-wins or -losses are observable, contestants typically cannot observe each others' efforts and may be privately informed about the, arguably, common value of the contested prize. In such situations, a stage-loss (-win) represents good (bad) news about the contest's prize, because the likelihood of a loss (win) is increasing (decreasing) in the opponent's effort which correlates with his private information. For example, in an R&D race, an early breakthrough may be the consequence of a large but unobservable investment by a rival company whose market-

[1] This effect has also been denoted as *momentum effect*, to distinguish it from the purely *static* discouragement resulting from differences in contestants' abilities (Drugov and Ryvkin, 2023).

research has revealed a profitable future for the contested innovation. In the presence of private information, it is therefore no longer clear whether in a dynamic contest aggregate incentives are reduced by the observation of intermediate outcomes and whether losers of early stages will exert smaller efforts than their rivals.

To understand why both issues are relevant from an applied perspective, note that in settings—such as labor contests and promotion tournaments—where the maximization of *aggregate* incentives constitutes an objective of the contest's designer, feedback policies might prove more beneficial when contestants' learning about prizes—such as the value of a promotion—is accounted for.[2] Moreover, in situations—such as the regulation of innovative industries—where competitive balance is a valid concern, accounting for contestants' learning about prizes–such as the value of a patent—can mitigate concerns that initial leaders will become too dominant.[3] Symbolic for the relevance of such learning effects is the U.S. victory in the "race to the moon", in spite of the U.S.S.R's advantage in rocket propulsion technology. Logsdon (2010) argues that it was the news about Juri Gagarin's first orbital flight that induced Kennedy to make NASA a 400,000 employee agency.

To shed light on the effects of learning on incentives in dynamic contests, Section 2 introduces a stylized model where two homogeneous contestants compete by exerting costly efforts in three sequential battles. We allow for a generic class of mappings between efforts and battle outcomes, including the frequently employed Tullock (1980) success function as a special case. The winner of the best-of-three contest obtains a prize whose common value is uncertain, either positive or zero.[4] At the start of the contest, contestants receive private, independent, and identically distributed signals, that are informative about the contest's prize, and whose realizations can be either good or bad. In each battle, contestants learn whether they win or lose but cannot observe their rival's effort.

Our model owes its tractability to the assumption that the underlying information structure is partially conclusive. In particular, we assume that, while conditional on the

[2]Evidence for incentive improving effects of feedback has been documented not only for labor settings (Blanesl and Nossol, 2011), but also for education (Azmat and Iriberri, 2010) and innovation (Lemus and Marshall, 2021) where aggregate effort matters at least partially.

[3]Evidence that initial breakthroughs do not necessarily lead to market-dominance abound in the pharmaceutical industry. For instance, cholesterol lowering compounds, so-called statins, were first discovered by the Japanese company Sankyo, but it was Merck who invested heavily into the development of the best-selling drug Zocor (Endo, 2010).

[4]In a sports-, labor-, or campaign-setting, the contest's prize may consist of the continuation value of reaching the next higher level, which might be reduced, potentially to zero, by information about an invincible future opponent. Note that, in our model, a zero prize is equivalent to prohibitively high costs of effort, which arise in an R&D setting when a potential innovation, e.g. a vaccine, proves to be infeasible.

prize being zero, either a good or a bad signal can be observed, both contestants will receive a good signal when the prize is positive.[5] It follows that contestants will conclude from the observation of a bad signal that the contest's prize must be zero and that it is optimal to refrain from exerting effort. The characterization of a Perfect Bayesian equilibrium—a challenging task for models combining dynamic competition with private information—is thus simplified to the description of the contestants' effort choices, conditional on the receipt of a good signal.

In Section 3.1 we show that, in accordance with the aforementioned intuition, players' learning reduces the gap between the leader's and the follower's efforts in the Perfect Bayesian equilibrium. Moreover, when the battles' rate of rent dissipation is sufficiently low, the follower's effort may exceed the leader's, making it *less* likely that the contest is decided after two rather than three battles. In empirical studies, contests that are decided within a few battles have been interpreted as evidence for the discouragement effect. Our finding, that in the presence of private information, the discouragement effect is mitigated means that long-lasting fights will be more frequently observed and need not be an indication for the absence of discouragement.[6]

The balancing effect of learning on a leader's and a follower's incentives turns out to have important implications for incentives on aggregate. In Section 3.2 we first show that aggregate effort in the Perfect Bayesian equilibrium is higher than in the public information benchmark where all signals are observed publicly, as long as learning improves the contest's competitive balance. Learning raises effort on aggregate because it mitigates the asymmetry of incentives that unfolds during the course of a dynamic contest by raising expectations of players who have fallen behind while lowering expectations of those in the lead. In a dynamic contest, private information thus improves incentives by *leveling the playing field.* Naturally, because efforts are costly, a direct implication of this result is that private information can be harmful from the contestants' perspective, i.e. asymmetric information may lead to "fighting for lemons".

Given its potential to improve incentives, a natural question to ask is whether learning can be strong enough to more than compensate for the discouragement effect that arises

[5] This information structure is called the "bad news" model in the literature on strategic experimentation (e.g. Keller and Rady, 2015; Bonatti and Hörner, 2017). "Good news" models (e.g. Keller et al., 2005) produce different investment and learning dynamics. This suggests an investigation of the effects of good news in dynamic contests which is left for future research. For a discussion of a model where information is non-conclusive—neither good news nor bad news—see Section 7.

[6] Ferrall and Smith (1999) argue that in basketball-, hockey-, and baseball-playoffs "a simple model in which players do not give up [...] best explains the outcome of the championship series." Similarly, Zizzo (2002) denotes the lack of evidence for discouragement in experimental patent race data as "a puzzle from the perspective of patent race theory."

from the dynamic nature of competition. We show that, perhaps surprisingly, aggregate incentives in the Perfect Bayesian equilibrium can be *higher* than in the static competition benchmark, where all battles take place simultaneously. This happens, when learning tips competitive balance in favor of the follower, inducing the follower to exert a higher effort than the leader. The updating of beliefs in opposite directions creates a "learning-based" incentive to exert additional effort because establishing a lead allows a player to remain skeptic about the contest's prize, thereby insuring him against excessive future effort costs. Our result thus establishes that the common wisdom, that incentives are reduced by the dynamic nature of competition, needs not hold in contests that are subject to private information.

In Section 4 we relate our work to a nascent literature on information design in contests by characterizing the contest's optimal (partly-conclusive) information structure.[7] Our analysis reveals a dichotomy of optimal contest designs with the optimal information structure selecting between two modes of competition. Either, contestants are induced to fight hard to establish themselves as leaders of the competition and leaders are likely to become final winners. Or, incentives are relatively weak at the start of the contest but fighting is likely to last until the very end. Our theory can thus be used to characterize the circumstances under which competition can be expected to be fierce but short or mild but long-lasting.

In Section 5, we extend our model by introducing heterogeneity in the contestants' valuation of the contest's prize. A valid concern is that private information, although beneficial for aggregate incentives, may have a negative impact on a contest's selective efficiency. In particular, because a low-valuing contestant is more likely to be lagging behind, narrowing the gap between a leader's and a follower's effort through learning may have the adverse effect of reducing the likelihood with which the high-valuing contestant can claim the contest's prize. We argue that this intuition is incomplete and show that, instead, private information can have a *positive* effect not only on aggregate incentives but also on a contest's selective efficiency.

The robustness of our results is discussed in Section 6 where we use a common value model familiar from auction theory to argue that the balancing effect of learning on incentives continues to exist for more general information structures and sets of potential prizes. Section 7 concludes with a discussion of longer horizons. All formal proofs can be found in the Appendix.

[7] The existing literature on information design has mostly restricted attention to static contests. For a detailed discussion of this literature see Section 4.

Related literature

The discouragement effect has made its first appearance in the literature on R&D races, where it can take the particularly severe form of ϵ-preemption (Fudenberg et al., 1983): Even the smallest innovation advantage can obstruct the investment of rival firms. The seminal model of an R&D race (Harris and Vickers, 1987) as well as more recent extensions (Cao, 2014) take the format of a best-of-N contest and its battle-components are strategically equivalent to a Tullock contest when investments are lump-sum (Baye and Hoppe, 2003). Our results thus apply and they suggest that, due to the inherently uncertain value of innovation, the dynamic nature of R&D-competition is not an obstacle but may in fact promote investment on aggregate, because firms' become encouraged by the success of their rivals. This finding resonates well with the idea of Choi (1991) that a rival's success may improve a firm's belief in the feasibility of a contested innovation (see also Malueg and Tsutsui, 1997 and Bimpikis et al., 2019) and that hiding information about failures can be optimal for incentives when prize-sharing rules are appropriately adjusted (Halac et al., 2017). An important difference is that in our setting, firms may hold differing beliefs from the beginning, which means that the observation of progress can have a balancing effect, augmenting the investment incentives of lagging firms while reducing the investment incentives of leading firms. Our results thus suggest that, in an R&D setting, private information induces a closer but also longer race for innovation.[8]

Our theory combines a dynamic contest framework with private information and it thereby contributes to two, mostly separate branches of the literature. The first branch investigates the role of information in static contests, where a different form of discouragement arises from potential differences in players' prize-valuations or abilities. While for private-value environments, asymmetric information is found to have a positive effect on aggregate incentives (Morath and Münster, 2008; Dubey, 2013; Wasser, 2013; Fu et al., 2014; Serena, 2021), in common-value settings, more akin to ours, private information typically has a negative or no effect (Hurley and Shogren, 1998; Wärneryd, 2003; Einy et al., 2017). Our analysis of the static competition benchmark in Section 2 shows that, in our model, private information has an influence on aggregate effort only when the contest is dynamic, thus identifying contestants' learning as the origin of the identified incentive-gains.

The second branch of the literature characterizes incentives for various types of dy-

[8]Further effects of private information include the possibility of homogeneous investment-behavior by heterogeneous firms (Moscarini and Squintani, 2010) and of an information-backlash due to firms' ability to learn from a better informed rival (Awaya and Krishna, 2021).

namic contests with perfect information. Konrad and Kovenock (2009) provide the seminal analysis of a best-of-N contest, with individual battles modeled as all-pay auctions, where the rate of rent-dissipation and hence the discouragement effect are extreme. For more moderate rates of rent dissipation, the characterization of equilibrium in a best-of-N contest has proven rather elusive. Ferrall and Smith (1999) determine a mixed-strategy equilibrium when battles take the form of an additive tournament with normally distributed noise and show, numerically, that the players' likelihood to provide positive effort falls towards zero when the contest reaches an asymmetric state. For standard Tullock-battles, a characterization of equilibrium for a best-of-N contest has been obtained by Klumpp and Polborn (2006). They take the predicted discouragement as an argument in favor of the sequential format of US presidential primaries, where efforts consist of wasteful campaign spending.[9] We contribute to this literature by providing a characterization of equilibrium for generic tournaments with multiplicative noise, including the Tullock specification as a special case.

The few articles that combine dynamic contests and incomplete information belong to a growing literature about the desirability of intermediate performance feedback in labor tournaments (Gershkov and Perry, 2009; Aoyagi, 2010; Goltsman and Mukherjee, 2011; Ederer, 2010) or cryptocurrency mining protocols (Ely et al., 2023). Feedback can induce fierce competition when the contest is close, but has a discouraging effect when large performance differences are revealed. As our static competition benchmark is strategically equivalent to a situation where players compete sequentially without knowledge of the individual battles' outcomes, our theory contributes to this literature. In particular, our results imply that, in the presence of private information about the contest's prize (e.g. the value of becoming promoted), intermediate performance feedback is detrimental when contestants are very poorly or very well informed but can improve incentives when information is of moderate quality.

On a more abstract level, our results resonate well with the general idea that, in strategic common-value settings, dynamics and private information, although each detrimental on their own, can be beneficial in combination. For example, in a common value auction revenue is no less in an (dynamic) English auction than in a (static) first-price

[9] By introducing multiplicative biases into a best-of-three version of the Klumpp and Polborn (2006) model, Barbieri and Serena (2022) show that aggregate effort can be increased by favoring the loser of battle one, thereby extending the logic of leveling the playing field from a static to a dynamic setting. While we share with Barbieri and Serena the finding that, in battle two, efforts are maximal when winning probabilities are equalized, in our setting, maximization of effort on aggregate requires the playing field to be "unleveled". Private information acts differently than a multiplicative bias because it influences the players' valuations rather than their probabilities of winning.

auction because bidders' learning of their rivals' drop-out prices mitigates the winner's curse (Milgrom and Weber, 1982; Levin et al., 1996).[10] Similarly, in a preemption game, where players aim to be the first to invest but only when investment is lucrative, private information can be welfare improving by counteracting the players motive to invest earlier than in the social optimum (Hopenhayn and Squintani, 2011; Bobtcheff et al., 2021). Finally, in a strategic experimentation setting (Bolton and Harris, 1999), where players can learn from the experimentation of others, private information can mitigate the players' free-riding problem (Heidhues et al., 2015; Dong, 2016; Wagner and Klein, 2022). We share with these articles the finding that private information can be beneficial but only in our setting benefits arise from players' beliefs moving in *opposite* directions which helps to reinstall the symmetry and hence competitiveness of a dynamic contest.[11]

2 Model

We consider two homogeneous, risk-neutral players engaged in a dynamic contest for a single prize of common value.[12] The prize can take two values, $V \in \{0, 1\}$, and we denote by $\omega \in (0, 1)$ the likelihood that $V = 0$ and by $\mathbb{E}[V] = 1 - \omega$ the expected prize.[13] The contest consists of three identical, consecutive battles and the prize is awarded to the first player achieving a total number of two battle victories.[14] In each battle $t \in \{1, 2, 3\}$, the two players $i \in \{1, 2\}$ choose an effort $e_{it} \geq 0$ simultaneously. A player's payoff equals his prize winnings minus his effort costs aggregated over all battles, i.e. we abstract from discounting. The costs of effort are identical across players and battles and are assumed to be linear, i.e. $C(e_{it}) = e_{it}$. Linearity facilitates comparison with an auction setting and is a natural assumption in contexts where "efforts" consist of financial outlays, such as investments in an R&D race. More importantly, we show below that with linear costs,

[10] In our setting, both auctions would create the same revenue (see footnote 25), ruling out the winner's curse as a determinant of potential differences in aggregate effort.

[11] An exception are multi-unit common value "second"-price auctions where the interplay of a winner's *and* a loser's curse facilitates information aggregation, because losing (winning) the auction indicates that a sufficient number of bidders must have values above (below) the equilibrium price (Pesendorfer and Swinkels, 1997). A "winner's blessing" can also emerge in an all-pay auction with an unknown number of bidders (Lauermann and Speit, 2023).

[12] The possibility of heterogeneity in prize valuations is introduced in Section 5.

[13] While normalizing $V = 1$ is without loss of generality, assuming $V = 0$ lends tractability to our model, as will become clear below. We show in Section 6 that the balancing effect of private information on incentives is robust with respect to this assumption.

[14] Klein and Schmutzler (2017) provide an incentive-based rationale for why competition may take the format of a best-of-N contest akin to our model. A discussion of the effects of extending the contest to more than three battles is postponed until Section 7.

expected aggregate effort is independent of the players' information in both the static competition benchmark and the public information benchmark. This allows us to focus on the effect of information on incentives that arises from the players' learning rather than from the curvature of their cost functions.

Competition. We model competition as a multiplicative tournament, featuring the frequently employed Tullock (1980) model as a special case (see below). Besides adding robustness, this general approach allows for a more intuitive understanding of our results. More specifically, we assume that each battle t is won by the player with the highest performance (with ties broken randomly) and player i's performance in battle t is given by the product of his effort e_{it} and an individual noise component $x_{it} > 0$.[15] Individual noise is distributed identically and independently across battles and players. Denoting by $H(.)$ the cumulative distribution function of the ratio of individual noise $y_t = \frac{x_{jt}}{x_{it}}$, player i's probability of winning battle t is thus given by $H(\frac{e_{it}}{e_{jt}})$. If in any battle both players exert zero effort, the battle is equally likely to be won by either player. As equilibrium will be fully determined by the distribution of the ratio of individual noise, we make assumptions directly on the corresponding probability density $h = H'$.[16] Note that from symmetry it follows that $H(y) = 1 - H(\frac{1}{y})$ and differentiating both sides leads to $yh(y) = \frac{1}{y}h(\frac{1}{y})$. The function $yh(y)$, which will play an important role in our analysis of incentives, must thus have a minimum or a maximum at $y = 1$. To guarantee that $y = 1$ constitutes a global maximum and that a pure-strategy equilibrium exists we make the following assumption.

Assumption 1. *The density h of the ratio of individual noise is continuously differentiable and strictly decreasing and the function $yh(y)$ is unimodal with* $\lim_{y\to 0} yh(y) = 0$.

Unimodality is a common assumption in models where performance is additive in effort and noise (e.g. Lazear and Rosen, 1981).[17] A family of densities that satisfy our

[15]By logarithmic transformation, our multiplicative specification arises from the additive model of Lazear and Rosen (1981) if efforts are allowed to be negative and $C(-\infty) = 0$. Owing to this greater flexibility, the multiplicative model includes an effort level that reduces a player's chance of winning to zero (c.f. zero investment in an R&D race), thereby facilitating Bayesian updating after a loss.

[16]Note that two different individual noise distributions, f and $\tilde{f}$, can give rise to the same ratio distribution h, even when f and $\tilde{f}$ differ in their "shape". For example, the distribution of $\frac{x_1}{x_2}$ is given by $h(\frac{x_1}{x_2}) = \frac{1}{(1+\frac{x_1}{x_2})^2}$ when x_1, x_2 are distributed according to $f(x_i) = \exp(-x_i)$ *and* when x_1, x_2 are distributed according to $\tilde{f}(x_i) = \frac{1}{x_i^2}\exp(-\frac{1}{x_i})$, although f is monotone decreasing whereas $\tilde{f}$ has a unique positive mode. It it therefore sensible to consider h as the primitive of our model and to make assumptions about the shape of h rather than the shape of f.

[17]Hodges and Lehmann (1954) show that the distribution of the difference of two unimodal noise distributions must itself be unimodal. Using this result, a straight forward logarithmic transformation shows that $yh(y)$ must be unimodal when the underlying distribution of individual noise is unimodal.

distributional assumptions is given by

$$h_{(d,r)}(y) = \frac{r\Gamma(2\frac{d}{r})}{\Gamma(\frac{d}{r})^2} \frac{y^{-d-1}}{(1+y^{-r})^{2\frac{d}{r}}}, \quad d \in (0,1], \quad r > 0. \tag{1}$$

For $d = r$, these ratio distributions generate the generalized Tullock contest success function $H_r(\frac{e_1}{e_2}) = \frac{e_1^r}{e_1^r + e_2^r}$ (Jia, 2008). They arise when individual noise follows a generalized Gamma distribution (Malik, 1967). Assumption 1 thus not only allows individual noise to follow an exponential ($r = d = 1$) or Weibull distribution ($r = d < 1$), as in the Tullock model, but also accommodates distributions such as the Chi ($r = 2, d < 1$), Chi-squared ($r = 1, d < 1$), or folded-normal ($r = 2, d = 1$), to name just a few. Note that a special feature of a multiplicative tournament is that a player with a zero effort cannot win against a player with a positive effort, which simplifies Bayesian updating in the dynamic contest considerably. While this property seems realistic in many settings (e.g. innovation requiring investment), it distinguishes our framework from those models where effort and noise are substitutes rather than complements (e.g. Lazear and Rosen, 1981).

Information. Our model captures situations in which contestants have private information about the common value of a contested prize and may learn about their rival's information via the observation of intermediate outcomes. In particular, we assume that after each battle, players observe the identity of the battle's winner, while neither individual performances nor the rival's effort are observable. For example, in procurement contests, such as the Small Business Innovation Research program of the U.S. Department of Defense, organizers often inform firms about the identity of their preferred supplier(s) at intermediate phases, while the scores of firms' proposals as well as the time it took to prepare them remains undisclosed (Bhattacharya, 2021). Similarly, in promotion tournaments, intermediate performance feedback often takes the form of an ordinal rather than cardinal ranking and individual efforts are commonly considered as unobservable (Meyer, 1991).

To model the players' private information about the contest's prize, we assume that prior to the first battle, each player i obtains a private signal, $s_i \in \{B, G\}$, that is informative about the value of V. Signals are independent draws from the same conditional probability distribution $\text{Prob}(s_i|V)$ specified by Table 1. The parameter $\sigma \in (0,1)$ measures the informativeness of the players' signals. In particular, for $\sigma \to 1$ players become perfectly informed about the value of the prize, whereas for $\sigma \to 0$ signals become completely uninformative. Note that implicit in this formulation is the assumption that a "bad" signal $s_i = B$ is conclusive, as it can only be received when $V = 0$. For example,

Prob($s_i|V$)	$V=0$	$V=1$
$s_i = B$	σ	0
$s_i = G$	$1-\sigma$	1

Table 1: Partially conclusive information structure.

workers competing for a promotion may learn that the position will be filled with an outsider, so that outperforming their internal rival has no value. This assumption, together with the fact that, in this state of the world, the prize has *zero* value, greatly simplifies the analysis because it implies that efforts must be zero upon the observation of a bad signal.[18] Based on the players' prior ω and the above signal structure, parametrized by σ, two variables will play an important role for our analysis. In particular, we let

$$\beta_1 \equiv \text{Prob}(s_j = G|s_i = G) = \frac{1-\omega+\omega(1-\sigma)^2}{1-\omega+\omega(1-\sigma)} \tag{2}$$

denote a player's belief that, conditional on having received a good signal, the rival's signal is also good, and we let

$$V^G \equiv \mathbb{E}[V|s_1 = s_2 = G] = \frac{1-\omega}{1-\omega+\omega(1-\sigma)^2} \tag{3}$$

be the contest's expected prize, conditional on both signals being good.

Equilibrium. Our setting constitutes a dynamic Bayesian game, with players' "types" given by their signals. We use Perfect Bayesian equilibrium as our solution concept and focus our analysis on symmetric equilibria in pure strategies. In equilibrium, players who observe a bad signal will conclude that the contest's prize is zero and hence exert zero effort. A symmetric, pure-strategy Perfect Bayesian equilibrium – in the remainder simply denoted as "an equilibrium" – is thus fully characterized by a vector of efforts $(e_1^*, e_L^*, e_F^*, e_3^*)$ which players exert conditional on having observed a good signal and the corresponding beliefs about their rival's type. Here e_1^* and e_3^* denote a player's efforts during the first and the third battle, respectively, whereas e_L^* and e_F^* denote a player's effort in the second battle depending on whether the player has become the *leader* (L) or *follower* (F). Note that effort in the third battle is independent of the sequencing of battle-outcomes (win-loss, loss-win) because in equilibrium a player with a good signal, who exerted effort in the previous battle, will conclude that his rival's signal is also good, given that both parties were capable of winning one battle.

[18]Our results are robust to pre-play communication if we assume that signals are non-verifiable, because players have an incentive to report a bad signal, independently of their true signal, making all communication uninformative. With verifiable signals, private information would unravel, because only players with a good signal have an incentive to conceal.

Benchmarking

We now discuss two variations of our model that will serve as benchmarks. In the *static competition benchmark*, all battles take place simultaneously rather than sequentially, ruling out the possibility that players may learn about their rival's signal.[19] In the *public information benchmark* all signals are public rather than private, making learning obsolete.

The following lemma determines expected effort, aggregated over all players and battles, for these two benchmarks. It shows that, in both benchmarks, expected aggregate effort is independent of the contest's information structure and can be fully characterized by the intensity of competition in the contest's individual battle components.

Note, for this purpose, that if the contest consisted of a single battle, and information was symmetric, so that all players had the same expectations about the contest's prize, say $\mathbb{E}[V]$, then in equilibrium, efforts would be given by $e^* = \arg\max_{e \geq 0} H(\frac{e}{e^*})\mathbb{E}[V] - e = h(1)\mathbb{E}[V]$ and each player would expect the payoff $U^* = [\frac{1}{2} - h(1)]\mathbb{E}[V]$.[20] Hence we can determine a single battle's *rate of rent dissipation* as

$$R \equiv \frac{\mathbb{E}[V] - 2U^*}{\mathbb{E}[V]} = 2h(1). \tag{4}$$

As we are interested in aggregate effort—or equivalently rent dissipation—in contests combining several individual battles, the rate of rent dissipation of the component battle, R, constitutes a useful way to parametrize our analysis. Rent dissipation is high when the impact of effort on performance is strong relative to the impact of noise, which is the case when the distribution of the ratio of individual noise is rather concentrated around $y = 1$. For example, in the Tullock model, where $h(1) = \frac{r}{4}$, rent dissipation is linearly increasing in the Tullock parameter $r > 0$, which provides an inverse measure of the contest's noisiness.

Lemma 1 (Benchmarks)**.** *In the static competition benchmark, if a symmetric pure-strategy Nash equilibrium exists (which requires that $h(1) < \frac{1}{3}$), expected aggregate effort is $E^S = \frac{3}{2}R \cdot \mathbb{E}[V]$. In the public information benchmark, expected aggregate effort in the unique pure-strategy Subgame Perfect equilibrium is strictly smaller $E^P = [R + (1 -$*

[19] In applications, an alternative *no feedback* benchmark can be relevant, where battles are sequential but learning is ruled out because battle outcomes are unobservable. We show in the Appendix that expected aggregate effort in the no feedback benchmark is the same as under static competition. In particular, whether battle 3 takes place for sure or only when battles 1 and 2 result in a draw, has no influence on aggregate incentives.

[20] This payoff is positive because it follows from Assumption 1 that the function $H(y) - yh(y)$ is strictly increasing, converges to zero for $y \to 0$, and equals $\frac{1}{2} - h(1)$ for $y = 1$.

$R)^2 h(\frac{1-R}{1+R})] \cdot \mathbb{E}[V] < E^S$, i.e. there exists a discouragement effect. In both benchmarks, aggregate incentives depend on the individual battle's rate of rent dissipation R but not on the informativeness, σ, of the players' signals. An increase in R augments the relative loss in incentives that is due to the discouragement effect, i.e. $\frac{dE^P/E^S}{dR} < 0$.

Note that in the public information benchmark, players exert positive efforts only when both signals turn out to be good. More specifically, as shown in the proof of Lemma 1 in the Appendix, expected aggregate effort can be written as

$$E^P \quad = \quad \text{Prob}(s_1 = s_2 = G)[2e_1^P + e_L^P + e_F^P + 2H(\frac{1-R}{1+R})e_3^P], \tag{5}$$

with $(e_1^P, e_L^P, e_F^P, e_3^P)$ denoting players' efforts *conditional* on the observation of two good signals. The signal's informativeness, σ, affects not only the likelihood of two good signals being observed but also the players' expectation of the contest's prize and hence the *level* of efforts conditional on this event. It is due to the linearity of players' effort costs that the two effects cancel, making expected aggregate effort independent of σ. Hence, an important implication of Lemma 1 is that, in our framework with private information, any effect of σ on aggregate incentives must be due to the players' learning. Our objective to understand the effect of learning on incentives thus justifies our focus on linear effort costs.

Also note that aggregate incentives in the static competition benchmark are given by E^S, no matter whether signals are private or public. By showing that $E^P < E^S$, Lemma 1 thus proves the existence of a discouragement effect for our setting. Although an increase in the battles' rate of rent dissipation increases aggregate incentives in both settings, according to Lemma 1, the *relative* loss in incentives due to discouragement is strictly increasing in R. This is intuitive because for larger R, a greater part of the expected prize winnings of a potential third battle are dissipated in form of future effort costs. This reduces aggregate effort not only directly, by decreasing the follower's incentive to obtain a draw in the second battle, but also indirectly because the leader anticipates the follower's effort reduction and responds strategically.[21] Although the total effect of R on the leader's effort is ambiguous, the effect of an increase in R on the *sum* of the leader's and the follower's efforts is negative. Due to the discouragement effect, both the follower's effort and the *sum* of the follower's and the leader's effort are lower than the corresponding values in the static competition benchmark.

[21] Formally, equations (42) and (43) in the Appendix show that the follower's effort e_F^P decreases with R and that the leader's effort e_L^P depends positively on e_F^P.

Finally, an alternative reasoning for why aggregate effort under dynamic competition might be lower is that the simultaneous choice of effort for all three battles induces players to expend effort in battles that may happen to have no impact on the contest's overall outcome. To meet this concern, in the proof of Lemma 1 we also consider an alternative benchmark in which competition is dynamic so that battle 3 is fought *only* if the contest is tied after battles 1 and 2, but battle outcomes are not observable. Expected aggregate effort turns out to be the same as in the static competition benchmark, which shows that the reduction in incentives $E^P < E^S$ is entirely because the observation of battle outcomes induces players to act as a leader and a follower.

3 Equilibrium characterization

This section characterizes the unique symmetric pure-strategy Perfect Bayesian equilibrium of the dynamic contest with private information. Sufficient conditions for the existence of such an equilibrium are given by the following:

Lemma 2 (Equilibrium existence)**.** *A symmetric pure-strategy Perfect Bayesian equilibrium exists and it is unique when the contest is sufficiently noisy, i.e. when* $h(1)$ *is sufficiently small, or when players are sufficiently informed/uninformed, i.e. when* σ *is sufficiently close to* 0 *or* 1*. For the ratio distribution* $h_r(y) = \frac{ry^{r-1}}{(1+y^r)^2}$ *generating the Tullock contest success function with parameter* $r \leq 1$*, existence and uniqueness are guaranteed for all* $\sigma \in (0, 1)$*.*

Note that while Lemma 2 deals with equilibrium existence in the full dynamic game, we show in the proof that in the Bayesian "sub-games" consisting of battles 2 and 3, the existence of a unique pure-strategy equilibrium is guaranteed by Assumption 1. While Proposition 1 is concerned only with those sub-games, Proposition 4 deals with sufficiently informed/uninformed players and existence thus follows from Lemma 2. The statements in Propositions 2 and 3 apply to the set of contests for which a symmetric pure-strategy equilibrium exists for all $\sigma \in (0, 1)$. Given Lemma 2, this set is non-empty, and includes the frequently studied Tullock family as well as all those contests that are sufficiently noisy.

To derive the unique *candidate* for a symmetric pure-strategy Perfect Bayesian equilibrium, first note that the analysis of battle 3 is straightforward, because in the last battle players must have symmetric beliefs about the contest's prize. This is because, in equilibrium battle 3 can only be reached when players' signals coincide. If signals differ, then the player with the bad signal and zero effort will lose battles 1 and 2 against the player with the good signal and positive effort, making battle 3 obsolete. In analogy to our single battle analysis in Section 2, players' efforts (conditional on both signals being good and the last battle being reached) are thus given by

$$e_3^* = h(1)V^G \tag{6}$$

and the continuation value of reaching the last battle is given by

$$U_3 = [\frac{1}{2} - h(1)]V^G > 0. \tag{7}$$

We proceed in two steps. Section 3.1 determines effort levels in battle 2 with a focus on the difference between the leader's and the follower's incentives. Section 3.2 analyzes incentives in battle 1 and derives the implications for aggregate effort.

3.1 The balancing effect

In battle 2, a player $i \in \{1, 2\}$ with signal $s_i = G$ updates his belief β_1 about the rival's signal $s_j \in \{B, G\}$ based on whether he has become the contest's leader or follower by winning or losing the previous battle, respectively.

If player i has lost battle 1 with effort $e_1 > 0$, he will conclude that his opponent has observed a good signal. Had his opponent observed a bad signal he would have exerted zero effort and could not have defeated him. Hence, the follower will update his belief in battle 2 from β_1 upwards to

$$\beta_F^* \equiv \text{Prob}(s_j = G | s_i = G, i \text{ lost battle 1}) = 1 > \beta_1, \tag{8}$$

i.e. losing the first battle represents "good news". If instead, player i has won battle 1 with effort $e_1 > 0$, then he does not know whether he was simply lucky or whether his opponent failed to provide effort upon observation of a bad signal. More specifically, assuming his opponent employed the equilibrium strategy of exerting effort $e_1^* > 0$ upon observation of a good signal and zero effort after observation of a bad signal, player i would have won the first battle with probability $H(\frac{e_1}{e_1^*})$ in the case where $s_j = s_i = G$ and with certainty in the case where $s_j = B \neq G = s_i$. Hence the leader will update his belief in battle 2 from β_1 downwards to

$$\beta_L(e_1) \equiv \text{Prob}(s_j = G | s_i = G, i \text{ won battle 1}) = \frac{\beta_1 H(\frac{e_1}{e_1^*})}{\beta_1 H(\frac{e_1}{e_1^*}) + 1 - \beta_1} < \beta_1, \tag{9}$$

i.e. winning the first battle represents "bad news". Note that, generally, the leader's updated belief in battle 2 depends on the level of effort e_1 he exerted in battle 1.[22] However, its equilibrium value

$$\beta_L^* \equiv \beta_L(e_1^*) = \frac{1-\omega+\omega(1-\sigma)^2}{1-\omega+\omega(1-\sigma)^2+2\omega\sigma(1-\sigma)} \tag{10}$$

is determined entirely by the values of ω and σ. In our model, a central role is taken by the *ratio* of the players' equilibrium beliefs, $\frac{\beta_L^*}{\beta_F^*}$, and although $\beta_F^* = 1$ due to the partial conclusiveness of our information structure, we keep β_F^* in the equations to highlight the fact that *both* players' beliefs matter. Note that as a function of σ, $\frac{\beta_L^*}{\beta_F^*}$ is U-shaped with a minimum value

$$\min_{\sigma\in(0,1)} \frac{\beta_L^*}{\beta_F^*} = \frac{\sqrt{1-\omega}-(1-\omega)}{1-\sqrt{1-\omega}} \in (0,1) \quad \text{at} \quad \hat{\sigma}(\omega) \equiv \frac{1-\sqrt{1-\omega}}{\omega} \in (0,1), \tag{11}$$

and converges to one in the limits where signals become perfectly informative or perfectly uninformative, i.e. for $\sigma \to 1$ or $\sigma \to 0$.

In equilibrium, effort choices (e_L^*, e_F^*) must satisfy:

$$e_L^* \in \arg\max_{e_L\geq 0} \beta_L^* \left[U_3 + H(\frac{e_L}{e_F^*})(V^G - U_3)\right] - e_L \tag{12}$$

$$e_F^* \in \arg\max_{e_F\geq 0} \beta_F^* H(\frac{e_F}{e_L^*})U_3 - e_F. \tag{13}$$

By Assumption 1, the above objectives are concave and the corresponding first order conditions lead to the equilibrium values

$$e_F^* = \frac{1+R}{2}\beta_L^* h(\frac{\beta_L^*}{\beta_F^*}\frac{1+R}{1-R})V^G \tag{14}$$

$$e_L^* = \frac{1-R}{2}\beta_F^* h(\frac{\beta_F^*}{\beta_L^*}\frac{1-R}{1+R})V^G. \tag{15}$$

The probabilities of winning the second battle depend on the ratio of the follower's and the leader's efforts which takes the following simple form:

$$\frac{e_F^*}{e_L^*} = \frac{\beta_F^*}{\beta_L^*}\frac{1-R}{1+R}. \tag{16}$$

The ratio $\frac{e_F^*}{e_L^*}$ thus inherits its shape from $\frac{\beta_F^*}{\beta_L^*}$ and is inverse-U-shaped with respect to σ, with a maximum at $\hat{\sigma}$. More specifically, in the Appendix we prove the following result:

[22] The fact that a deviation from e_1^* to $e_1 \neq e_1^*$ influences the informativeness of the first battle's outcome will be taken into account in the determination of the equilibrium effort level e_1^* in Section 3.2.

Proposition 1 (Balancing Effect)**.** *Private information counteracts the discouragement effect in battle 2 by moving competitive balance towards the follower:*

- *Private information increases the probability that the follower catches up with the leader above the public information benchmark:*

$$\frac{e_F^*}{e_L^*} > \lim_{\sigma \to 0} \frac{e_F^*}{e_L^*} = \lim_{\sigma \to 1} \frac{e_F^*}{e_L^*} = \frac{e_F^P}{e_L^P} = \frac{1-R}{1+R} \quad \textit{for all} \quad \sigma \in (0,1). \tag{17}$$

- *Private information maximizes the sum of the leader's and the follower's expected efforts when it restores "competitive balance", i.e.* $\frac{e_F^*}{e_L^*} = 1$*, which happens when* $\frac{\beta_L^*}{\beta_F^*} = \frac{1-R}{1+R}$ *or equivalently*

$$R \leq \overline{R}(\omega) \equiv \frac{2 - \omega - 2\sqrt{1-\omega}}{\omega} \in (0,1), \quad \textit{and} \tag{18}$$

$$\sigma = \sigma_{\pm} \equiv \frac{1+R}{2} \pm \sqrt{(\frac{1+R}{2})^2 - \frac{R}{\omega}} \in (0,1). \tag{19}$$

- *Private information makes the follower even more likely to win the second battle than the leader, i.e.* $\frac{e_F^*}{e_L^*} > 1$*, if and only if rent dissipation is low, i.e.* $R < \overline{R}(\omega)$*, and the players' information is neither too precise, nor too imprecise, i.e.* $\sigma_- < \sigma < \sigma_+$*.*

The intuition for Proposition 1 can be obtained from (16). In the limits where players are symmetrically informed or uninformed, i.e. for $\sigma \to 1$ or $\sigma \to 0$, the leader's and the follower's updated equilibrium beliefs converge, i.e. $\frac{\beta_L^*}{\beta_F^*} \to 1$. As in the public information benchmark, the leader then exerts a higher effort than the follower and the contest is more likely to end after two rather than three battles. The follower is discouraged from exerting effort because winning the overall contest requires not only winning the current but also the future battle. Intuitively, the follower's discouragement is increasing in the battle's rate of rent dissipation, $R \in (0, 1)$.

In contrast, in the presence of private information, i.e. for $\sigma \in (0, 1)$, the leader and the follower update their beliefs about their rival's signal in opposite directions because winning represents bad news whereas losing represents good news, i.e. $\frac{\beta_L^*}{\beta_F^*} < 1$. Private information thus has a balancing effect on incentives by counteracting the follower's discouragement. Our finding that expected aggregate effort in battle 2 is maximized when private information "levels the playing field" resonates well with similar results from the literature on static contests. Note, however, that for private information to be capable of leveling the playing field, rent dissipation should not be too high. More specifically, condition (18) guarantees that the term under the square root in (19) is non-negative and competitive balance then becomes restored for $\sigma = \sigma_-$ or for $\sigma = \sigma_+$.

Proposition 1 also provides conditions under which the informational effect is strong enough to tip competitive balance in favor of the follower. Private information makes the follower more likely to win the second battle than the leader if rent dissipation is low ($R < \overline{R}(\omega)$) and players' information is sufficiently asymmetric ($\sigma_- < \sigma < \sigma_+$). Note that the threshold $\overline{R}(\omega)$ is strictly increasing in the players' prior, ω, and $\lim_{\omega \to 1} \overline{R}(\omega) = 1$. This means that when players are rather pessimistic about the contest's prize, asymmetric information can induce followers to exert more effort than leaders for arbitrarily high degrees of rent dissipation.

Summarizing, the results in this section suggest that in the presence of private information, dynamic competition does not have to suffer from deteriorating incentives. Followers might be as motivated or, in fact, more motivated than leaders. An important implication for *R&D* races is that long-standing concerns about ϵ-preemption (Fudenberg et al., 1983) and suboptimal investments (Harris and Vickers, 1987) might not be warranted.

3.2 Aggregate incentives

Our results in the previous section suggest that in dynamic contests, private information may have a *positive* effect on incentives. Private information increases the likelihood that the contest's final battle is reached, giving players the opportunity to exert additional efforts. Moreover, for low rates of rent dissipation, private information can level the playing field in an intermediate battle between a leader and a follower, thereby maximizing the sum of their efforts. To understand the effect of private information on incentives *on aggregate*, we now complete our characterization of equilibrium by determining the players' effort choice e_1^* in the contest's opening battle. We then compare aggregate incentives in the equilibrium with the benchmarks of public information and static competition.

In battle 1, a player with a good signal believes that with probability β_1 the rival observed a good signal, and hence the rival's effort is e_1^*, whereas with probability $1-\beta_1$ the rival observed a bad signal, and hence the rival's effort is zero. Denoting the continuation values of the leader and the follower, *conditional on the rival's signal* $s \in \{G, B\}$, by U_L^s and U_F^s, respectively, the players' equilibrium effort in battle 1 must therefore satisfy:

$$e_1^* \in \arg\max_{e_1 > 0} \beta_1 \left\{ H(\frac{e_1}{e_1^*}) U_L^G(e_1) + [1 - H(\frac{e_1}{e_1^*})] U_F^G \right\} + (1-\beta_1) U_L^B(e_1) - e_1. \tag{20}$$

Here we have used the fact that, conditional on the rival's signal being bad, a player exerting a positive effort $e_1 > 0$ must win battle 1 with certainty.[23] Moreover, it is important to note that the continuation values of becoming the leader, depend on the player's effort choice e_1 through its influence on the leader's belief $\beta_L(e_1)$ in (9). A deviation from e_1^* changes the player's belief about the rival's signal after winning battle 1 and will thus induce him to adjust his effort e_L in battle 2 optimally. In the proof of Lemma 2 we can thus employ the envelope theorem to show that the first-order condition corresponding to (20) takes the following simple form:

$$e_1^* = \beta_1 h(1)[U_L^G - U_F^G] = \beta_1 h(1)[H(\frac{e_L^*}{e_F^*})V^G - (e_L^* - e_F^*)]. \tag{21}$$

Equation (21) shows that when players choose their efforts in battle 1, they evaluate continuation values conditional on their rival having received a good signal, thereby correctly anticipating the winner's curse that the contest's prize is zero when the rival's signal is bad. Incentives in battle 1 derive from the fact that an early success leads to the opportunity to secure overall victory already in the intermediate battle. Early success happens with probability $H(\frac{e_L^*}{e_F^*})$ but comes at the expense of the future effort differential $e_L^* - e_F^*$.

Having completed our characterization of equilibrium efforts $(e_1^*, e_L^*, e_F^*, e_3^*)$ we are now ready to consider aggregate incentives, i.e. the expected sum of efforts aggregated over all battles and all players. We already know from Proposition 1 that in battle 2, the sum of the leader's and follower's effort is maximized when private information levels the playing field so that $e_F^* = e_L^*$. In fact, expected aggregate effort in battle 2 can be written as

$$E_2^* = \text{Prob}(s_1 = s_2 = G)(e_L^* + e_F^*) + \text{Prob}(s_1 \neq s_2)e_L^* = \mathbb{E}[V]\frac{e_L^*}{e_F^*}h(\frac{e_L^*}{e_F^*}), \tag{22}$$

and it follows from (16) and Assumption 1 that E_2^* is a unimodal function of the ratio of players' beliefs $\frac{\beta_L^*}{\beta_F^*}$ with a maximum at $\frac{\beta_L^*}{\beta_F^*} = \frac{1-R}{1+R}$. Figure 1 depicts E_2^* as a function of $\frac{\beta_L^*}{\beta_F^*}$ when rent dissipation is sufficiently low for this maximum to be attained.

The following lemma characterizes expected aggregate effort in the remaining battles, battle 1 and battle 3.

Lemma 3. *Expected aggregate effort in the symmetric battles 1 and 3 is given by*

$$E_1^* + E_3^* = R \cdot \mathbb{E}[V]\left[1 + \frac{1}{V^G}(e_F^* - e_L^*)\right], \tag{23}$$

and it is maximized when private information induces the follower to exert larger effort than the leader in the asymmetric battle 2.

[23] The possibility of a deviation to $e_1 = 0$ must be checked separately, because in that case a player will lose battle 1 against a rival with a bad signal with probability $\frac{1}{2}$. See the proof of Lemma 2 for details.

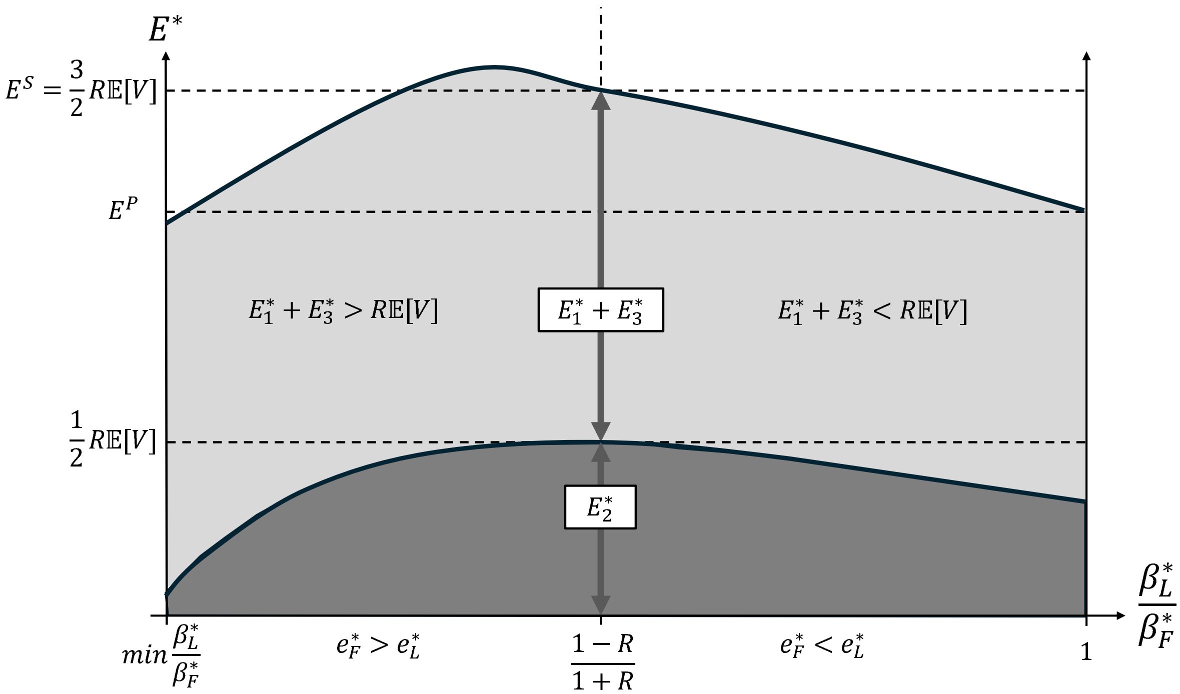

Figure 1: **Aggregate Effort (Schematic)**: E^* as a function of the ratio of equilibrium beliefs $\frac{\beta_L^*}{\beta_F^*}$ for $R < \overline{R}(\omega)$. The figure separates efforts in battle 2 from efforts in battles 1 and 3. E_2^* is unimodal and maximized at $\frac{\beta_L^*}{\beta_F^*} = \frac{1-R}{1+R}$, when learning restores competitive balance so that $e_F^* = e_L^*$. $E_1^* + E_3^*$ is larger (smaller) than $R\mathbb{E}[V]$ when learning induces the follower to exert higher (lower) effort than the leader. Our formal analysis shows that E^* is strictly decreasing for all $\frac{\beta_L^*}{\beta_F^*} \in [\frac{1-R}{1+R}, 1)$ and takes the benchmark values $E^S = \frac{3}{2}R\mathbb{E}[V]$ and $E^P = \frac{1}{2}R\mathbb{E}[V]$ on the boundaries of this interval. Whether or not E^* dips below E^S and E^P for low belief ratios depends on parameters as shown by the numerical examples in Figure 2.

Lemma 3 shows that aggregate effort in battles 1 and 3 varies with the difference between the leader's and the follower's efforts in battle 2 but is independent of the likelihood with which battle 2 is decided in favor of one or the other player. This is because a change in battle 2 efforts affects the likelihood $[1 - H(\frac{e_L^*}{e_F^*})]$ that battle 3 is reached by the same absolute amount as it influences the likelihood $H(\frac{e_L^*}{e_F^*})$ that securing leadership in battle 1 results in an early victory. As a consequence, any potential gain in aggregate effort that is due to a higher likelihood that battle 3 is reached is exactly compensated by an equally sized loss in battle 1 effort resulting from a reduction of the benefits of becoming the contest's leader.

Further note from (14) and (15) that $E_1^* + E_3^*$ in (23) depends on information only through its influence on the players' equilibrium beliefs β_L^* and β_F^*. Beliefs that induce a positive (negative) effort-differential between the follower and the leader in the asymmetric battle 2 augment (diminish) incentives in the symmetric battles 1 and 3 by making it less (more) costly to become the leader rather than the follower (see Figure 1).

From these observations and the unimodality of second-stage effort it follows immediately that over the range $[\frac{1-R}{1+R}, 1]$, aggregate effort is maximized at $\frac{\beta_L^*}{\beta_F^*} = \frac{1-R}{1+R}$. Further details of the dependence of aggregate effort on the underlying information structure σ are the subject of Section 4. In the remainder of this section, we compare aggregate incentives in the Perfect Bayesian equilibrium with the two benchmarks of public information, E^P, and static competition, E^S, characterized in Lemma 1.

Comparison with benchmarks

Start by considering the limits where signals become either perfectly uninformative ($\sigma \to 0$) or perfectly informative ($\sigma \to 1$). In both cases, private information ceases to play a role because learning about the rival's signal cannot improve upon a player's information. In both limits, the leader's and the follower's beliefs converge and the ratio of their equilibrium efforts converges to the public information benchmark, i.e. $\frac{e_F^*}{e_L^*} \to \frac{e_F^P}{e_L^P} = \frac{1-R}{1+R}$. As a consequence, players' continuation values and thus their incentives to exert effort in the first battle, as well as the likelihood that the last battle is reached all become the same as under public information. In the limit where $\frac{\beta_L^*}{\beta_F^*} \to 1$, aggregate incentives in the Perfect Bayesian equilibrium therefore converge to aggregate incentives under public information and are thus strictly smaller than when competition is static, i.e. $E^* \to E^P < E^S$, as depicted in Figure 1.

Off the limit, that is for $\frac{\beta_L^*}{\beta_F^*} < 1$, the comparison of aggregate incentives with the benchmarks becomes most transparent when we distinguish between three cases. These cases differ in the extent to which learning can affect the dynamic contest's competitive balance and can be classified according to the individual battles' degree of rent dissipation.

High Rent Dissipation: $R > \overline{R}$. When rent dissipation is high, learning improves competitive balance but cannot restore it completely. In spite of the diametrical updating of beliefs, the discouragement effect then induces the follower to exert a lower effort than the leader, i.e. $\frac{e_F^P}{e_L^P} < \frac{e_F^*}{e_L^*} < 1$, *independently* of the signals' precision. The effect of private information on aggregate incentives can be seen in the numerical example in Figure 2. A reduction in $\frac{\beta_L^*}{\beta_F^*}$ has an unambiguously positive effect on aggregate incentives because it makes it less costly to become the leader and can only improve competitive balance in the intermediate battle. Formally, we show in the proof of Proposition 2 that E^* is strictly decreasing in $\frac{\beta_L^*}{\beta_F^*}$ for all $\frac{\beta_L^*}{\beta_F^*} > \frac{1-R}{1+R}$. To understand why, for $R > \overline{R}$, aggregate incentives fall short of the static competition benchmark, consider the limit $R \to \overline{R}$. For $\frac{\beta_L^*}{\beta_F^*} = \min_\sigma \frac{\beta_L^*}{\beta_F^*} \to \frac{1-R}{1+R}$, learning exactly neutralizes the discouragement effect and competitive balance is restored perfectly, making the contest equally likely to be decided after two or three battles. This means that the only difference between winning or losing the first battle is the corresponding change in the contest's intermediate score, making incentives at every stage of the dynamic contest become equal to static incentives. Hence $E^* \to E^S$ for $\frac{\beta_L^*}{\beta_F^*} \to \frac{1-R}{1+R}$ and because, as argued above, aggregate incentives are strictly decreasing in $\frac{\beta_L^*}{\beta_F^*}$, it follows that $E^* < E^S$ for all $\frac{\beta_L^*}{\beta_F^*} > \frac{1-R}{1+R}$.

Moderate rent dissipation: $\underline{R} \leq R < \overline{R}$. When rent dissipation is moderate, learning can tip competitive balance in favor of the follower. More specifically, if $R < \overline{R}$ then $\min_\sigma \frac{\beta_L^*}{\beta_F^*} < \frac{1-R}{1+R}$ and for all belief ratios $\frac{\beta_L^*}{\beta_F^*} < \frac{1-R}{1+R}$, learning will induce the follower to exert a strictly higher effort than the leader. We show in the proof of Proposition 2 that tipping competitive balance in favor of the follower has a strictly positive effect on aggregate effort. Intuitively, tipping competitive balance marginally has no first-order effect on E_2^*, because incentives in the asymmetric intermediate battle are maximized when the playing field is leveled. However, tipping the balance in favor of the follower raises $E_1^* + E_3^*$ by reducing the cost $C(e_L^*) - C(e_F^*) = e_L^* - e_F^*$ of becoming the contest's leader. Because $E^* = E^S$ for $\frac{\beta_L^*}{\beta_F^*} = \frac{1-R}{1+R}$, a straightforward but important implication is that for $R < \overline{R}$, learning can increase aggregate incentives above the static competition benchmark (see Figure 2). Note that although aggregate incentives might decline and dip below E^S when the ratio of beliefs approaches its minimum, for $R \geq \underline{R}$ aggregate incentives are guaranteed to stay above the less demanding public information benchmark, $E^P < E^S$. The reason is that, for $R \geq \underline{R}$, the contest's competitive imbalance can never be *more than reversed*, i.e. $\frac{e_L^*}{e_F^*} \geq \frac{e_F^P}{e_L^P}$ for all $\sigma \in (0,1)$. To understand why this guarantees that $E^* > E^P$, remember that due to the model's symmetry $yh(y) = \frac{1}{y}h(\frac{1}{y})$, which means that, given (22), reversing competitive imbalance would leave E_2^* unchanged. However, reversing competitive imbalance would have a positive effect on $E_1^* + E_3^*$, because, as we have seen, a positive cost differential $C(e_L^P) - C(e_F^P) = e_L^P - e_F^P > 0$ creates disincentives to exert effort whereas a negative cost differential $C(e_L^*) - C(e_F^*) = e_L^* - e_F^* < 0$ creates incentives.

Low rent dissipation: $R < \underline{R}$. Finally, when rent dissipation is low, learning effects can be strong enough to *reduce* competitive balance relative to the public information benchmark. As argued above, this requires that the contest's competitive imbalance becomes reversed, i.e. for some $\sigma \in (0,1)$ it has to hold that $\frac{e_L^*}{e_F^*} < \frac{e_F^P}{e_L^P}$ which, given (16),

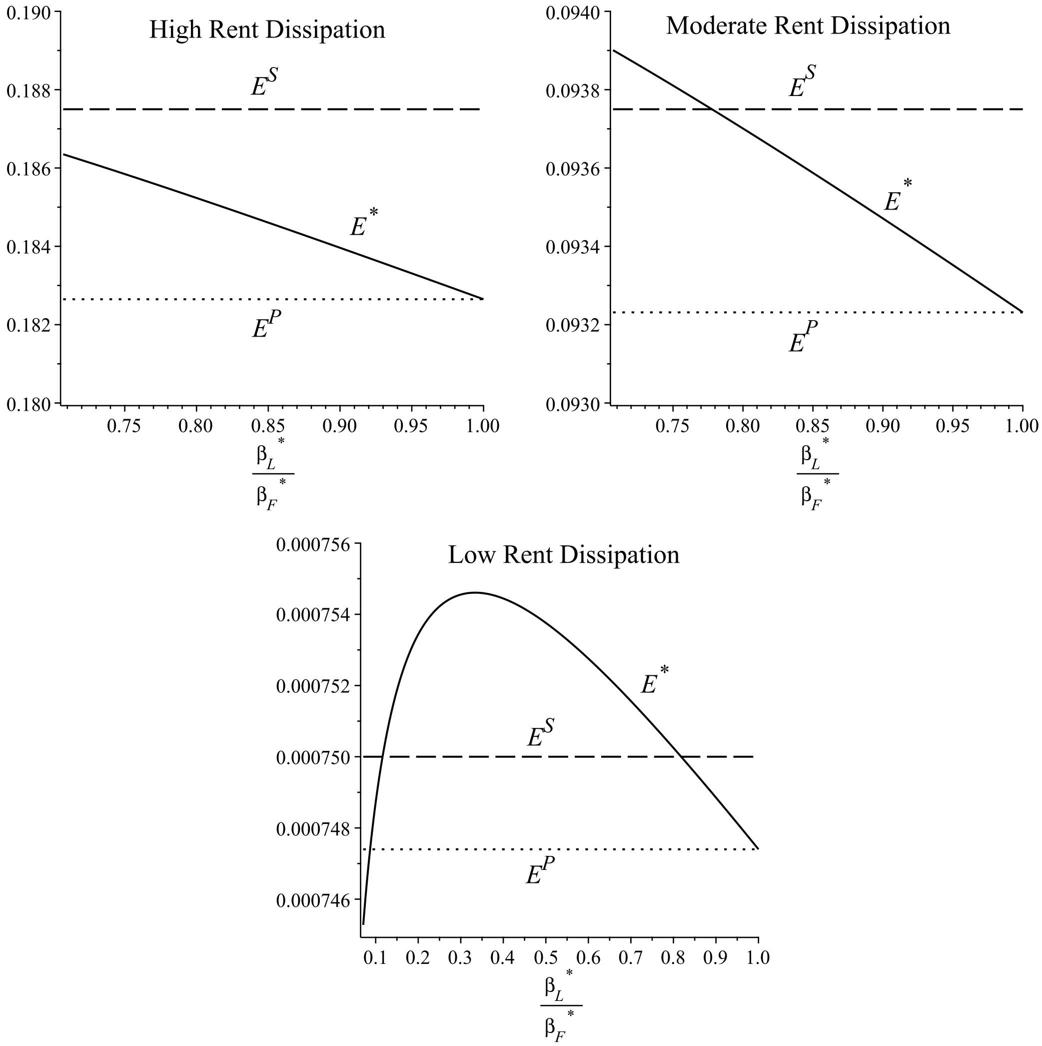

Figure 2: **Aggregate Effort (Numerical Examples)**: E^* in dependence of the ratio of equilibrium beliefs $\frac{\beta_L^*}{\beta_F^*} \in (\min_\sigma \frac{\beta_L^*}{\beta_F^*}, 1)$ for the ratio distribution $h_r(y) = \frac{ry^{r-1}}{(1+y^r)^2}$ generating the Tullock contest success function with parameter r. Learning leads to updating of beliefs in opposite directions ($\beta_L^* < \beta_1 < \beta_F^*$) which increases aggregate incentives above the public information benchmark E^P and the static competition benchmark E^S, unless updating becomes extreme. Parameter values are: (High Rent Dissipation) $r = 0.5$, $\omega = 0.5 \Rightarrow R = 0.25 > \overline{R} \approx 0.17$; (Moderate Rent Dissipation) $r = 0.25$, $\omega = 0.5 \Rightarrow \underline{R} \approx 0.08 < R = 0.125 < \overline{R} \approx 0.17$; (Low Rent Dissipation) $r = 0.2$, $\omega = 0.995 \Rightarrow R = 0.125 < \underline{R} \approx 0.58$.

is equivalent to $\min_\sigma \frac{\beta_L^*}{\beta_F^*} < (\frac{1-R}{1+R})^2$ or

$$R < \underline{R}(\omega) \equiv \frac{\omega - 2\sqrt{(2-\omega)\sqrt{1-\omega} - 2(1-\omega)}}{2 - \omega - 2\sqrt{1-\omega}} \in (0, \overline{R}(\omega)). \tag{24}$$

In the final panel of Figure 2, we depict an example where the resulting incentive effects are strong enough to reduce aggregate incentives *below* the public information benchmark. Our formal result below shows that this always happens when $R < \underline{R}(\omega)$ and the likelihood ω of the contest's prize being zero is sufficiently large. When players' are sufficiently pessimistic about the contest's prize, observation of the first battle's outcome can induce players to hold beliefs about their rival's signal that are approximately diametrical, i.e. $\min_\sigma \frac{\beta_L^*}{\beta_F^*} \to 0$. Private information is then harmful for incentives because the bad news of a battle win induces the leader to refrain from exerting further efforts in spite of his lead.

The following proposition summarizes our results about the comparison of aggregate incentives with the benchmarks of public information and static competition:

Proposition 2 (Aggregate Incentives - Benchmarking)**.** *The comparison of aggregate effort in the unique equilibrium of the dynamic contest with private signals, E^*, with the public information benchmark, E^P, and the static competition benchmark, E^S, depends on the individual battles' rate of rent dissipation $R = 2h(1)$ as follows:*

- *If rent dissipation is high, $R > \overline{R}(\omega)$, learning increases aggregate incentives but cannot make up for the discouragement stemming from the dynamics of competition, i.e. $E^P < E^* < E^S$ for all $\sigma \in (0, 1)$.*

- *If rent dissipation is moderate, $\underline{R}(\omega) < R < \overline{R}(\omega)$, learning increases aggregate incentives and can raise aggregate incentives above the static competition benchmark by tipping competitive balance in favor of the follower, i.e. $E^* > E^P$ for all $\sigma \in (0, 1)$ and $E^* > E^S$ for some $\sigma \in (0, 1)$ inducing $e_F^* > e_L^*$.*

- *If rent dissipation is low, $R < \underline{R}(\omega)$, learning increases aggregate incentives as long as it improves competitive balance but can reduce aggregate incentives when competitive imbalance is more than reversed, i.e. for all ω sufficiently close to 1 it holds that $E^* < E^P$ for some $\sigma \in (0, 1)$ inducing $\frac{e_L^*}{e_F^*} < \frac{e_F^P}{e_L^P}$.*

Proposition 2 shows that in the dynamic contest with private information, learning has two effects: (1) it will increase incentives above the public information benchmark as long as learning improves competitive balance; and (2) it will raise aggregate effort above its static competition level if it can tip competitive balance in favor of the follower. While the first part of this result resonates well with the general idea that in contests incentives are maximized when the playing field is leveled (e.g. Barbieri and Serena, 2022), the second part contrasts with the common wisdom that the dynamic nature of competition must be harmful for incentives (e.g. Klumpp and Polborn, 2006).

Our results thus establish that in a contest, dynamics need not necessarily be harmful for incentives when players are endowed with private information. Intuitively, the static contest can be improved upon, because in the dynamic contest, additional incentives can be created by establishing an appropriate link between battle outcomes and the players' learning of their rival's signal. In particular, if $e_F^* > e_L^*$, then winning battle 1 not only establishes a lead, but allows the winning player to maintain the belief that his rival's signal might be bad, thereby reducing his future effort cost.

On a more abstract level, the positive influence of dynamics on incentives described by Proposition 2 is reminiscent of Milgrom and Weber's (1982) result that in a common value setting with affiliated signals, expected revenue is (weakly) higher in a dynamic, English auction than in a static, first-price auction.[24] Note, however, that in our environment, both auction formats generate the same expected revenue given by $\mathbb{E}[V]$.[25] This shows that, while, generally, dynamic formats benefit both auction- and contest-designers through enabling players to learn about each others' private information, they do so for different reasons. While in dynamic auctions, learning mitigates the winner's curse that induces bidders to shade their bids when bidding is static, in dynamic contests learning reduces the future effort costs of early winners, thereby raising the players' incentives to establish a lead.

4 Information design

In this section, we characterize the signal quality that maximizes aggregate incentives in dependence of the contest's rate of rent dissipation, R, and the contestants' prior, ω. Our analysis shows that, when information can be used as a design variable, a contest designer will use it to fine-tune the likelihood with which initial leaders will become final winners, thereby resolving a basic trade-off between fights that are short and fierce and fights that are mild but long-lasting.

While our results contribute to a nascent but growing literature on information de-

[24]Important here is that, as in our setting, competition is dynamic but for a single object. If privately informed bidders compete in a series of auctions for multiple objects, bidders will reduce their bids to appear as having low valuation, and this effect can make a one-shot sale of all objects preferable for the seller (Hörner and Jamison, 2008).

[25] Given our informational assumptions, player i will bid zero upon observation of $s_i = B$ and t upon observation of $s_i = G$. Using arguments familiar from the all-pay auction literature (Baye et al., 1996) it is straightforward to show that in the unique symmetric equilibrium of a first-price sealed bid auction t is distributed according to the cdf $F(t) = \frac{\omega(1-\sigma)\sigma t}{1-\omega-[1-\omega+\omega(1-\sigma)^2]t}$ and expected revenue equals $\mathbb{E}[V] = 1 - \omega$. Because expected revenue in an English auction is weakly higher due to the linkage principle, expected revenues in both auctions formats have to be identical.

sign in contests (discussed below), we should emphasize that our assumption of partially conclusive signals poses a restriction on the set of posteriors that can be induced.[26] More specifically, besides the usual requirement of Bayes plausibility, in our model posteriors satisfy $\text{Prob}(V = 1|s_i = B) = 0$. In our setting an "information structure" is therefore fully determined by the posterior $\text{Prob}(V = 1|s_i = G) = \frac{1-\omega}{1-\omega+\omega(1-\sigma)}$, parametrized by the signal quality σ, and our following result characterizes the information structure that maximizes aggregate incentives within the set of all such partially conclusive information structures.[27] Importantly, this set contains fully informative ($\sigma = 1$) and fully uninformative ($\sigma = 0$) information structures as extreme cases, which means that with respect to our subsequent claims about the optimality of a partially revealing information structure, our assumption is without loss of generality.

Proposition 3 (Information design)**.** *In the dynamic contest with private signals, the signal quality σ^* that maximizes aggregate incentives, $E^*(\sigma)$, depends on the contest's rate of rent dissipation, $R = 2h(1)$ and the contestants' prior $\omega = Prob(V = 0)$ as follows:*

- *If $R \geq \overline{R}(\omega)$ then $E^*(\sigma)$ has inverted U-shape and the optimal signal is $\sigma^* = \hat{\sigma}(\omega)$ as defined in (11). Optimal signals induce the contest to be more likely to be decided after two rather than three battles, i.e. $e_L^* > e_F^*$. More pessimistic priors require more accurate information, i.e. $\hat{\sigma}(\omega)$ is strictly increasing with $\lim_{\omega\to 0} \hat{\sigma}(\omega) = \frac{1}{2}$ and $\lim_{\omega\to 1} \hat{\sigma}(\omega) = 1$.*

- *If $R < \overline{R}(\omega)$ then $E^*(\sigma)$ is strictly increasing in $(0, \sigma_-]$ and strictly decreasing in $[\sigma_+, 1)$ and any signal quality σ^* that maximizes incentives satisfies $\sigma^* \in (\sigma_-, \sigma_+)$. Optimal signals thus induce the contest to be more likely to be decided after three rather than two battles, i.e. $e_L^* < e_F^*$.*

The threshold $\overline{R}(\omega)$ is strictly increasing with $\lim_{\omega\to 0} \overline{R}(\omega) = 0$ and $\lim_{\omega\to 1} \overline{R}(\omega) = 1$.

Proposition 3 is illustrated in Figure 3. The figure reveals a dichotomy of optimal contest designs. For high rates of rent dissipation and relatively optimistic priors, optimal signals create fierce but short fighting. The incentive maximizing contest is characterized by a large likelihood ($> 50\%$) to be decided within only two battles and high initial

[26] An alternative simplification of the information design problem can be achieved by assuming information to be verifiable and to focus on the designer's choice between disclosure and concealment (e.g. Serena, 2021).

[27] In the seminal contribution of Kamenica and Gentzkow (2011) and related articles, the optimal information structure turns out to be partially conclusive.

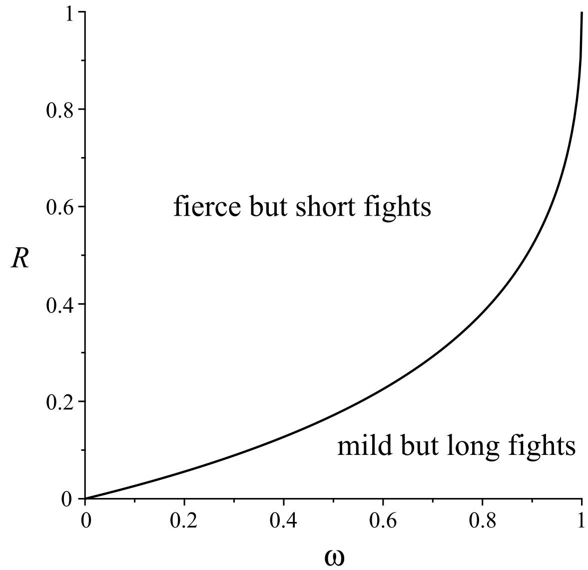

Figure 3: **Information Design**: The incentive maximizing signal quality σ^* induces either short and fierce fighting or fighting that is mild but long-lasting, depending on the individual battles' rate of rent dissipation $R = 2h(1)$ and the contestants' prior $\omega = \text{Prob}(V = 0)$. The diagram depicts the critical value $\overline{R}(\omega) = \frac{2-\omega-2\sqrt{1-\omega}}{\omega}$.

efforts e_1^* to become the contest's leader. In contrast, for low rates of rent dissipation and relatively pessimistic priors, optimal signals lead to longer lasting but less fierce fighting.

Note that, independently of the contest's rate of rent dissipation and the contestants' prior, dynamic incentives are maximized when private information is neither perfectly informative nor perfectly uninformative. This is a direct implication of Proposition 2 which has shown that learning raises aggregate incentives above the public information benchmark as long as it improves competitive balance. As for $\sigma \to 0$ and $\sigma \to 1$, the distinction between public and private information becomes obsolete, aggregate incentives must be maximized by some $\sigma^* \in (0, 1)$, i.e. the optimal information structure is *partially revealing*. The reason for this result is that for learning to have a balancing effect on incentives, contestants have to update their beliefs about the contest's prize in opposite directions, which is impossible when information is symmetric.

Proposition 3 adds to an ongoing discussion about the potential optimality of partially revealing information structures in contests. While the seminal article by Zhang and Zhou (2016) has mainly advocated fully informative or fully uninformative information structures, more recent articles have rationalized the use of information structures that are only partially revealing (Chen, 2021; Clark and Kundu, 2021a; Clark and Kundu, 2021b;

Kuang et al., 2019; Melo-Ponce, 2020). Our setting distinguishes itself from most existing work by allowing asymmetric information to be two-sided and the contest to be dynamic. Probably most closely related is Antsygina and Teteryatnikova (2022) who consider a two-player static all-pay auction with linear costs where both players' valuations are binary and ex ante uncertain. They allow for information technologies that send messages to players privately or publicly and show that the optimal information structure features private signals and induces symmetric beliefs. This structure reveals the state whenever both players valuations are identical but employs noisy and correlated signals when valuations differ. Intuitively, the designer tries to make players believe that their valuations are likely to be equal, because effort is largest when valuations are identical. As in our setting, information is thus used to "level the playing field", but the incentive-deteriorating heterogeneity emerges from exogenous differences in prize valuations rather than endogenous score-differences along the course of a dynamic contest.

5 Selective efficiency

Besides the provision of incentives, *selective efficiency* is an important objective in many competitive settings. It is achieved when a contest is won by the contestant with the highest valuation for the contest's prize or—when prize-valuations are homogenous—by the contestant with the lowest marginal cost of effort. For instance, in an R&D setting, granting a patent to the highest valuing firm may benefit consumers. Similarly, an organization's success may depend on whether a contest for promotion is won by the most "able" candidate.

Our theory has shown how in dynamic contests learning can improve incentives by motivating contestants who lag behind. However, because low-valuation contestants are more likely to be lagging behind than high-valuation contestants, one might be concerned that learning could have an adverse effect on selective efficiency. In this section, we introduce heterogeneity into our model, to show that the gain in aggregate incentives from learning does not necessarily come at the cost of a reduction in selective efficiency.

More specifically, we now extend our model by assuming that costs of effort are $C^i(e_{it}) = c^i e_{it}$ for contestant $i \in \{l, h\}$ and that one contestant has a lower marginal cost than the other, i.e. we let $\frac{c^h}{c^l} \equiv \gamma > 1$. A super-index will be used throughout the analysis to denote the contestants' cost-types. To keep our model tractable, we assume that contestants observe whether they are the low-cost contestant l or the high-cost contestant h only *after* they have competed once by exerting effort in the first battle.[28] In some applications, such as promotion tournaments, where workers are ignorant of their abilities relative to their rivals initially, this assumption may be a reasonable starting point. In other settings, where abilities are known right from the start, our subsequent results remain valid when ability differences are sufficiently small.

Selective efficiency, i.e. the probability that the low-cost (high-ability) contestant wins the contest is given by

$$S \equiv \frac{1}{2} \cdot [H(\frac{e_L^l}{e_F^h}) + H(\frac{e_F^h}{e_L^l})H(\frac{e_3^l}{e_3^h})] + \frac{1}{2} \cdot H(\frac{e_F^l}{e_L^h})H(\frac{e_3^l}{e_3^h}). \tag{25}$$

The two terms represent the cases where the low-cost type has won or lost the first battle, respectively. Both cases are equally likely because, given our assumptions, contestants will exert identical efforts in the first battle.

Efforts and expected payoffs in battle 3 are straightforward to calculate and given by

$$e_3^l = \gamma \cdot \frac{V^G}{c^l} h(\gamma) > \frac{V^G}{c^l} h(\gamma) = e_3^h, \tag{26}$$

$$U_3^l = [H(\gamma) - \gamma h(\gamma)]V^G > [H(\frac{1}{\gamma}) - \gamma h(\gamma)]V^G = U_3^h. \tag{27}$$

In the second battle, we have to distinguish between two cases. If the low-cost contestant has become the leader, equilibrium efforts must solve

$$e_L^l \in \arg\max_{e \geq 0} U_3^l + \beta_L^*(V^G - U_3^l)H(\frac{e}{e_F^h}) - c^l e \tag{28}$$

$$e_F^h \in \arg\max_{e \geq 0} U_3^h H(\frac{e}{e_L^l}) - c^h e \tag{29}$$

and it follows that

$$\frac{e_L^l}{e_F^h} = \gamma \beta_L^* \frac{V^G - U_3^l}{U_3^h}. \tag{30}$$

Similarly, if the high-cost contestant has become the leader, we get

$$\frac{e_L^h}{e_F^l} = \frac{1}{\gamma} \beta_L^* \frac{V^G - U_3^h}{U_3^l}. \tag{31}$$

Substitution of (27), (30), and (31) into (25) gives a closed form expression for selective efficiency $S(\gamma, \sigma)$ in dependence of the contestants' cost differential γ and their signals' informativeness σ:

$$S(\gamma, \sigma) = \frac{1}{2}[H(\gamma) + H\left(\beta_L^*(\sigma)\gamma \frac{V^G - U_3^l}{U_3^h}\right) H(\frac{1}{\gamma})] + \frac{1}{2} H\left(\frac{\gamma}{\beta_L^*(\sigma)} \frac{U_3^l}{V^G - U_3^h}\right) H(\gamma). \tag{32}$$

[28] When contestants are heterogeneous in battle 1 they will exert differing efforts, so that equilibrium beliefs in battle 2 will depend on past efforts, which means that the model can no longer be solved recursively.

We obtain the following result:

Proposition 4 (Selective Efficiency)**.** *Private information can improve the contest's selective efficiency. Formally, for all $\gamma > 1$ there exist $\sigma^{min}, \sigma^{max} \in (0,1)$ such that for all $\sigma \in (0, \sigma^{max}) \cup (\sigma^{min}, 1)$, selective efficiency $S(\gamma, \sigma)$ is strictly larger than in the public information benchmark.*

To understand the intuition for this result consider the effect of a marginal reduction in the leader's equilibrium belief β_L^* starting from its public information benchmark value $\beta_L^* = 1$. This can be achieved via the introduction of private information, either by making signals marginally informative, $\sigma \in (0, \sigma^{max})$, or marginally uninformative, $\sigma \in (\sigma^{min}, 1)$. Lowering the leader's equilibrium belief raises the likelihood with which the second battle is won by the lagging contestant. This decreases selective efficiency when the low-cost contestant is in the lead but increases selective efficiency when the low-cost contestant has fallen behind. From (32), the resulting change in selective efficiency is $\Delta S = \Delta S^+ - \Delta S^-$ where

$$\Delta S^+ = \frac{1}{2}\gamma \frac{U_3^l}{V^G - U_3^h} h(\gamma \frac{U_3^l}{V^G - U_3^h}) H(\gamma) > 0, \tag{33}$$

$$\Delta S^- = \frac{1}{2}\gamma \frac{V^G - U_3^l}{U_3^h} h(\gamma \frac{V^G - U_3^l}{U_3^h}) H(\frac{1}{\gamma}) > 0. \tag{34}$$

In the proof of Proposition 4 we show that $\Delta S^+ > \Delta S^-$. Intuitively, learning has a stronger effect on incentives for a lagging low-cost contestant than for a lagging high-cost contestant. When the low-cost contestant is equally likely to be lagging as the high-cost contestant, which happens when abilities are initially unknown or when differences in abilities are small, the overall effect is an increase in selective efficiency.

6 Robustness

Our theory has shown that, in a dynamic contest with private information, learning will induce leaders and followers to update their beliefs in opposite directions which can have powerful effects on incentives. In our model, learning takes a particularly simple form because losing a battle (with effort) allows a player to become perfectly informed about his rival's signal. This is the consequence of players choosing zero effort conditional on the observation of a bad signal, which is optimal because a bad signal is conclusive about the contest's prize being zero. In this section we show that the balancing effect of private information on incentives highlighted in Section 3.1 is not driven by these simplifying assumptions. More specifically, we show that, even when both players remain imperfectly informed because a player's effort and hence his chance of winning is always positive, learning is capable of balancing a leader's and a follower's incentives. Our analysis in this section demonstrates that the mechanism responsible for the increase in aggregate incentives identified in Section 3.2 exists for rather general specifications of potential prize-valuations and information-structures.

In the following, we adopt a symmetric common value model (Krishna, 2010) by assuming that each player receives a signal $s_i \in \Re$ and that the contest's prize $V(s_1, s_2)$ is a symmetric function that is non-negative and strictly increasing in each signal. Signals are identically and independently distributed with density $g(s_i)$ and cumulative distribution $G(s_i)$. While we impose no further restrictions on the contest's potential prizes and the players' information structure, we simplify our model in two other dimensions. First, we assume efforts to be binary, i.e. $e_{it} \in \{e^l, e^h\}$ where $0 < e^l < e^h$. Second, we restrict the dynamic contest to consist of only two stages, by assuming that in case of a draw after two battles, each player receives the contest's prize with probability $\frac{1}{2}(1-\rho)$. We thus refrain from modeling players' behavior following a draw, and introduce the parameter $\rho \in (0,1)$ to captures the rate of rent-dissipation that would arise if, following a draw, players' had to keep competing. Finally, to abbreviate notation, we let $c = C(e^h) - C(e^l)$ denote the incremental cost of providing high effort rather then low effort, and we denote by $\eta = H(\frac{e^h}{e^l}) \in (\frac{1}{2}, 1)$ the likelihood that a high effort player wins a battle against a low effort player.

In each battle, players will adopt a *threshold-strategy* by choosing high effort if and only if their signal exceeds a threshold. Focusing on a symmetric equilibrium, we let $\bar{s}_1$ denote the players' threshold in battle 1. In battle 2, the players' thresholds depend on whether they lost or won battle 1 and, through updating, on whether they lost or won with low or with high effort. As the players' incentives to exert effort increase from battle 1 to battle 2, the relevant thresholds $\bar{s}_L$ and $\bar{s}_F$ of the leader and the follower, respectively, are smaller than $\bar{s}_1$. They can thus be determined from the requirement that after winning (losing) battle 1 with low effort $e_1 = e^l$, a leader (follower) with signal $s_L = \bar{s}_L$ ($s_F = \bar{s}_F$) must be indifferent between low and high effort in battle 2. The derivation of the thresholds $\bar{s}_L$ and $\bar{s}_F$ is simplified by the fact that exerting high effort rather than low effort increases a player's expected payoff by an amount that is independent of the opponent's effort.

More specifically, note that, for any value, $V(s_1, s_2)$, of the contest's prize, the leader's gain in expected payoff from exerting high rather than low effort is $\frac{1}{2}(1+\rho)(\eta-\frac{1}{2})V(s_1, s_2)$, whereas the corresponding value for the follower is $\frac{1}{2}(1-\rho)(\eta-\frac{1}{2})V(s_1, s_2)$. For any *realized* value of the contest's prize, the follower thus has a weaker incentive to exert effort than the leader. There exists a discouragement effect, which in this stylized two stage model arises from the fact that a draw leads to rent dissipation, parameterized by ρ.

However, private information has a balancing effect on incentives, because it induces the leader and the follower to have different expectations about the contest's prize. To see this let b_L and b_F denote the players' updated beliefs—after winning or losing battle 1—that their rival's signal lies above the threshold $\bar{s}_1$. From Bayesian updating these beliefs are given by

$$b_L = \frac{2(1-\eta)[1-G(\bar{s}_1)]}{G(\bar{s}_1)+2(1-\eta)[1-G(\bar{s}_1)]} \tag{35}$$

$$b_F = \frac{2\eta[1-G(\bar{s}_1)]}{G(\bar{s}_1)+2\eta[1-G(\bar{s}_1)]}, \tag{36}$$

and it holds that $0 < b_L < 1 - G(\bar{s}_1) < b_F < 1$, i.e. the follower is more optimistic about his rival's signal than the leader. The thresholds $\bar{s}_L$ and $\bar{s}_F$ solve

$$b_L \mathbb{E}[V(\bar{s}_L, s_F)|s_F > \bar{s}_1] + (1-b_L)\mathbb{E}[V(\bar{s}_L, s_F)|s_F < \bar{s}_1] = \frac{c}{(\eta - \frac{1}{2})\frac{1}{2}(1+\rho)} \tag{37}$$

$$b_F \mathbb{E}[V(\bar{s}_F, s_L)|s_L > \bar{s}_1] + (1-b_F)\mathbb{E}[V(\bar{s}_F, s_L)|s_L < \bar{s}_1] = \frac{c}{(\eta - \frac{1}{2})\frac{1}{2}(1-\rho)}, \tag{38}$$

and the leader's and the follower's incentives are balanced, i.e. $\bar{s}_F = \bar{s}_L = \bar{s}_2$, if and only if

$$1 = \frac{b_F \mathbb{E}[V(\bar{s}_2, s)|s > \bar{s}_1] + (1-b_F)\mathbb{E}[V(\bar{s}_2, s)|s < \bar{s}_1]}{b_L \mathbb{E}[V(\bar{s}_2, s)|s > \bar{s}_1] + (1-b_L)\mathbb{E}[V(\bar{s}_2, s)|s < \bar{s}_1]} \frac{1-\rho}{1+\rho} \tag{39}$$

Equation (39) is the equivalent of (16) and it shows that, as stated in Proposition 1, players' learning about the first battle's outcome can induce the follower to exert more effort than the leader when the contest's rate of rent dissipation is not to high. Our analysis in this section thus demonstrates that the fundamental mechanism, responsible for the beneficial effect of private information on incentives, continues to be present in a generic setting where players' information is inconclusive and the contest's prize can take arbitrary values.

In addition, the above setup allows us to discuss the effects of *intermediate prizes* on players' learning and incentives. Assume, for this purpose, that, on top of the contest's overall prize V, there exists an intermediate prize $v > 0$ that is awarded to the winner of the first battle. For simplicity, we continue to assume that players face uncertainty about V, but we let the size of v be certain and common knowledge. Intuitively, the presence of an intermediate prize creates an additional incentive for players to exert effort in battle

1, leading to a reduction in the threshold $\bar{s}_1$. As a consequence, both the leader's and the follower's beliefs, b_L and b_F, that their rival exerted high effort following a sufficiently high signal, increase. Notably, the wedge between b_L and b_F, that arises from the fact that winning represents bad news whereas losing represents good news about the rival's signal, is largest when $\bar{s}_1$ is such that $F(\bar{s}_1) = \frac{1}{2}$. In other words, the balancing effect of learning on incentives is strongest when, in the initial battle, the intermediate prize is such that it induces each player to face maximum uncertainty with respect to his rival's effort. While this insight identifies the potentially beneficial effects on players' learning as a rational for the emergence of intermediate prizes, a thorough analysis of the incentive-maximizing prize allocation across battles is beyond the scope of this paper.

7 Discussion and conclusion

In this article, we have identified the balancing effect of private information on incentives as an important aspect of dynamic competition. Our modeling approach has kept the dynamics as simple as possible by limiting the contest to at most three battles. Before we summarize our main message and its implications, we now discuss potential generalizations to longer horizons.

In a best-of-three contest, the gap between the leader and the follower can take only one value. Empirical studies have found that the discouragement effect increases in the disparity of intermediate outcomes. As a consequence, one would expect the discouragement effect to have a heavier toll on incentives in contests with longer horizons. Given that, in equilibrium, players will conclude that their rival's signal is good as soon as they have been defeated in a single battle, our model maintains its tractability when its horizon is extended.

Surprisingly, in a best-of-five Tullock contest, the effect of private information on aggregate incentives turns out to be even more positive than in a best-of-three contest. More specifically, we have confirmed that, for high and moderate rates of rent dissipation, the relative gain in incentives due to information being private rather than public, $\frac{E^*-E^P}{E^P}$, is larger in a best-of-five contest than in a best-of-three contest, *independently* of the signals' informativeness.

We have also analyzed a contest with a potentially infinite horizon where the prize is awarded if and only if a player has established a two-battle lead. Assuming future payoffs to be discounted with discount factor $\delta \in (0,1)$, and denoting by e_0^* players' equilibrium efforts when their score is equal and by e_{+1}^* and e_{-1}^* players' efforts when one player has taken a one-battle lead, it is straight forward to show that $\frac{e^*_{+1}}{e^*_{-1}} = \frac{\beta^*_{+1}}{\beta^*_{-1}}\frac{1+R}{1-R}$ and $e^*_0 = \delta h(1)[H(\frac{e^*_{+1}}{e^*_{-1}})V^G - (e^*_{+1} - e^*_{-1})]$. This means that the main equations of our model, i.e. (16) and (21), remain largely unchanged which indicates that our results do not arise from end-of-horizon effects.

We are thus confident to conclude, that, by balancing incentives, learning has a positive effect on aggregate effort in a dynamic contest. In the presence of private information, the discouraging effect of falling behind is mitigated by the fact that leaders and follower update their beliefs in opposite direction following their observation of intermediate outcomes. As an important consequence, the common concern that dynamics will be harmful for incentives, may not be justified. In the presence of private information, aggregate incentives in a dynamic contest can be even greater than in the static benchmark. Our results contrast with the existing literature on dynamic contests that has mostly abstracted from the potential privacy of information. They shed new light on a variety of applications, by showing, for example: that lagging firms can be more motivated to invest in an R&D race than leading firms; that wasteful campaign spending can be reduced if presidential primaries were held simultaneously rather than sequentially; and that feedback policies in labor tournaments can have a positive effect not only on workers' incentives but also on the likelihood of promoting the most able candidate. Although, admittedly, each one of those applications contains elements that would require our model to be more adapted, we hope that the simplicity of our approach is helpful to shed light on the interplay between learning and dynamic incentives that is a common feature of a broad variety of settings.

Acknowledgments. We are grateful to Andrea Prat, Giacomo Calzolari, Mikhail Drugov, Ian Jewitt, Jorge Lemus, Meg Meyer, Marco Serena, and participants of the European Summer Symposium in Economic Theory for helpful comments and suggestions. Special thanks go to Igor Letina for the fruitful title suggestion. Financial Support from Swiss National Science Foundation Grant no. 207874 is gratefully acknowledged.

Appendix

Proof of Lemma 1. This proof determines equilibrium effort choices in the public-information and static-competition benchmarks as well as for a dynamic contest with unobservable battle outcomes, before considering comparative statics.

Public information benchmark: In the following, we characterize the unique pure-strategy Subgame Perfect equilibrium of the public information benchmark. We use $(e^P_1, e^P_L, e^P_F, e^P_3)$ to denote players' effort levels conditional on $s_1 = s_2 = G$. Our characterization can restrict attention to first order conditions because our assumption that h is decreasing guarantees the concavity of players' objectives. Using backwards induction, equilibrium in battle 3 can be described in analogy to our single-battle analysis in Section 2. As players expect the prize to be V^G given by (3) we thus have $e_3^P = h(1)V^G$ and a player's continuation payoff from reaching battle 3 is $U_3 = [\frac{1}{2} - h(1)]V^G$, which is strictly positive due to Assumption 1 (see footnote 20). In battle 2, the follower's valuation of winning is given by U_3 whereas the leader's valuation of winning battle 2 is $V^G - U_3 = [\frac{1}{2} + h(1)]V^G > U_3$. In a Subgame Perfect equilibrium (e_L^P, e_F^P) must therefore solve

$$e_L^P \in \arg\max_{e_L \geq 0} U_3 + (V^G - U_3)H(\frac{e_L}{e_F^P}) - e_L \tag{40}$$

$$e_F^P \in \arg\max_{e_F \geq 0} U_3[1 - H(\frac{e_L^P}{e_F})] - e_F. \tag{41}$$

The first order conditions following from (40) and (41) have a unique solution given by

$$e_F^P = \frac{1+R}{2}h(\frac{1+R}{1-R})V^G \tag{42}$$

$$e_L^P = \frac{1+R}{1-R}e_F^P, \tag{43}$$

with R given by (4). The corresponding continuation payoffs from entering battle 2 as the leader or the follower are

$$U_L^G = U_3 + [H(\frac{1+R}{1-R}) - \frac{1-R}{1+R}h(\frac{1-R}{1+R})](V^G - U_3) > U_3 \tag{44}$$

$$U_F^G = [H(\frac{1-R}{1+R}) - \frac{1-R}{1+R}h(\frac{1-R}{1+R})]U_3 > 0, \tag{45}$$

where the inequalities follow from the fact that $H(\frac{1+R}{1-R}) > H(\frac{1-R}{1+R})$ and because $H(y) > yh(y)$ for all $y > 0$ by Assumption 1 (see footnote 20). Finally, in battle 1 players' have identical valuations of winning, $U_L^G - U_F^G$, and choose their effort to solve

$$e_1^P \in \arg\max_{e_1 \geq 0} U_F^G + H(\frac{e_1}{e_1^P})(U_L^G - U_F^G) - e_1 \tag{46}$$

leading to

$$e_1^P = (U_L^G - U_F^G)h(1) = [H(\frac{1+R}{1-R}) - 2h(1)\frac{1-R}{1+R}h(\frac{1-R}{1+R})]h(1)V^G > 0. \tag{47}$$

The corresponding equilibrium payoff is strictly positive because each player can guarantee himself a payoff of $U_F^G > 0$ by choosing $e_1 = 0$. Aggregating expected efforts over all three battles and both players gives

$$\begin{aligned} E^P &= \text{Prob}(s_1 = s_2 = G)[2e_1^P + e_L^P + e_F^P + 2H(\frac{1-R}{1+R})e_3^P] \\ &= [R + (1-R)^2 h(\frac{1-R}{1+R})] \cdot \mathbb{E}[V]. \end{aligned} \tag{48}$$

Static competition benchmark: Klumpp and Polborn (2006) show that for a Tullock contest where $H = H_r$ with $r \in (0, 1]$, all equilibria are payoff-equivalent to a pure-strategy equilibrium in which players choose the same effort in each battle. In the following we derive a necessary conditions for such an equilibrium for H satisfying Assumption 1. Suppose that conditional on having received a good signal, players exert effort $e^S > 0$ in each battle. From the viewpoint of a player with a good signal, the contest's prize can have non-zero value only when the rival's signal is good as well, which happens with probability β_1 given by (2). Moreover, in any particular battle, a player's effort influences his chance of winning only if there is a draw in the remaining two battles, which, given symmetry, happens with probability $\frac{1}{2}$. In equilibrium e^S must therefore solve

$$e^S \in \arg\max_{e\geq 0} \frac{1}{2}\beta_1 H(\frac{e}{e^S})V^G - e, \tag{49}$$

with V^G given by (3). Taking the first order condition of (49) and setting $e = e^S$ gives $e^S = \frac{1}{2}\beta_1 V^G h(1)$ as the unique candidate for a symmetric pure-strategy equilibrium. Summing efforts over both players and all battles, and multiplying with the probability that $s_i = G$ gives $3h(1)\mathbb{E}[V]$ as the corresponding expected aggregate effort. Note that the corresponding equilibrium payoff of each player is $\frac{1}{2}\beta_1 V^G[1 - 3h(1)]$, i.e. existence of a pure-strategy equilibrium in the static contest requires $h(1) < \frac{1}{3}$. If signals were observed publicly rather than privately, then, conditional on $s_1 = s_2 = G$ both players would exert efforts $e = \frac{1}{2}V^G h(1)$ and it follows from the fact that $\text{Prob}(s_1 = s_2 = G) = \text{Prob}(s_i = G)\beta_1$ that expected aggregate effort again equals E^S.

Unobservable battle outcomes. Consider a variation of our dynamic contest model with private signals where battle outcomes are unobservable. In the following, we determine the equilibrium effort levels e_1^U, e_2^U, and e_3^U, which players, conditional on having observed a good signal, choose in battles 1, 2, and 3, respectively. Since battle 3 is reached only when both players have won exactly one battle, efforts in battle 3 are the same as in the model with observable battle outcomes, i.e. $e_3^U = e_3^*$, given by (6). In battle 2, a player with a good signal is uncertain whether he has won or lost the first battle. His effort therefore solves

$$e_2^U \in \arg\max_{e_2\geq 0} \beta_1\{\frac{1}{2}[H(\frac{e_2}{e_2^U})(V^G - U_3) + U_3] + \frac{1}{2}H(\frac{e_2}{e_2^U})U_3\} - e_2, \tag{50}$$

with U_3 denoting the continuation payoff from reaching battle 3. Solving this program leads to the same effort as in the static benchmark, i.e. $e_2^U = e^S$. Finally, in battle 1, a player with a good signal chooses effort to solve

$$e_1^U \in \arg\max_{e_1\geq 0} \beta_1\{H(\frac{e_1}{e_1^U})U_L^G + [1 - H(\frac{e_1}{e_1^U})]U_F^G\} + (1 - \beta_1)U_L^B - e_1, \tag{51}$$

where $U_L^G = \frac{1}{2}V^G + \frac{1}{2}U_3 - e_2^U$, $U_F^G = \frac{1}{2}U_3 - e_2^U$, and $U_L^B = -e_2^U$, denote the continuation payoffs of reaching battle 2 as the leader or follower (without knowing it), conditional on the rival's signal. Solving this program we again find $e_1^U = e^S$. Expected aggregate effort is thus given by

$$E^U = 2\text{Prob}(s_i = G)(e_1^U + e_2^U) + 2\text{Prob}(s_1 = s_2 = G)\frac{1}{2}e_3^U = 3h(1)\mathbb{E}[V] = E^S. \tag{52}$$

Hence, with unobservable battle outcomes, expected aggregate effort is the same as in the static competition benchmark.

Comparative statics: To see that $E^P < E^S$ use $R = 2h(1)$ to obtain

$$E^S - E^P = \left[h(1) - (1-R^2)\frac{1-R}{1+R}h(\frac{1-R}{1+R})\right]\mathbb{E}[V] \tag{53}$$

and note that $h(1) > \frac{1-R}{1+R}h(\frac{1-R}{1+R})$ because $yh(y)$ is maximized at $y = 1$ (Assumption 1). To see that $\frac{E^P}{E^S}$ is decreasing in R note that

$$\frac{E^P}{E^S} = \frac{2}{3}\left[1 + \frac{1-R^2}{R}\cdot\frac{1-R}{1+R}h(\frac{1-R}{1+R})\right], \tag{54}$$

i.e. $\frac{E^P}{E^S}$ is the product of two decreasing positive-valued functions (Assumption 1 guarantees that $yh(y)$ increases in y for all $y < 1$). □

Proof of Lemma 2. We first show that, in battle 1, the first order condition corresponding to the players' objective function (20) takes the simple form in (21). To see this write the first order condition as

$$0 = \beta_1\frac{h(1)}{e_1^*}[U_L^H(e_1^*) - U_F^G] - 1 + \frac{d}{de_1}[H(1)\beta_1 U_L^G(e_1) + (1-\beta_1)U_L^B(e_1)]|_{e_1=e_1^*}. \tag{55}$$

Substituting continuation values $U_L^G(e_1)$ and $U_L^B(e_1)$, the remaining derivative can be written as

$$[\beta_1 H(1) + 1 - \beta_1]\frac{d}{de_1}\{\beta_L(e_1^*)[U_3 + H(\frac{e_L(e_1)}{e_F^*})(V^G - U_3)] - e_L(e_1)\}|_{e_1=e_1^*} = 0. \tag{56}$$

The term in parentheses equals the battle 2 objective of a player who deviated in battle 1 by choosing e_1 and happened to become the leader. Since $e_L(e_1)$ is chosen to maximize this objective, it follows from the envelope theorem that its derivative equals zero. It follows that e_1^* has to satisfy (21). Together with the analysis contained in Section 3.1, this shows that $(e_1^*, e_L^*, e_F^*, e_3^*)$ defined by (6), (14), (15), and (21), is the unique candidate for a symmetric pure-strategy Perfect Bayesian equilibrium.

A comment is in order concerning the fact that the maximization program in (20) restricts the players' choice to strictly positive effort levels $e_1 > 0$. We now show that a deviation to $e_1 = 0$ is dominated by a deviation to $e_1 = \epsilon$ for $\epsilon > 0$ sufficiently small, which implies that neglecting the possibility of zero effort in (20) comes without loss of generality. Treating the possibility of zero effort separately is necessary because Bayesian updating in the case where $e_1 = 0$ differs from Bayesian updating in the case where $e_1 > 0$. More precisely, consider an equilibrium with $e_1^* > 0$, and suppose a player deviates to $e_1 = 0$. If the deviating player wins the first battle he learns that his rival must have received the signal B, i.e. $\beta_L^0 = 0$. Instead, if the deviating player loses the first battle, he will update his belief to $\beta_F^0 = \frac{\beta_1}{\beta_1 + \frac{1}{2}(1-\beta_1)}$ and then choose an effort $e_F^0 \in \arg\max_{e_F} \beta_F^0 H(\frac{e_F}{e_L^*})U_3 - e_F$. The payoff from a deviation to zero effort in battle 1 is thus given by

$$U_1^0 = \beta_1[H(\frac{e_F^0}{e_L^*})U_3 - e_F^0] - (1-\beta_1)\frac{1}{2}e_F^0. \tag{57}$$

Instead, a deviation to $e_1 = \epsilon$ gives the payoff

$$U_1^\epsilon = \beta_1\{H(\frac{\epsilon}{e_1^*})U_L^G(\epsilon) + [1 - H(\frac{\epsilon}{e_1^*})]U_F^G\} + (1-\beta_1)U_L^B(\epsilon) - \epsilon. \tag{58}$$

After winning battle 1, a player who deviated from an equilibrium $e_1^* > 0$ by exerting only a small effort in battle 1 must be nearly certain that his rival has observed a bad signal. Formally, for $\epsilon \to 0$ it holds that $\beta_L(\epsilon) \to 0$ and thus $e_L(\epsilon) \to 0$. Hence, for $\epsilon \to 0$, it holds that

$$U_1^\epsilon \to \beta_1 U_F^G = \beta_1[H(\frac{e_F^*}{e_L^*})U_3 - e_F^*] \geq U_1^0, \tag{59}$$

and the inequality follows from the fact that $e_F^* \in \arg\max_{e_F} H(\frac{e_F}{e_L^*})U_3 - e_F$. Intuitively, although a player can achieve that a win in battle 1 reveals the rival's signal perfectly by choosing $e_1 = 0$, the player can do even better because when choosing an infinitesimal effort $e_1 = \epsilon$, the rival's signal becomes revealed not only by a win (approximately) but also by a loss in battle 1.

Finally, to prove existence of equilibrium it remains to consider second order conditions. We first consider the case where the distribution of the ratio of noise is given by $h_r = \frac{ry^{-r-1}}{(1+y^{-r})^2}$ generating the generalized Tullock contest success function with parameter r. Nti (1999) shows that in a static Tullock contest a pure strategy equilibrium exists if and only if $r \leq 1 + v^r$ where $v \in (0, 1]$ denotes the contestants' ratio of valuations of winning. Our contest is dynamic rather than static, but using continuation values we were able to write each battle in the form of a static Tullock contest. The contestants have identical valuations of winning in battles 1 and 3, i.e. valuations differ only in battle 2 where $v = \frac{U_3}{\beta_L^*(V^G - U_3)}$. v is minimized when signals are public, i.e. for $\beta_L^* = 1$. Note that in contrast to Nti (1999), our contest features imperfect information. However, because contestants exert zero efforts after observing a bad signal, the conditions for a pure strategy Perfect Bayesian equilibrium are just an analogue of the equilibrium conditions in Nti (1999). Since for h_r we find $U_3 = (\frac{1}{2} - \frac{r}{4})V^G$ and $V^G - U_3 = (\frac{1}{2} + \frac{r}{4})V^G$, a pure strategy Perfect Bayesian equilibrium thus exists for all σ if and only if

$$r \leq 1 + (\frac{2-r}{2+r})^r. \tag{60}$$

As this inequality is satisfied for all $r \leq 1$ we have thus shown existence of equilibrium for the family of Tullock contest success functions with parameters $r \leq 1$. The equilibrium is unique and can be determined in closed form as:

$$\begin{aligned} e_3^* &= \frac{rV_G}{4} && (61)\\ e_L^* &= \frac{rV_G}{4}\beta_L^*(2+r)\chi && (62)\\ e_F^* &= \frac{rV_G}{4}(2-r)\chi && (63)\\ e_1^* &= \frac{rV_G}{4}\beta_1\chi\{(\beta_L^*\frac{2+r}{2-r})^r + 1 - \frac{r}{4}[\beta_L^*(2+r) - 2 + r]\} && (64) \end{aligned}$$

where we abbreviated notation by defining $\chi \equiv \frac{(\beta_L^*)^r(2+r)^r(2-r)^r}{[(\beta_L^*)^r(2+r)^r+(2-r)^r]^2}$.

While for the Tullock family, equilibrium existence is guaranteed for all $\sigma \in [0,1]$, that is, *independently* of the informativeness of the contestants' signals, for general distributions of the ratio of noise, existence is harder to establish. In the remainder of this proof we show that, under the conditions of Assumption 1, an equilibrium exists when the contest is "sufficiently noisy", i.e. if $h(1)$ is sufficiently small, or contestants' information is "sufficiently public", that is when σ is sufficiently close to 0 or 1.

To see this, first note that the players' objective in battle 3, as well as the leader's and the follower's objectives in battle 2, given by (12) and (13), are globally concave because $h = H'$ is assumed to be strictly decreasing. For the remaining battle 1, the second order condition requires

$$\frac{d}{de_1}\{\frac{\beta_1}{e_1^*}h(\frac{e_1}{e_1^*})[U_L^G(e_1) - U_F^G] - 1\}|_{e_1=e_1^*} = \frac{\beta_1}{e_1^*}[\frac{h'(1)}{\beta_1 h(1)} + h(1)\frac{dU_L^G(e_1^*)}{de_1}] < 0 \tag{65}$$

with

$$\begin{aligned} U_L^G(e_1) &= U_3 + H(\frac{e_L(e_1)}{e_F^*})(V^G - U_3) - e_L(e_1), && (66)\\ e_L(e_1) &\in \arg\max_{e_L\geq 0} \beta_L(e_1)[U_3 + (V^G - U_3)H(\frac{e_L}{e_F^*})] - e_L. && (67) \end{aligned}$$

Note that because h is decreasing the second order condition is automatically satisfied when $h(1)$ is sufficiently small. To see that the second order condition is also satisfied in the limits $\sigma \to 0$ and $\sigma \to 1$, note that

$$\frac{dU_L^G(e_1^*)}{de_1} = [h(\frac{e_L^*}{e_F^*})(V^G - U_3) - 1]\frac{de_L(e_1^*)}{de_1} = \frac{1-\beta_L^*}{\beta_L^*}\frac{de_L(e_1^*)}{de_1} \tag{68}$$

where we have used the fact that e_L^* solves the first order condition $\beta_L^* h(\frac{e_L^*}{e_F^*})(V^G - U_3) = 1$. As $e_L(e_1)$ satisfies an analogue first order condition with β_L^* substituted by $\beta_L(e_1)$, we can employ the Implicit Function Theorem to get

$$\frac{de_L(e_1^*)}{de_1} = -\frac{h(\frac{e_L^*}{e_F^*})e_F^*}{h'(\frac{e_L^*}{e_F^*})\beta_L^*}\frac{d\beta_L(e_1^*)}{de_1}. \tag{69}$$

Note from (9) that $\frac{d\beta_L(e_1^*)}{de_1}$ is positive but tends to zero for $\sigma \to 0$ and for $\sigma \to 1$. As $\beta_L^* \to 1$ in both cases, we can thus conclude from $h' < 0$ that the second order condition in (65) must be satisfied when σ is sufficiently close to 0 or 1. ☐

Proof of Proposition 1. Given that $\beta_F^* = 1$,

$$\frac{e_L^*}{e_F^*} = \frac{1+R}{1-R}\beta_L^* = \frac{1+R}{1-R}\frac{1-\omega+\omega(1-\sigma)^2}{1-\omega+\omega(1-\sigma)^2+2\omega\sigma(1-\sigma)} \tag{70}$$

and it follows that

$$\lim_{\sigma\to 0}\frac{e_L^*}{e_F^*} = \lim_{\sigma\to 1}\frac{e_L^*}{e_F^*} = \frac{e_L^P}{e_F^P} = \frac{1+R}{1-R} > 1. \tag{71}$$

Moreover, the derivative

$$\frac{d}{d\sigma}[\frac{e_L^*}{e_F^*}] = \frac{1+R}{1-R}\frac{2\omega(2\sigma-1-\omega\sigma^2)}{(1-\omega\sigma^2)^2} \tag{72}$$

has a unique root in $(0,1)$ at $\sigma = \hat{\sigma}(\omega)$ defined in (11), is negative for $\sigma \in (0, \hat{\sigma}(\omega))$ and positive for $\sigma \in (\hat{\sigma}(\omega), 1)$. Hence $\frac{e_F^*}{e_L^*}$ has inverse U-shape with a maximum at $\sigma = \hat{\sigma}(\omega)$ and $\frac{e_F^*}{e_L^*} > \frac{e_F^P}{e_L^P}$ for all $\sigma \in (0,1)$. Its maximized value is

$$\frac{e_F^*}{e_F^*}|_{\sigma=\hat{\sigma}(\omega)} = \frac{1-R}{1+R}\frac{1-\sqrt{1-\omega}}{\sqrt{1-\omega}-(1-\omega)}. \tag{73}$$

It follows that $\frac{e_F^*}{e_L^*} > 1$ for a non-empty interval (σ_-, σ_+) if and only if

$$\frac{1+R}{1-R}\frac{\sqrt{1-\omega}-(1-\omega)}{1-\sqrt{1-\omega}} < 1 \Leftrightarrow R < \overline{R}(\omega), \tag{74}$$

with $\overline{R}(\omega)$ as defined in (18). The thresholds σ_- and σ_+ solve the equation $e_L^* = e_F^*$ and are given by (19). That the sum of the leader's and the follower's expected effort is maximized when $\sigma \in \{\sigma_-, \sigma_+\}$ follows directly from (22) and the fact that the function $yh(y)$ has a unique maximum at $y = 1$. ☐

Proof of Lemma 3. Given that in battle 1, a player exerts effort if and only if he observed a good signal, the expected sum of the two players' efforts in battle 1 is

$$E_1^* = 2\text{Prob}(s_1 = G)e_1^* = R \cdot \mathbb{E}[V]\left[H(\frac{e_L^*}{e_F^*}) - \frac{1}{V^G}(e_L^* - e_F^*)\right]. \tag{75}$$

As battle 3 is reached only when both players observe a good signal and when the follower wins the second battle, the expected sum of efforts in battle 3 is given by

$$E_3^* = \text{Prob}(s_1 = s_2 = G)\left[1 - H(\frac{e_L^*}{e_F^*})\right] \cdot 2e_3^* = R \cdot \mathbb{E}[V]\left[1 - H(\frac{e_L^*}{e_F^*})\right]. \tag{76}$$

Aggregating efforts over battles 1 and 3, we are thus left with the simple expression in (23). ☐

Proof of Proposition 2. To abbreviate notation, define $\phi \equiv \frac{1-R}{1+R}$. Consider

$$E^* = E_1^* + E_3^* + E_2^* = R\mathbb{E}[V][1 - \frac{1}{V^G}(e_L^* - e_F^*)] + \mathbb{E}[V]\frac{e_L^*}{e_F^*}h(\frac{e_L^*}{e_F^*}). \tag{77}$$

Substitution of e_F^* and e_L^* from (14) and (15) gives

$$E^* = \{R + \frac{\beta_L^*}{\phi}h(\frac{\beta_L^*}{\phi})[1 - \frac{\beta_L^* - \phi}{1+\phi}R]\}\mathbb{E}[V] \tag{78}$$

which shows that σ affects aggregate incentives only through its effect on β_L^*. In the following we can thus focus on the dependence of E^* on β_L^*. First note that $\lim_{\beta_L^* \to 1} E^* = E^P$, with E^P given by Lemma 1, because

$$\lim_{\beta_L^* \to 1} \frac{\beta_L^*}{\phi}h(\frac{\beta_L^*}{\phi})[1 - \frac{\beta_L^* - \phi}{1+\phi}R] = \phi h(\phi)[1 - \frac{1-\phi}{1+\phi}R] = (1-R)^2 h(\frac{1-R}{1+R}) \tag{79}$$

where we have used the fact that by symmetry $\frac{1}{\phi}h(\frac{1}{\phi}) = \phi h(\phi)$. Also note that

$$\lim_{\beta_L^* \to \phi} \frac{\beta_L^*}{\phi}h(\frac{\beta_L^*}{\phi})[1 - \frac{\beta_L^* - \phi}{1+\phi}R] = h(1) = \frac{1}{2}R \tag{80}$$

which shows that $\lim_{\beta_L^* \to \phi} E^* = E^S$, with E^S given by Lemma 1. Next, we show that $\frac{dE^*}{d\beta_L^*} < 0$ for all $\beta_L^* \geq \phi$. For this purpose, we define the function $g(y) \equiv yh(y)$ and denoting its derivative by g' we obtain

$$\frac{1}{\mathbb{E}[V]}\frac{dE^*}{d\beta_L^*} = \frac{1}{\phi}g'(\frac{\beta_L^*}{\phi})[1 - \frac{\beta_L^* - \phi}{1+\phi}R] - \frac{1}{1+\phi}g(\frac{\beta_L^*}{\phi})R \tag{81}$$

which is strictly negative for all $\beta_L^* \geq \phi$ because $g(.)$ is unimodal with a maximum at 1. Next, we argue that $E^* > E^P$ for all $\beta_L^* \in [\phi^2, \phi)$. To see this use (77) to obtain

$$E^* - E^P = R\mathbb{E}[V]\frac{1}{V^G}(e_L^P - e_F^P + e_F^* - e_L^*) + \mathbb{E}[V][\frac{e_L^*}{e_F^*}h(\frac{e_L^*}{e_F^*}) - \frac{e_L^P}{e_F^P}h(\frac{e_L^P}{e_F^P})] > 0. \tag{82}$$

Here the first term is positive because $e_L^P > e_F^P$ and $\beta_L^* < \phi$ implies that $e_F^* > e_L^*$ while the second term is positive because from $\beta_L^* \geq \phi^2$ it follows that $1 > \frac{e_L^*}{e_F^*} \geq \frac{e_F^P}{e_L^P}$ which given g's unique mode at one implies that

$$0 < \frac{e_L^*}{e_F^*}h(\frac{e_L^*}{e_F^*}) - \frac{e_F^P}{e_L^P}h(\frac{e_F^P}{e_L^P}) = \frac{e_L^*}{e_F^*}h(\frac{e_L^*}{e_F^*}) - \frac{e_L^P}{e_F^P}h(\frac{e_L^P}{e_F^P}). \tag{83}$$

Finally, note from

$$E^* - E^P = \mathbb{E}[V]\{\frac{\beta_L^*}{\phi}h(\frac{\beta_L^*}{\phi})[1 - \frac{\beta_L^* - \phi}{1+\phi}R] - (1-R)^2 h(\phi)\} \tag{84}$$

that for $\beta_L^* \to 0$, the difference $E^* - E^P$ converges to $-\mathbb{E}[V](1-R)^2 h(\phi) < 0$.

The Proposition then follows from the fact that $\beta_L^*(\sigma)$ is U-shaped with $\lim_{\sigma\to 0}\beta_L^* = \lim_{\sigma\to 1}\beta_L^* = 1$ and that its minimum value $\min_{\sigma\in(0,1)}\beta_L^* = \frac{\sqrt{1-\omega}-(1-\omega)}{1-\sqrt{1-\omega}}$ is smaller than ϕ if and only if $R < \overline{R}(\omega)$ and smaller than ϕ^2 if and only if $R < \underline{R}(\omega)$ with both thresholds converging to one for $\omega \to 1$ and to zero for $\omega \to 0$. □

Proof of Proposition 3. For $R \geq \overline{R}(\omega)$, $E^*(\sigma)$ inherits its shape from $\beta_L^*(\sigma)$, because, as shown in the proof of Proposition 2, $\frac{dE^*}{d\beta_L^*} < 0$ for $\beta_L^* \geq \phi = \frac{1-R}{1+R} \Leftrightarrow \frac{e_L^*}{e_F^*} > 1$ and because the follower cannot be induced to exert higher effort than the leader, independently of σ. For $R < \overline{R}(\omega)$, the proof of Proposition 2 has shown that $E^*(\sigma)$ must be maximized at a $\sigma^* \in (\sigma_-, \sigma_+)$ and since the thresholds σ_- and σ_+ are defined by the requirement that $e_L^* = e_F^*$, at $\sigma = \sigma^*$ it must hold that $e_F^* > e_L^*$. It thus remains to consider the comparative statics:

$$\frac{d\overline{R}}{d\omega} = \frac{2-\omega-2\sqrt{1-\omega}}{\omega^2\sqrt{1-\omega}} > 0 \tag{85}$$

because the nominator is increasing in ω for $\omega \in (0,1)$ and converges to zero for $\omega \to 0$. For the same reason it holds that

$$\frac{d\hat{\sigma}}{d\omega} = \frac{2-\omega-2\sqrt{1-\omega}}{2\omega^2\sqrt{1-\omega}} > 0. \tag{86}$$

□

Proof of Proposition 4. Consider $-\frac{\partial S}{\partial \beta_L^*}|_{\beta_L^*=1} = \Delta S^+ - \Delta S^-$ with ΔS^+ and ΔS^- given by (33) and (34), respectively. Note first that $\gamma > 1$ implies that $H(\gamma) > H(\frac{1}{\gamma})$. Remember that the function $yh(y)$ is unimodal with a unique maximum at $y = 1$ and that $yh(y) = \frac{1}{y}h(\frac{1}{y})$. As

$$\frac{V^G - U_3^l}{U_3^h} = \gamma\frac{H(\frac{1}{\gamma}) + \gamma h(\gamma)}{H(\frac{1}{\gamma}) - \gamma h(\gamma)} > 1 \tag{87}$$

it holds that $\gamma\frac{V^G-U_3^l}{U_3^h} > 1$ and it is thus sufficient for $\Delta S^+ > \Delta S^-$ that

$$[\gamma\frac{V^G - U_3^l}{U_3^h}]^{-1} < \gamma\frac{U_3^l}{V^G - U_3^h} < \gamma\frac{V^G - U_3^l}{U_3^h}. \tag{88}$$

The second inequality follows directly from

$$\frac{U_3^l}{V^G - U_3^h} = \frac{H(\gamma) - \gamma h(\gamma)}{H(\gamma) + \gamma h(\gamma)} < 1. \tag{89}$$

For the first inequality note that $\frac{U_3^l}{V^G-U_3^h} > \frac{U_3^h}{V^G-U_3^l}$ if and only if

$$H(\gamma) - \gamma h(\gamma) - [H(\gamma) - \gamma h(\gamma)]^2 > H(\frac{1}{\gamma}) - \gamma h(\gamma) - [H(\frac{1}{\gamma}) - \gamma h(\gamma)]^2. \tag{90}$$

This inequality is satisfied because the terms $H(\gamma) - \gamma h(\gamma)$ and $H(\frac{1}{\gamma}) - \gamma h(\gamma)$ lie between zero and one and the former is closer to $\frac{1}{2}$ than the latter. We have thus shown that $\Delta S^+ > \Delta S^-$, or equivalently, $\frac{\partial S}{\partial \beta_L^*}|_{\beta_L^*=1} < 0$. Using private information to reduce the leader's belief marginally below his belief in the public information benchmark has a positive effect on selective efficiency. $\square$

References

Antsygina, Anastasia and Mariya Teteryatnikova, "Optimal information disclosure in contests with stochastic prize valuations," *Economic Theory*, 2022.

Aoyagi, Masaki, "Information feedback in a dynamic tournament," *Games and Economic Behavior*, 2010, *70* (2), 242–260.

Awaya, Yu and Vijay Krishna, "Startups and Upstarts : Disadvantageous Information in R&D," *Journal of Political Economy*, 2021, *129* (2), 534–569.

Azmat, Ghazala and Nagore Iriberri, "The importance of relative performance feedback information: Evidence from a natural experiment using high school students," *Journal of Public Economics*, 2010, *94* (7-8), 435–452.

Barbieri, Stefano and Marco Serena, "Biasing dynamic contests between ex-ante symmetric players," *Games and Economic Behavior*, 2022, *136*, 1–30.

Baye, M, D Kovenock, and C de Vries, "The All-Pay Auction with Complete Information," *Economic Theory*, 1996, *8* (2), 1–328.

Baye, Michael R. and Heidrun C. Hoppe, "The strategic equivalence of rent-seeking, innovation, and patent-race games," *Games and Economic Behavior*, 2003, *44* (2), 217–226.

Bhattacharya, Vivek, "An Empirical Model of R&D Procurement Contests: An Analysis of the DOD SBIR Program," *Econometrica*, 2021, *89* (5), 2189–2224.

Bimpikis, Kostas, Shayan Ehsani, and Mohamed Mostagir, "Designing dynamic contests," *Operations Research*, 2019, *67* (2), 339–356.

Blanesl, Jordi and Mareike Nossol, "Tournaments without prizes: Evidence from personnel records," *Management science*, 2011, *57* (10), 1721–1736.

Bobtcheff, Catherine, Raphaël Levy, and Thomas Mariotti, "Negative Results in Science: Blessing or (Winner's) Curse?," *CEPR Discussion Paper*, 2021.

Bolton, Patrick and Christopher Harris, "Strategic experimentation," *Econometrica*, 1999, *67* (2), 349–374.

Bonatti, Alessandro and Johannes Hörner, "Learning to disagree in a game of experimentation," *Journal of Economic Theory*, 2017, *169*, 234–269.

Cao, Dan, "Racing under uncertainty: Boundary value problem approach," *Journal of Economic Theory*, 2014, *151* (1), 508–527.

Chan, William, Pascal Courty, and Li Hao, "Suspense : Dynamic Incentives in Sports Contests," *The Economic Journal*, 2009, *119* (534), 24–46.

Chen, Zhuoqiong, "Optimal information exchange in contests," *Journal of Mathematical Economics*, 2021, *96*, 102518.

Choi, Jay P, "Dynamic R&D Competition under " Hazard Rate " Uncertainty," *RAND Journal of Economics*, 1991, *22* (4), 596–610.

Clark, Derek J. and Tapas Kundu, "Competitive balance: Information disclosure and discrimination in an asymmetric contest," *Journal of Economic Behavior and Organization*, 2021, *184*, 178–198.

_ and _ , "Partial information disclosure in a contest," *Economics Letters*, 2021, *204*, 109915.

Dong, Miaomiao, "Strategic experimentation with asymmetric information," *Unpublished Manuscript*, 2016.

Drugov, Mikhail and Dmitry Ryvkin, "Hunting for the discouragement effect in contests," *Review of Economic Design*, 2023.

Dubey, Pradeep, "The role of information in contests," *Economics Letters*, 2013, *120* (2), 160–163.

Ederer, Florian, "Feedback and motivation in dynamic tournaments," *Journal of Economics and Management Strategy*, 2010, *19* (3), 733–769.

Einy, Ezra, Diego Moreno, and Benyamin Shitovitz, "The value of public information in common-value Tullock contests," *Economic Theory*, 2017, *63* (4), 925–942.

Ely, Jeffrey, George Georgiadis, Sina Moghadas Khorasani, and Luis Rayo, "Optimal Feedback in Contests," *Review of Economic Studies*, 2023, *90* (5), 2370–2394.

Endo, Akira, "A historical perspective on the discovery of statins," *Proceedings of the Japan Academy Series B: Physical and Biological Sciences*, 2010, *86* (5), 484–493.

Ferrall, C and A Smith, "A Sequential Game Model of Sports Championship Series : Theory and Estimation," *The Review of Economics and Statistics*, 1999, *81* (4), 704–719.

Fu, Qiang, Qian Jiao, and Jingfeng Lu, "Disclosure policy in a multi-prize all-pay auction with stochastic abilities," *Economics Letters*, 2014, *125* (3), 376–380.

Fudenberg, Drew, Richard Gilbert, Joseph Stiglitz, and Jean Tirole, "Preemption, Leapfrogging, and Competition in Patent Races," *European Economic Review*, 1983, *22*, 3–31.

Gershkov, Alex and Motty Perry, "Tournaments with midterm reviews," *Games and Economic Behavior*, 2009, *66* (1), 162–190.

Goltsman, Maria and Arijit Mukherjee, "Interim performance feedback in multistage tournaments: The optimality of partial disclosure," *Journal of Labor Economics*, 2011, *29* (2), 229–265.

Halac, Marina, Navin Kartik, and Qingmin Liu, "Contests for Experimentation Navin Kartik Qingmin Liu," *Journal of Political Economy*, 2017, *125* (5), 1523–1569.

Harris, Christopher and John Vickers, "Racing with uncertainty," *Review of Economic Studies*, 1987, *54* (1), 1–21.

Heidhues, Paul, Sven Rady, and Philipp Strack, "Strategic experimentation with private payoffs," *Journal of Economic Theory*, 2015, *159*, 531–551.

Hodges, J and E. Lehmann, "Matching in Paired Comparisons," *The Annals of Mathematical Statistics*, 1954, *25* (4), 787–791.

Hopenhayn, Hugo A. and Francesco Squintani, "Preemption games with private information," *Review of Economic Studies*, 2011, *78* (2), 667–692.

Hörner, Johannes and Julian Jamison, "Sequential Common-Value Auctions with Asymmetrically Informed Bidders," *Review of Economic Studies*, 2008, *75*, 475–498.

Hurley, Terrance M. and Jason F. Shogren, "Asymmetric information contests," *European Journal of Political Economy*, 1998, *14* (4), 645–665.

Iqbal, Hamzah and Alex Krumer, "Discouragement effect and intermediate prizes in multi-stage contests: Evidence from Davis Cup," *European Economic Review*, 2019, *118*, 364–381.

Jia, Hao, "A stochastic derivation of the ratio form of contest success functions," *Public Choice*, 2008, *135*, 125–130.

Judd, Kenneth, Karl Schmedders, and Sevin Yeltekin, "Optimal Rules for Patent Races," *International Economic Review*, 2012, *53* (1), 23–52.

Kamenica, Emir and Matthew Gentzkow, "Bayesian persuasion," *American Economic Review*, 2011, *101* (6), 2590–2615.

Keller, Godfrey and Sven Rady, "Breakdowns," *Theoretical Economics*, 2015, *10*, 175–202.

_ , _ , and Martin Cripps, "Strategic experimentation with exponential bandits," *Econometrica*, 2005, *73* (1), 39–68.

Klein, Arnd Heinrich and Armin Schmutzler, "Optimal effort incentives in dynamic tournaments," *Games and Economic Behavior*, 2017, *103*, 199–224.

Klumpp, Tilman and Mattias K. Polborn, "Primaries and the New Hampshire Effect," *Journal of Public Economics*, 2006, *90* (6-7), 1073–1114.

Konrad, Kai, *Strategy and Dynamics in Contests*, Oxford, UK: Oxford University Press, 2009.

_ and Dan Kovenock, "Multi-battle contests," *Games and Economic Behavior*, 2009, *66* (1), 256–274.

Krishna, Vijay, *Auction Theory*, London, UK: Elsevier Academic Press, 2010.

Kuang, Zhonghong, Hangcheng Zhao, and Jie Zheng, "Information Design in All-pay Auction Contests," *Unpublished Manuscript*, 2019.

Lauermann, Stephan and Andre Speit, "Bidding in Common-Value Auctions With an Unknown Number of Competitors," *Econometrica*, 2023, *91* (2), 493–527.

Lazear, Edward P . and Sherwin Rosen, "Rank-Order Tournaments as Optimum Labor Contracts," *The Journal of Political Economy*, 1981, *89* (5), 841–864.

Lemus, Jorge and Guillermo Marshall, "Dynamic tournament design: Evidence from prediction contests," *Journal of Political Economy*, 2021, *129* (2), 383–420.

Levin, Dan, John H Kagel, and Jean-Francois Richard, "Revenue Effects and Information Processing in English Common Value Auctions," *American Economic Review*, 1996, *86* (3), 442–460.

Logsdon, John, *John F. Kennedy and the Race to the Moon*, Springer, 2010.

Mago, Shalom D., Roman M. Sheremeta, and Andrew Yates, "Best-of-three contest experiments: Strategic versus psychological momentum," *International Journal of Industrial Organization*, 2013, *31* (3), 287–296.

Malik, Henrick John, "Exact distribution of the quotient of independent generalized gamma variables," *Canadian Mathematical Bulletin*, 1967, *10*, 463–465.

Malueg, D. and S. Tsutsui, "Dynamic R&D Competition with Learning," *Rand Journal of Economics*, 1997, *28* (4), 751–772.

Malueg, David and Andrew Yates, "Testing Contest Theory : Evidence from Best-of-Three Tennis Matches," *The Review of Economics and Statistics*, 2010, *92* (3), 689–692.

Melo-Ponce, Alejandro, "The Secret Behind The Tortoise and the Hare: Information Design in Contests," *Unpublished Manuscript*, 2020, pp. 1–77.

Meyer, Margaret A., "Learning from coarse information: Biased contests and career profiles," *Review of Economic Studies*, 1991, *58* (1), 15–41.

Milgrom, Paul R. and Robert J. Weber, "A Theory of Auctions and Competitive Bidding," *Econometrica*, 1982, *50* (5), 1089.

Morath, Florian and Johannes Münster, "Private versus complete information in auctions," *Economics Letters*, 2008, *101* (3), 214–216.

Moscarini, Giuseppe and Francesco Squintani, "Competitive experimentation with private information: The survivor's curse," *Journal of Economic Theory*, 2010, *145* (2), 639–660.

Nti, Kofi O, "Rent-Seeking with Asymmetric Valuations," *Public Choice*, 1999, *98* (3), 415–430.

Pesendorfer, Wolfgang and Jeroen Swinkels, "The Loser ' s Curse and Information Aggregation in Common Value Auctions," *Econometrica*, 1997, *65* (6), 1247–1281.

Serena, Marco, "Harnessing beliefs to optimally disclose contestants' types," *Economic Theory*, 2021.

Tullock, G., "Efficient rent seeking," in J. Buchanan, R. Tollison, and G. Tullcok, eds., *Toward a theory of the rent- seeking society*, College Station, Texas: A&M Press, 1980.

Wagner, Peter A. and Nicolas Klein, "Strategic investment and learning with private information," *Journal of Economic Theory*, 2022, *204*, 105523.

Wärneryd, Karl, "Information in conflicts," *Journal of Economic Theory*, 2003, *110* (1), 121–136.

Wasser, Cédric, "Incomplete information in rent-seeking contests," *Economic Theory*, 2013, *53* (1), 239–268.

Zhang, Jun and Junjie Zhou, "Information Disclosure in Contests: A Bayesian Persuasion Approach," *Economic Journal*, 2016, *126* (597), 2197–2217.

Zizzo, Daniel John, "Racing with uncertainty: A patent race experiment," *International Journal of Industrial Organization*, 2002, *20* (6), 877–902.